Predictions for multiple H production: comparing HEFT and SMEFT

TRIUMF, Vancouver, Canada
SMEFT meets χEFT
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JHEP 03 (2024) 037; PRD 106 (2022) 5, 5; Commun.Theor.Phys. 75 (2023) 9, 095202

A. Nasoni (Master Thesis 2025), S. Martellini (Master Thesis 2025) ,

A. Dobado, C. Quezada Calonge (PhD 2024)

A. Pich, I. Rosell, PRD 112 (2025) 3, 035009

Outline

1. EW Chiral Lagr for BSM: SM, SMEFT and HEFT

2. Applications: VBS

   2.a) $W_L W_L \rightarrow \ n \times h$ (**MAIN OBJECT OF THIS TALK**)

   2.b) $W_L W_L \rightarrow bosons$

   2.c) $W_L W_L \rightarrow fermions$

3. Other Chiral Lagr. applications including heavy BSM states:

   V & A mass bounds from S and T

1. INTRO:

Chiral Lagrangians for BSM

- **SM:**

USUAL INTRO

- Complex doublet H
- Renormalizable (canonical dim. $D \leq 4$)

$$\boxed{\mathcal{L}_{SM} = \mathcal{L}_{D\leq 4}}$$

- **SMEFT:**

- Complex doublet H
- Non-renormalizable (canonical dim. expan.)

$$\mathscr{L}_{SMEFT} = \mathscr{L}_{SM} + \sum_{n,i} \frac{c_i^{(n)}}{\Lambda^{n-4}} \mathcal{O}_i^{(n)}$$

- **HEFT**

(= EWChL = EWET)

- 3 EW Goldstones + 1 singlet Higgs h (indep.)
- Non-renormalizable (chiral expan.)

$$\boxed{\mathcal{L}_{HEFT} = \mathcal{L}_{p^2} + \mathcal{L}_{p^4} + \ \ldots}$$

[w/ $\mathcal{L}_{SM} \subset \mathcal{L}_{p^2}$]

GEOMETRY INTRO

Geometry of the scalar fields (and much more in B. Assi's talk)

SM,

SMEFT,

HEFT... and

geometry:

- R. Alonso, E. E. Jenkins, and A. V. Manohar,
  - "A Geometric Formulation of Higgs Effective Field Theory: Measuring the Curvature of Scalar Field Space," Phys. Lett. B754 (2016) 335–342, arXiv:1511.00724 [hep-ph].
  - "Sigma Models with Negative Curvature," Phys.Lett.B756,358(2016),arXiv:1602.00706 [hep-ph].
  - "Geometry of the Scalar Sector," JHEP 08 (2016) 101, arXiv:1605.03602 [hep-ph]." (Cohen et al., 2021, p. 95)
- T. Cohen, N. Craig, X. Lu, and D. Sutherland:
  - "Is SMEFT Enough?", JHEP 03, 237, arXiv:2008.08597 [hep-ph].
  - "Unitarity Violation and the Geometry of Higgs EFTs", (2021), arXiv:2108.03240 [hep- ph].

we now know that HEFT and SMEFT can be understood geometrically

Old works in 80's:

Boulware,Brown, Annals Phys. 138 (1982) 392

[thanks to M. Knecht for calling out my attention to this]

GEOMETRY INTRO

- Beautiful geometric connection to scalar loop corrections [(*)] provided by the curvature [(x)] of the scalar manifold metric $g_{ij}(\phi) = \begin{bmatrix} F(h)^2 g_{ab}(\varphi) & 0 \\ 0 & 1 \end{bmatrix}$, with $\mathcal{L} = \frac{1}{2} g_{ij} D_\mu \phi^i D^\mu \phi^j$

$$\mathcal{R}_4 = \left(1 - v^2 (F')^2\right) F^2 \quad = \quad \left(1 - \mathcal{K}^2/4\right) \mathcal{F}_C \,,$$

$$\mathcal{R}_2 = \left(1 - v^2 (F')^2\right) - \frac{v^2 F'' F}{(N_\varphi - 1)} \quad = \quad \left(1 - \mathcal{K}^2/4\right) - \frac{\mathcal{F}_C \Omega}{8} \,,$$

$$\mathcal{R}_0 = 2\mathcal{F}_C^{-1} \mathcal{R}_2 - \mathcal{F}_C^{-2} \mathcal{R}_4 \,,$$

$$F = \mathcal{F}_C^{1/2} \qquad N_\varphi = 3$$

with Λ^{-2} = the Riemann $\mathbb{R}_{ijmn} \propto \mathscr{R}_{0,2,4} / v^2$ *(loosely speaking, the curvature $\mathbb{R}$)*

- NDA gives you the suppression of individual diagrams $\sim 1/(4\pi v)^2$

  but the <u>full</u> loop suppression is $\sim \mathbb{R}^2 / (4\pi)^2$ and $\sim g^2 \mathbb{R} / (4\pi)^2$

  EFT as an expansion $\mathcal{M} \sim R\, \mathbf{p}^2 + \frac{R^2\, \mathbf{p}^4}{(4\pi)^2} + \frac{R^3\, \mathbf{p}^6}{(4\pi)^4} + \ldots$ in the curvature?

- **SM:** $\mathbb{R}_{ijmn} = 0$ ➔ No $O(p^4)$ renormalization

(*) Guo,Ruiz-Femenia,SC, PRD92 (2015) 074005
(x) Alonso,Jenkins,Manohar, PLB754 (2016) 335; PLB756 (2016) 358; JHEP 1608 (2016) 101

• Depending on the **choice of coordinates** cancellations/suppressions may not be obvious diagram by diagram

- **Example:** SO(5)/SO(4) MCHM tree LO computations for $\omega\omega \to \omega\omega$

** Agashe,Contino,Pomarol '05
** Contino et al. '12

HEFT MCHM

$$A(s,t,u)^{\omega\omega\to\omega\omega} = \frac{(1-a^2)\,s}{v^2} = \frac{s}{f^2}\,,$$

HEFT MCHM

$$A(s,t,u)^{\omega\omega\to hh} = \frac{(a^2-b)\,s}{v^2} = \frac{s}{f^2}$$

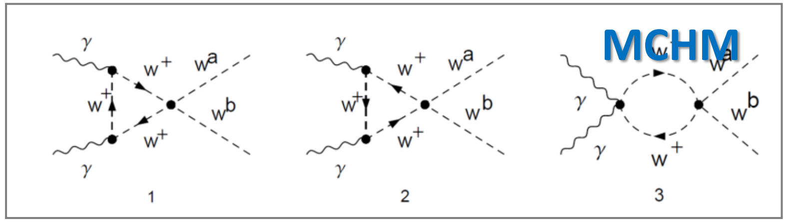
- **Example:** SO(5)/SO(4) MCHM 1-loop computations for $\gamma\gamma \to \omega\omega$

** Agashe,Contino,Pomarol '05
** Contino et al. '12

MCHM HEFT

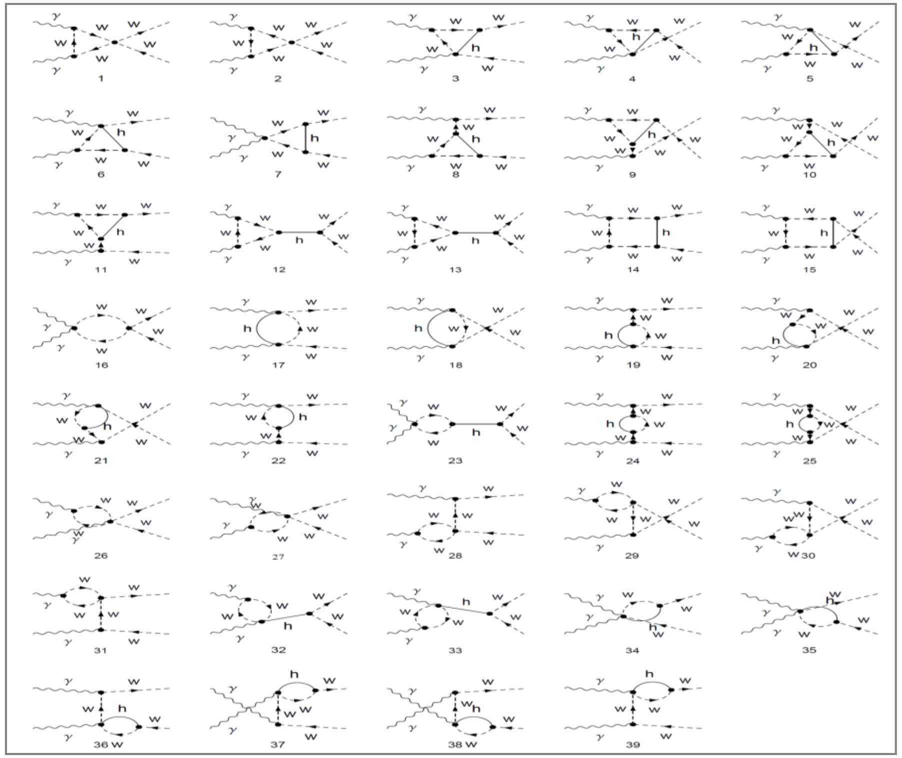
$$A(s,t,u)^{\gamma\gamma\to zz} = -\frac{1}{4\pi^2 f^2} = -\frac{(1-a^2)}{4\pi^2 v^2}$$

$$A(s,t,u)^{\gamma\gamma\to w^+w^-} = -\frac{1}{8\pi^2 f^2} = -\frac{(1-a^2)}{8\pi^2 v^2}$$

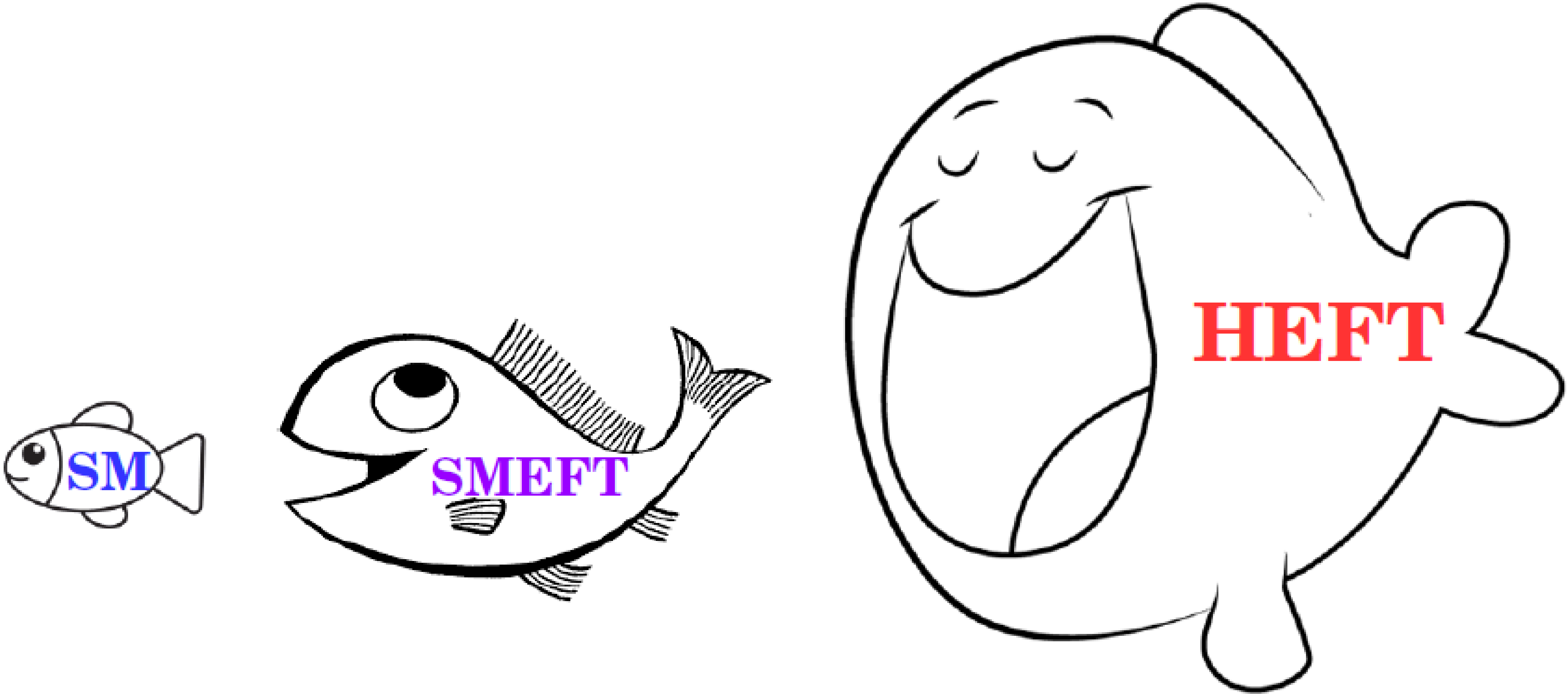


MCHM HEFT

** Delgado,Dobado,Herrero,SC, JHEP 07 (2014) 149



[by Felipe J. Llanes-Estrada]



(x) See, e.g., Alonso,Jenkins,Manohar, PLB 754 (2016) 335-342; PLB 756 (2016) 358-364; JHEP 08 (2016) 101; Cohen,Craig,Lu,Sutherland, JHEP 03 (2021) 237; JHEP 12 (2021) 003; Brivio,Corbett,Éboli,Gavela,González-Fraile,González-García,Merlo,Rigolin, JHEP 03 (2014) 024; Agrawal,Saha,Xu,Yu,Yuan, PRD 101 (2020) 7, 075023; Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; Commun.Theor.Phys. 75 (2023) 9, 095202; Dawson,Fontes,Quezada-Calonge,SC, 2311.16897 [hep-ph]; PRD 108 (2023) 5, 055034; Arco,Domenech,Herrero,Morales, PRD 108 (2023) 9, 095013;

• Chiral counting:

(See M.J. Herrero's talk for NLO corrections $O(p^4)$ or ask for backup)

$$\mathcal{L}_{HEFT} = \mathcal{L}_2 + \mathcal{L}_4 + \cdots$$

•LO Lagrangian, $O(p^2)$:

$$\begin{aligned}\mathcal{L}_2 = & -\frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_{\psi=q_L,l_L,u_R,d_R,e_R} \bar{\psi} i \not{D} \psi \\ & + \frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle\,(1+F_U(h)) + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) \\ & - v\left[\bar{q}_L\left(Y_u + \sum_{n=1}^{\infty} Y_u^{(n)}\left(\frac{h}{v}\right)^n\right) U P_+ q_R + \bar{q}_L\left(Y_d + \sum_{n=1}^{\infty} Y_d^{(n)}\left(\frac{h}{v}\right)^n\right) U P_- q_R \right. \\ & \left. + \bar{l}_L\left(Y_e + \sum_{n=1}^{\infty} Y_e^{(n)}\left(\frac{h}{v}\right)^n\right) U P_- l_R + \text{h.c.}\right] \end{aligned} \tag{II.2.164}$$

Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector - 1610.07922 [hep-ph]

F_π from χPT ➔ EW vev $v = \left(\sqrt{2}G_F\right)^{-1} = 246$ GeV

- Relevant LO Lagrangian for most of this talk:

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}(\partial_\mu h)^2 + \frac{v^2}{4}\mathcal{F}(h)\,\mathrm{Tr}\left\{\partial_\mu U^\dagger \partial^\mu U\right\}$$

w/ the singlet "Flare" function,

$$\mathcal{F}(h) = 1 + a_1\frac{h}{v} + a_2\left(\frac{h}{v}\right)^2 + a_3\left(\frac{h}{v}\right)^3 + a_4\left(\frac{h}{v}\right)^4 + \dots$$

Other usual notation in the bibliography: $\boldsymbol{\kappa_V} \equiv a \equiv a_1/2\,,\quad \boldsymbol{\kappa_{2V}} \equiv b \equiv a_2$

- Non-linear Goldstone realization: $U(\omega) = 1 + i\sigma^a\omega^a/v + \mathcal{O}(\omega^2)$

2. $W_L W_L$ VBS:

multi-H production & more

2. a.I) $W_L W_L \to n \times h$ VBS

This work's approx. in HEFT: $W_L W_L \to 2h, 3h, 4h \ldots$

- kinematics well over WW threshold: $s \gg m_W^2 \sim m_h^2$

- Mass corrections neglected

- Chiral LO: only $O(\partial^2)$ derivative operators

- Equivalence theorem appr.: $W_L W_L \to n \times h \;\approx\; \omega\omega \to n \times h$

[I know: 3h, 4h, etc. looks like science-fiction nowadays]

- Specific $\omega\omega \to n \times h$ stand-alone Mathematica code [link]
- FeynRules + FeynCalc chiral model file @ LO + NLO [link1] [link2]

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

- Relevant HEFT Lagrangian at LO, O(p²):

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}(\partial_\mu h)^2 + \frac{v^2}{4}\mathcal{F}(h)\,\text{Tr}\left\{\partial_\mu U^\dagger \partial^\mu U\right\}$$

w/ the SU(2)-singlet "Flare" function,

$$\mathcal{F}(h) = 1 + a_1\frac{h}{v} + a_2\left(\frac{h}{v}\right)^2 + a_3\left(\frac{h}{v}\right)^3 + a_4\left(\frac{h}{v}\right)^4 + \dots$$

NOTICE: $\boldsymbol{\kappa_V} \equiv a \equiv a_1/2\,,\quad \boldsymbol{\kappa_{2V}} \equiv b \equiv a_2$

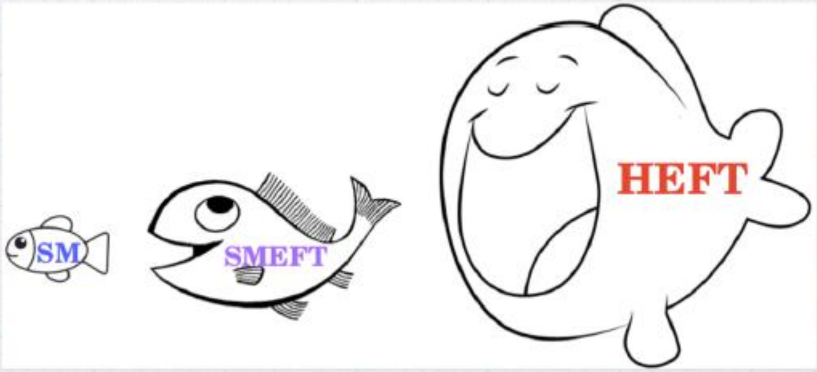
- Non-linear Goldstone realization: $U(\omega) \;=\; 1 + i\sigma^a\omega^a/v + \mathcal{O}(\omega^2)$

The Flare Function

* In HEFT: $\mathcal{F}(h)_{HEFT} = 1 + a_1 \frac{h}{v} + a_2 \left(\frac{h}{v}\right)^2 + a_3 \left(\frac{h}{v}\right)^3 + \ldots$
* In the SM: $\mathcal{F}(h)_{SM} = \left(1 + \frac{h}{v}\right)^2$
* In SMEFT?

* Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; Commun.Theor.Phys. 75 (2023) 9, 095202
* Domenech,Herrero,Morales,Salas-Bernárdez, 2506.21716 [hep-ph]

$$
\begin{aligned}
\mathcal{F}(h_1) &= \left(1 + \frac{h(h_1)}{v}\right)^2 \\
&= 1 + \left(\frac{h_1}{v}\right)\left(2 + 2\frac{c^{(6)}_{H\Box} v^2}{\Lambda^2} + 3\frac{(c^{(6)}_{H\Box})^2 v^4}{\Lambda^4} + 2\frac{c^{(8)}_{H\Box} v^4}{\Lambda^4}\right) + \left(\frac{h_1}{v}\right)^2\left(1 + 4\frac{c^{(6)}_{H\Box} v^2}{\Lambda^2} + 12\frac{(c^{(6)}_{H\Box})^2 v^4}{\Lambda^4} + 6\frac{c^{(8)}_{H\Box} v^4}{\Lambda^4}\right) \\
&+ \left(\frac{h_1}{v}\right)^3\left(8\frac{c^{(6)}_{H\Box} v^2}{3\Lambda^2} + 56\frac{(c^{(6)}_{H\Box})^2 v^4}{3\Lambda^4} + 8\frac{c^{(8)}_{H\Box} v^4}{\Lambda^4}\right) + \left(\frac{h_1}{v}\right)^4\left(2\frac{c^{(6)}_{H\Box} v^2}{3\Lambda^2} + 44\frac{(c^{(6)}_{H\Box})^2 v^4}{3\Lambda^4} + 6\frac{c^{(8)}_{H\Box} v^4}{\Lambda^4}\right) \\
&+ \left(\frac{h_1}{v}\right)^5\left(88\frac{(c^{(6)}_{H\Box})^2 v^4}{15\Lambda^4} + 12\frac{c^{(8)}_{H\Box} v^4}{5\Lambda^4}\right) + \left(\frac{h_1}{v}\right)^6\left(44\frac{(c^{(6)}_{H\Box})^2 v^4}{45\Lambda^4} + 2\frac{c^{(8)}_{H\Box} v^4}{5\Lambda^4}\right) + \mathcal{O}(\Lambda^{-6}).
\end{aligned}
$$



SMEFT: $\omega\omega \to 2h, 3h, 4h \ldots$
multi-H VERTEX suppression

- SMEFT⇔HEFT relations for the Higgs couplings:

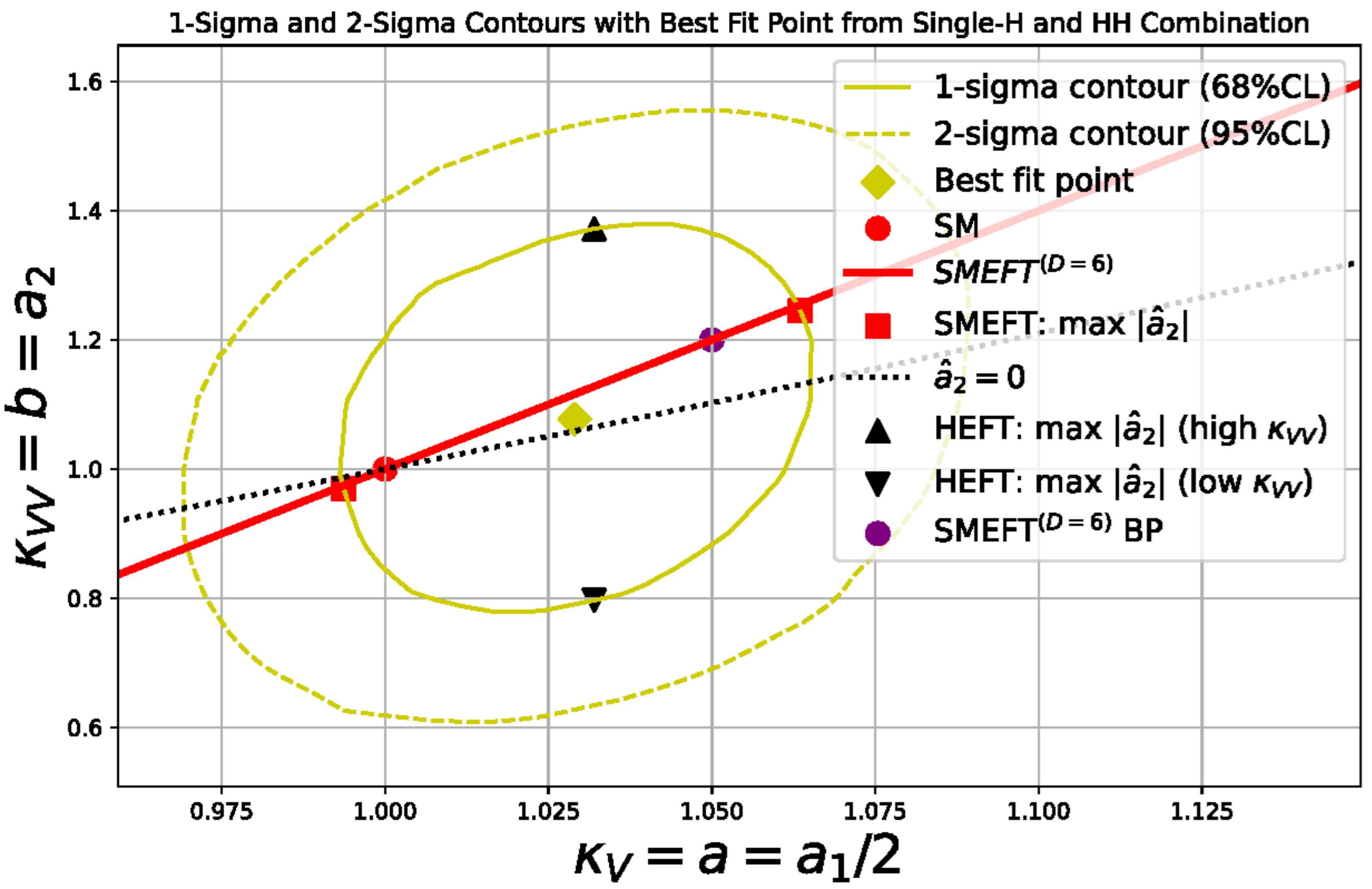
$$
\begin{aligned}
a_1/2 &= a = 1 + \frac{d}{2} + \frac{d^2}{2}\left(\frac{3}{4}+\rho\right) + \mathcal{O}\left(d^3\right) \\
a_2 &= b = 1 + 2d + 3d^2\left(1+\rho\right) + \mathcal{O}\left(d^3\right) \\
a_3 &= \frac{4}{3}d + d^2\left(\frac{14}{3}+4\rho\right) + \mathcal{O}\left(d^3\right) \\
a_4 &= \frac{1}{3}d + d^2\left(\frac{11}{3}+3\rho\right) + \mathcal{O}\left(d^3\right)
\end{aligned}
$$

[4] a_5 and a_6 can be found in the paper. a_n for $n \geq 7$ vanishes at order $1/\Lambda^4$.

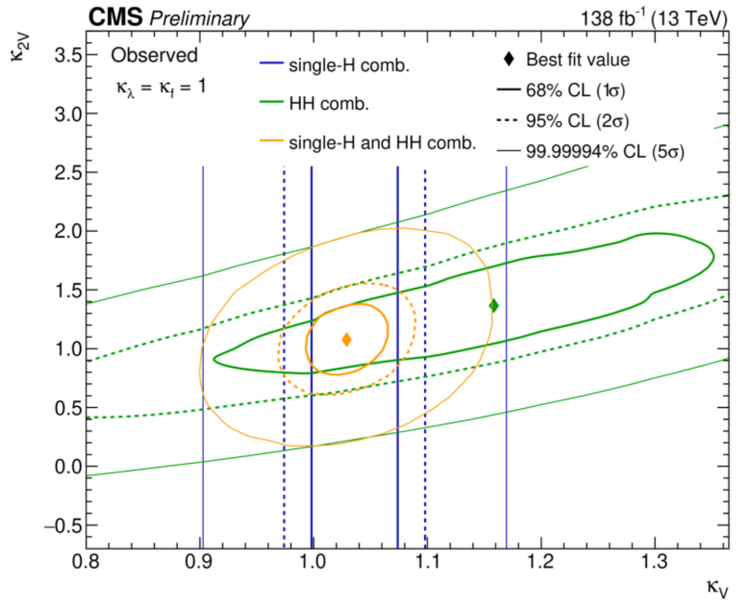
$$
d = \frac{2v^2 c_{H\Box}^{(6)}}{\Lambda^2} \quad , \qquad \rho = \frac{c_{H\Box}^{(8)}}{2(c_{H\Box}^{(6)})^2}
$$

* Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC,
PRD 106 (2022) 5, 5;
Commun.Theor.Phys. 75 (2023) 9, 095202
* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

(x) Custodial breaking still keeps this structure:
See Domenech,Herrero,Morales,Salas-Bernárdez, 2506.21716 [hep-ph]

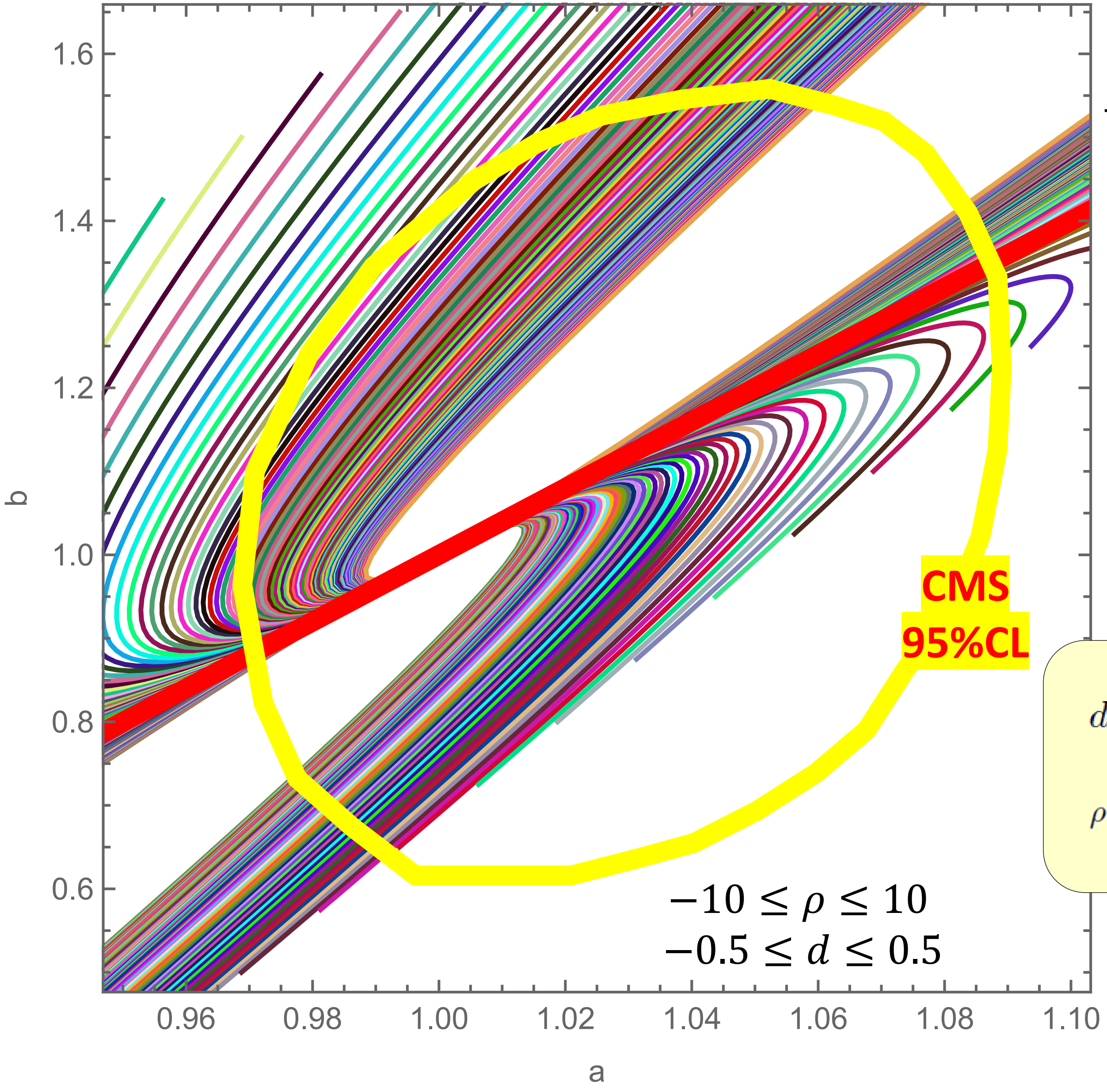


Data from CMS-PAS-HIG-23-006:

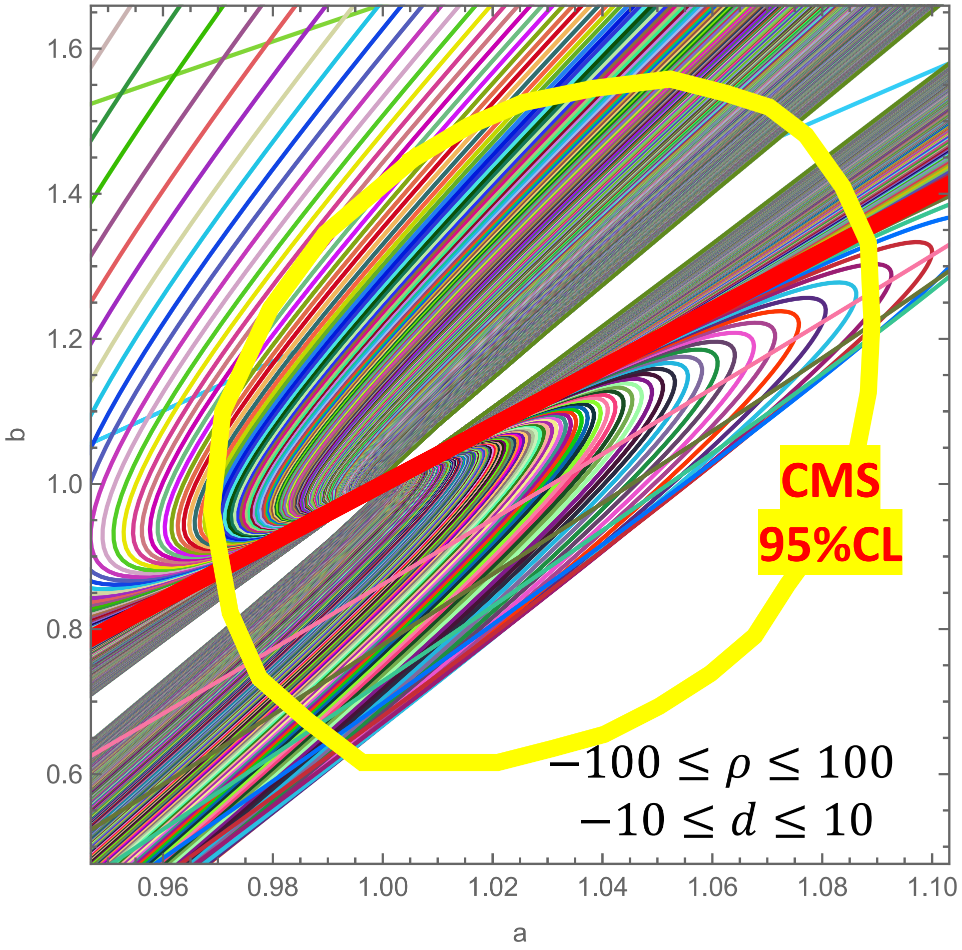


What about D=8 corrections to SMEFT? ➔

- The line turns into a “thick line”/area
- But still can’t go everywhere
- Actually lines (2D surfaces) within $\mathbb{R}^4$ for D=6 ($\mathbb{R}^6$ for D=8)
- Volunteers can play with D=10 corrections



$$d = \frac{2v^2 c_{H\Box}^{(6)}}{\Lambda^2}$$
$$\rho = \frac{c_{H\Box}^{(8)}}{2\left(c_{H\Box}^{(6)}\right)^2}$$



$$\boxed{\omega\omega \to 2h}$$

$$T_{\omega\omega\to 2h} = -\frac{\hat{a}_2 s}{v^2}$$

$$\sigma_{\omega\omega\to 2h} = \frac{8\pi^3\, \hat{a}_2^2}{s}\left(\frac{s}{16\pi^2 v^2}\right)^2$$

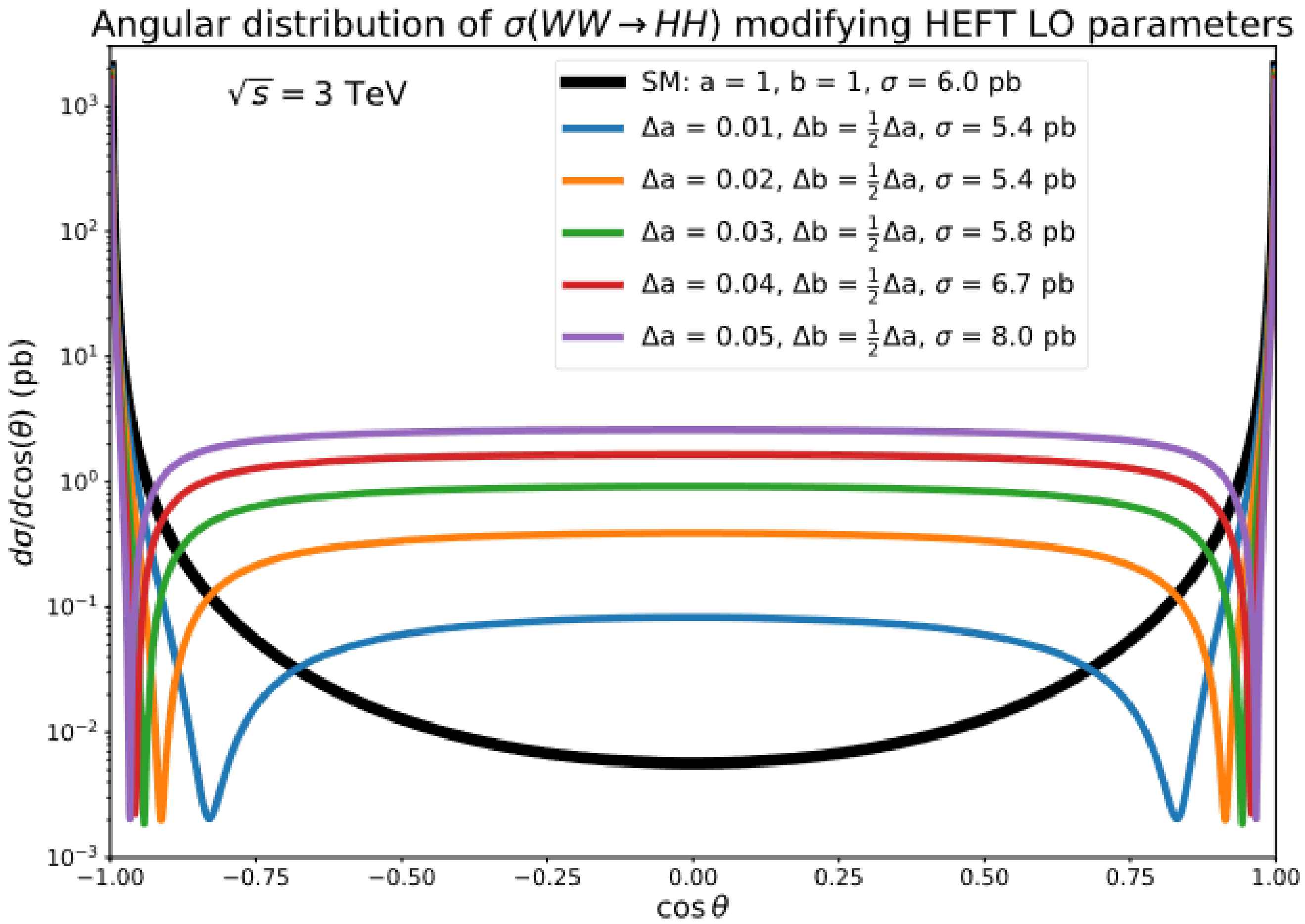
- Relevant combination: $\boxed{\hat{a}_2 = a_2 - a_1^2/4 = b - a^2}$

- Pure s-wave (J=0) ➔ critical angular information

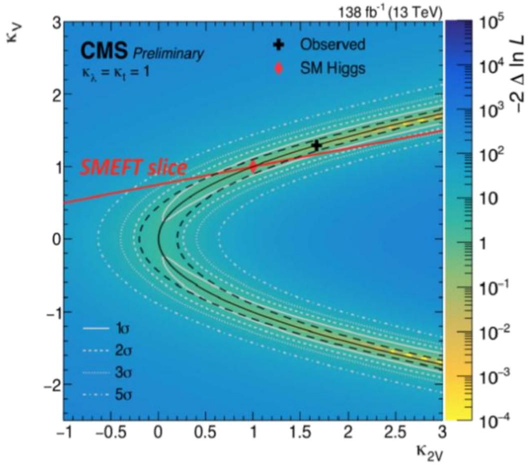
* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

(x) Gonzalez-Lopez,Herrero,Martinez-Suarez, EPJ C 81 (2021) 3, 260

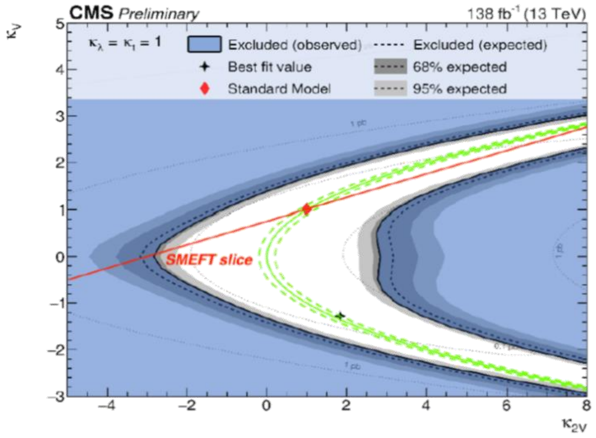
- **IR finite**
- Equiv. Theorem implies a **pure s-wave**
- This HEFT behaviour approximately observed with **real W's** (x) [vs **SM** angular distribution]



(x) Dávila,Domenech,Herrero,Morales, EPJC 84 (2024) 5, 503.
(x) See also the recent LHC análisis on $\hat{a}_2 = (b - a^2)$: Domenech,García-Mir,Herrero, 2507.20988 [hep-ph]



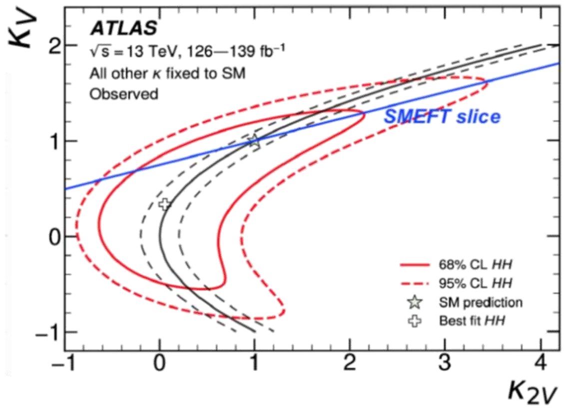
(a)



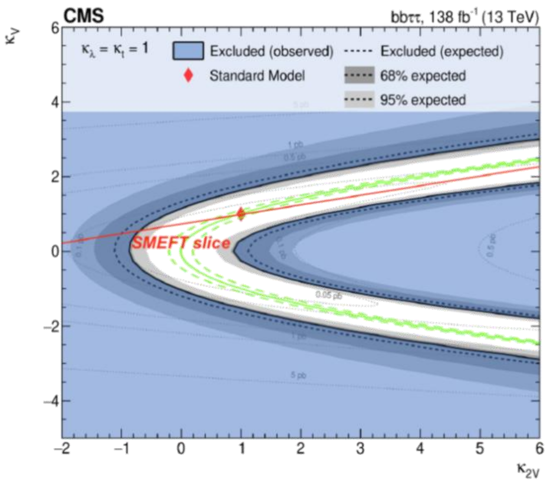
(b)

- Exp. data on hh-production at LHC show an important correlation between (a, b)

[notation: $a = a_1/2 = \kappa_V$, $b = a_2 = \kappa_{2V}$]



(c)



(d)

- NOTE we have superimposed:

  - Parabolles w/ constant $\hat{a}_2 = a_2 - \frac{a_1^2}{4}$

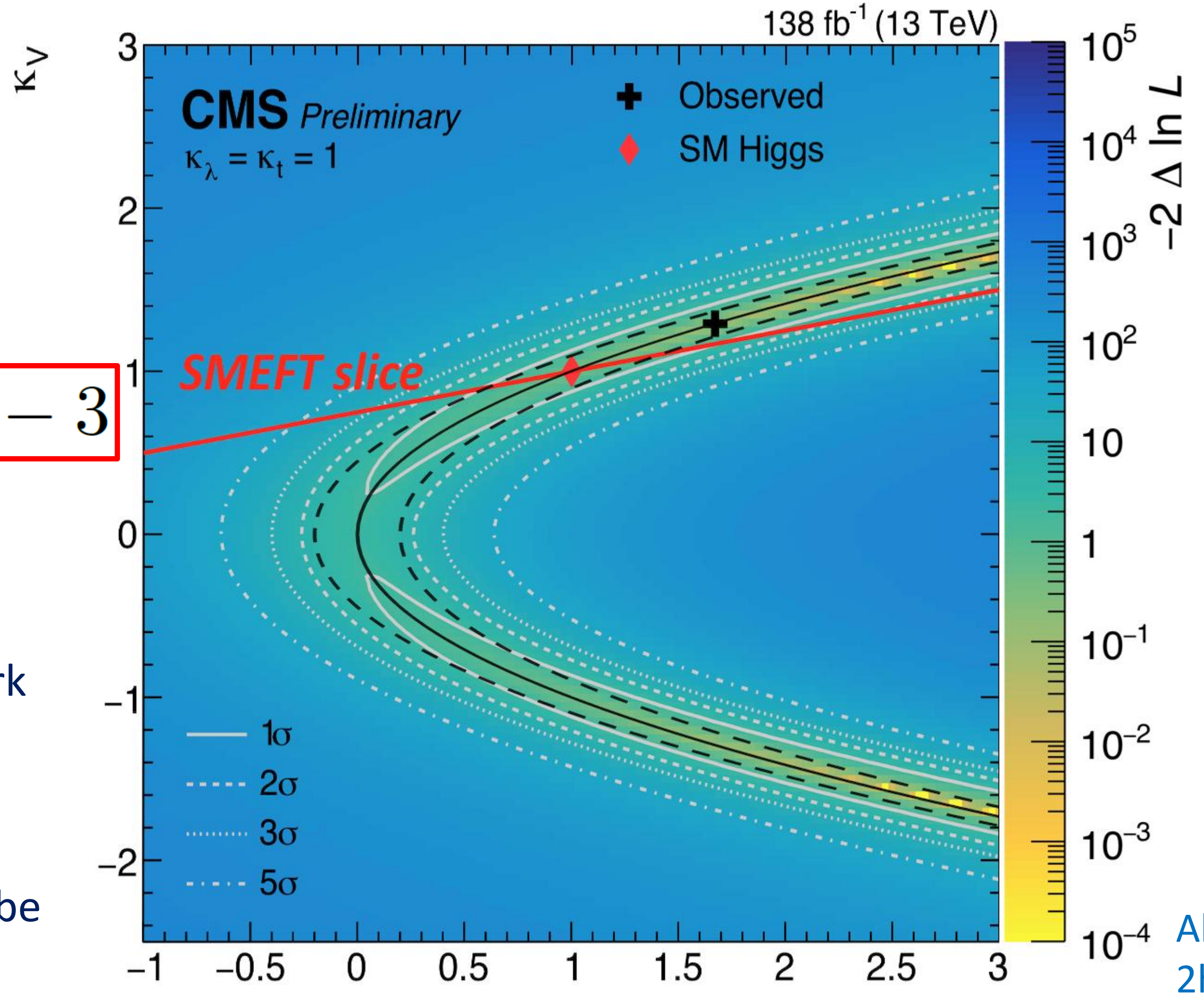
  - D=6 SMEFT prediction $a_2 = 2\,a_1 - 3$

... + some recente important improvement from HH + H: **CMS PAS HIG-23-006**

(a) Phys. Rev. Lett. 131 (2023) 041803 [2205.06667] **(b)** CMS-PAS-HIG-21-005 **(c)** ATLAS-CONF-2022-050 **(d)** Phys. Lett. B 842 (2023) 137531 [2206.09401]. NOTE: $\kappa_V = a_1/2$, $\kappa_{2V} = a_2$.

(a)

$$a_2 = 2a_1 - 3$$



$$\boxed{\hat{a}_2 = \pm 0.2}$$

The equivalence theorem approximations in this work seem to be in agreement with hh-production data

Indications that we might be O(10%) close to the SM in (a, b)

Figure 2. a) CMS experimental confidence regions for the hWW coupling $\kappa_V = a = a_1/2$ and for the $hhWW$ coupling $\kappa_{2V} = b = a_2$, from a non-resonant hh production search with each Higgs boson decaying into a highly boosted $b\bar{b}$ pair [77] (white lines and colour map in figure 11 from the additional material in CMS-B2G-22-003). b) CMS confidence regions from processes with one Higgs boson

Also previous theoretical 2h-production for LHC* noted important correlations between (κ_V, κ_{2V})

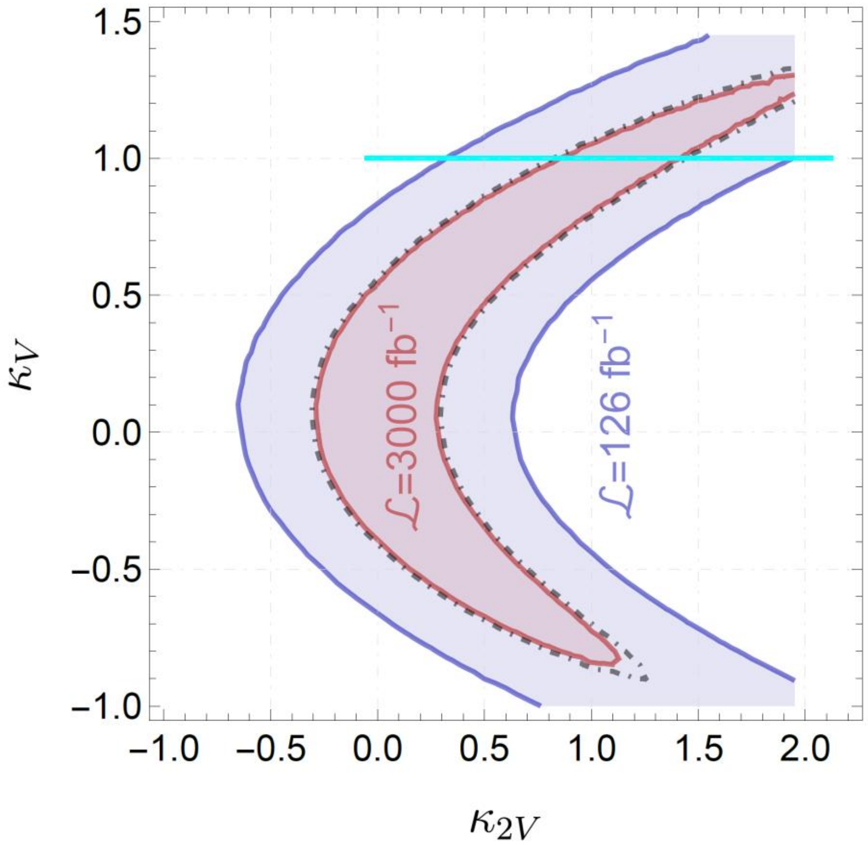
["banana" plots, as M.J. Herrero calls them]

(a) Phys. Rev. Lett. 131 (2023) 041803 [2205.06667]

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

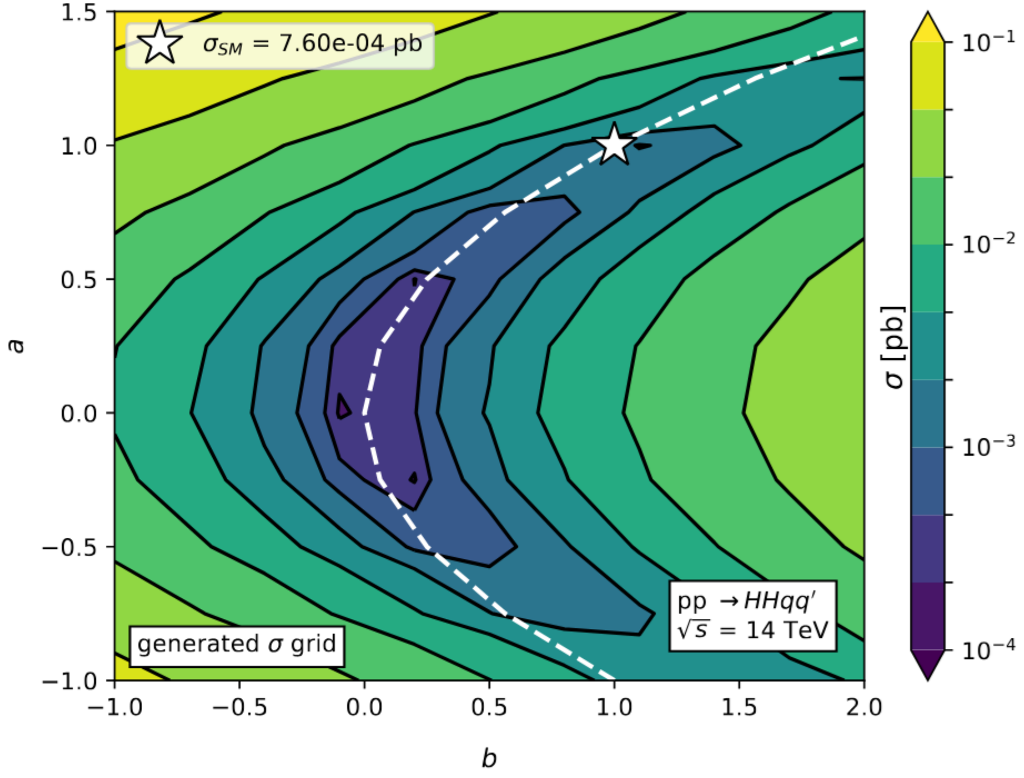
* Anisha, Atkinson, Bhardwaj, Englert, Stylianou, JHEP 10 (2022) 172

* Anisha, Atkinson, Bhardwaj, Englert, Stylianou, JHEP 10 (2022) 172



- Also previous and posterious theoretical hh-production simulations for LHC* noted an important correlation between (a, b)

** Domenech,García-Mir,Herrero, 2507.20988 [hep-ph]



$$\boxed{\omega\omega \to 3h}$$

V. Brigljevic et al.,
"HHH Whitepaper"
2407.03015 [hep-ph]

$$T_{\omega\omega\to 3h} = -\frac{3\hat{a}_3 s}{v^3}$$

$$\sigma_{\omega\omega\to 3h} = \frac{12\pi^3\,\hat{a}_3^2}{s}\left(\frac{s}{16\pi^2 v^2}\right)^3$$

- Relevant combination: $\boxed{\hat{a}_3 = a_3 - \frac{2}{3}a_1\left(a_2 - a_1^2/4\right) = a_3 - \frac{4}{3}a\left(b - a^2\right)}$

- Pure s-wave (J=0) ➔ critical angular information

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037 ** For a thorough LHC analysis: Anisha,Domenech,Englert,Herrero,Morales, PRD 111 (2025) 5, 055004

$\omega\omega \to 4h$

1-crossed-propagator dimensionless angular function

[BACKUP SLIDES]

$$T_{\omega\omega\to 4h} = -\frac{4s}{v^4}\left(3\hat{a}_4 + \hat{a}_2^2\,(B-1)\right)$$

$$\sigma_{\omega\omega\to 4h} = \frac{8\pi^3}{9s}\left(\frac{s}{16\pi^2 v^2}\right)^4\left[\left(3\hat{a}_4 - \hat{a}_2^2\right)^2 + 2\left(3\hat{a}_4 - \hat{a}_2^2\right)\hat{a}_2^2\chi_1 + \hat{a}_2^4\chi_2\right]$$

numerical integration constants $\chi_{1,2}$: MaMuPaXS [link]

- Relevant combination:

$$\boxed{\hat{a}_4 = a_4 - \frac{3}{4}a_1 a_3 + \frac{5}{12}a_1^2\left(a_2 - a_1^2/4\right) = a_4 - \frac{3}{2}a\,a_3 + \frac{5}{3}a^2\left(b - a^2\right)}$$

$$\boxed{\hat{a}_2 = a_2 - a_1^2/4 = b - a^2}$$

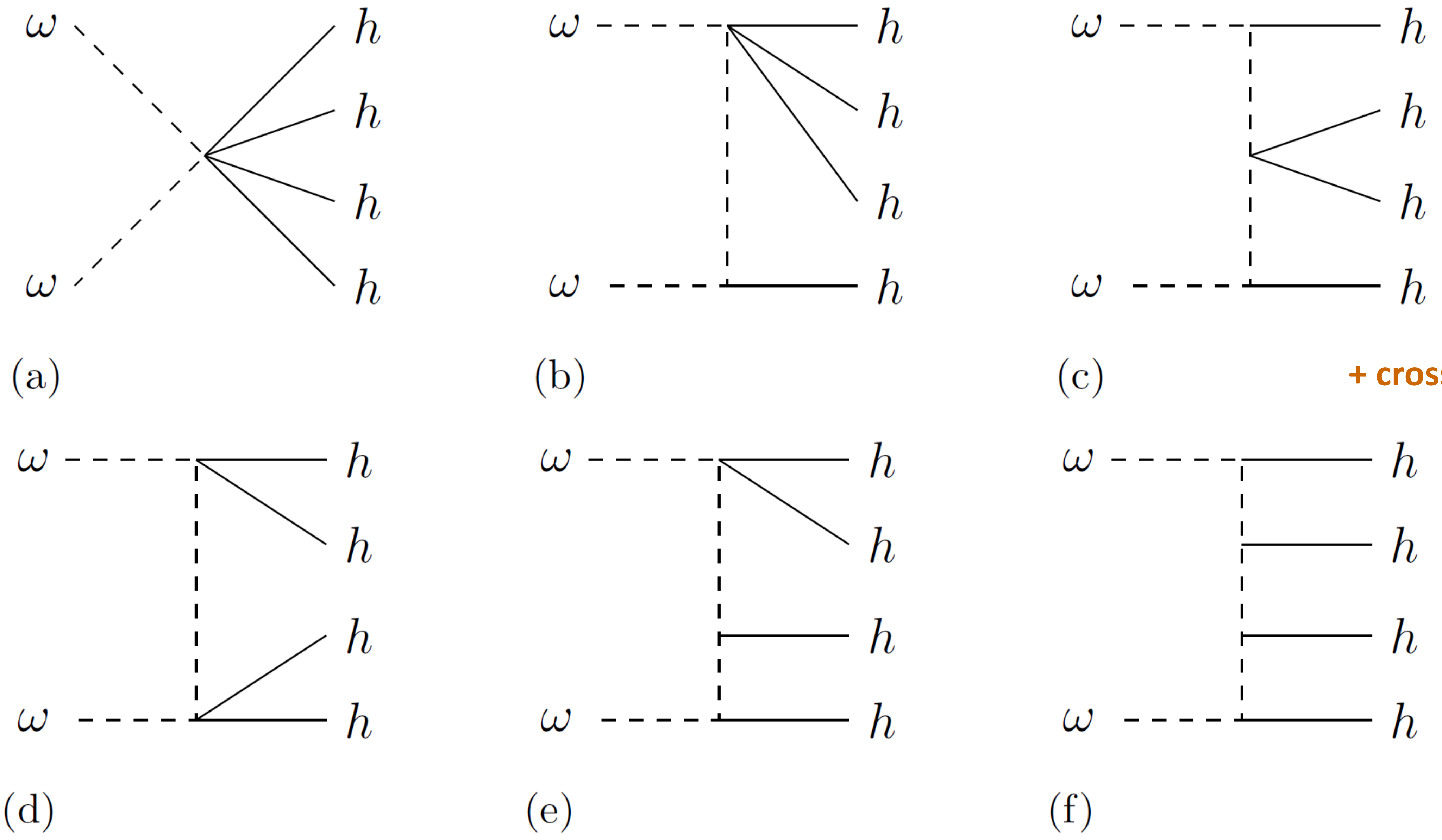
[exactly same combination as in $\omega\omega \to 2h$]

- Almost s-wave (J=0) [$\chi_1 = -0.12,\ \chi_2 = 0.019$] ➔ critical angular information

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

[with permutations: 75 diagrams]

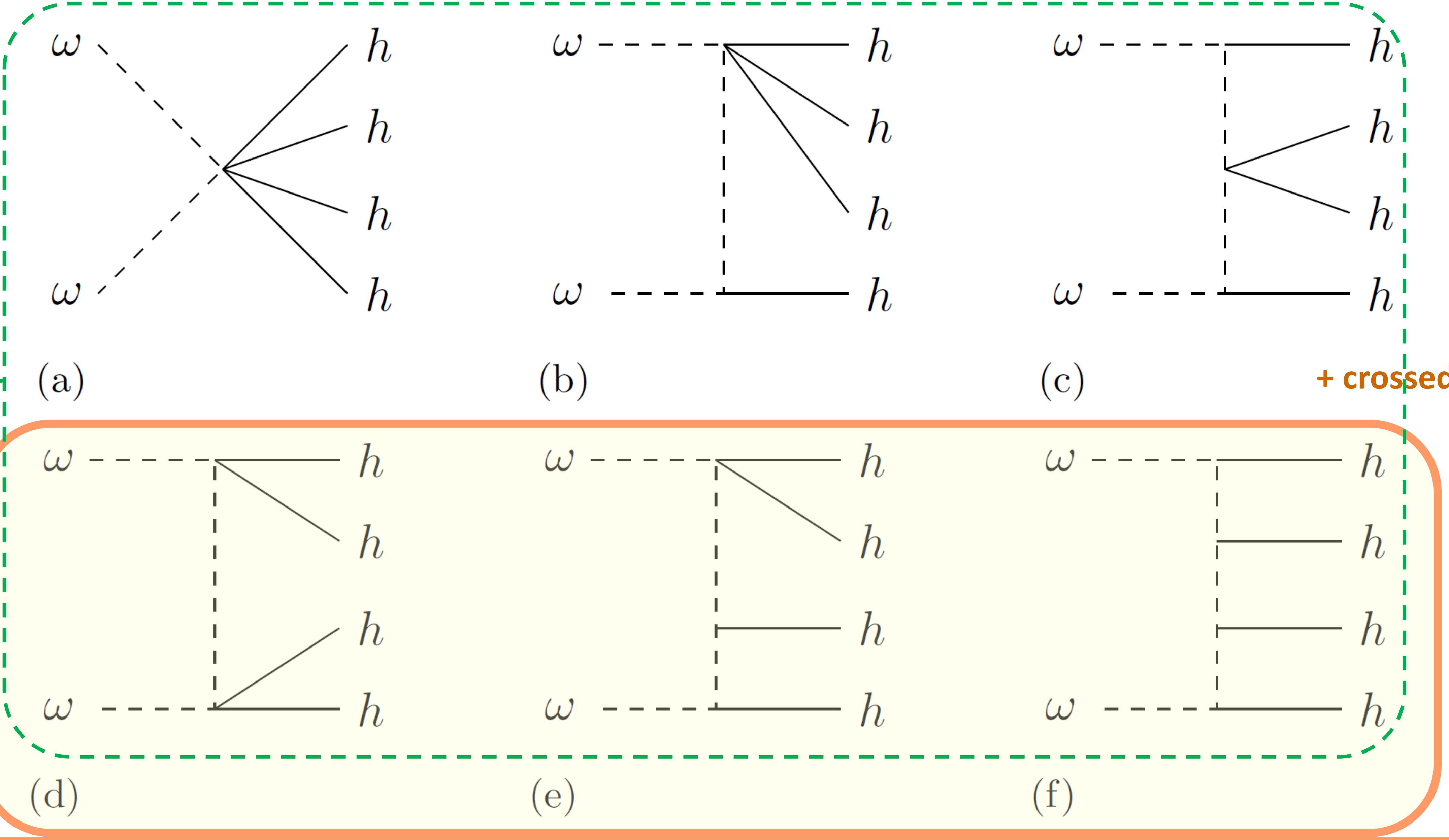
$$T_{\omega\omega\to 4h} = -\frac{4s}{v^4}\left(3\hat{a}_4 + \hat{a}_2^2(B-1)\right)$$



+ crossed

[with permutations: 75 diagrams]

$$T_{\omega\omega\to 4h} = -\frac{4s}{v^4}\left(3\hat{a}_4 + \hat{a}_2^2(B-1)\right)$$



2. a.II) $W_L W_L \to n\, h$ VBS

understanding simple structures

Understanding the $T_{\omega\omega\to n\times h}$ structure: field redefinitions

HEFT Lagrangian[1]

[Appelquist et al. - Phys. Rev. D 22 (1980) 200 , Longhitano et al. - Phys. Rev. D 22 (1980) 1166]

$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\mathcal{F}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a \;+ \mathcal{O}(\omega^4)$$

Flare function[2]

[Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez and Sanz-Cillero - 2204.01762]

$$\mathcal{F}(h) = 1 + a_1 \frac{h}{v} + a_2\left(\frac{h}{v}\right)^2 + a_3\left(\frac{h}{v}\right)^3 + a_4\left(\frac{h}{v}\right)^4 + \mathcal{O}(h^5)$$

$$a \equiv \frac{a_1}{2}\,,\; a_2 \equiv b \quad \text{with} \quad a_{1,\mathrm{SM}} = 2\,,\; a_{2,\mathrm{SM}} = 1\,,\; a_{3,\mathrm{SM}} = 0\,,\; a_{4,\mathrm{SM}} = 0$$

- Consider field redefinitions of the form (other coordinates for the fields)

$$\omega^a \to \tilde{\omega}^a(\omega, h) \equiv \omega^a + G_1(h)\,\omega^a$$

$$h \to \tilde{h}(\omega, h) \equiv h + f_1(h)\frac{\omega^2}{v}$$

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

** Alessia Nasoni, Master Thesis 2025; Martellini,Nasoni,SC, forthcoming

HEFT lagrangian

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\mathcal{F}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a \; + \mathcal{O}(\omega^4)$$

Field redefinition

$$\omega^a \to \omega^a + g(h)\,\omega^a\,, \qquad h \to h + \mathcal{N}\,(1+g(h))\,\frac{\omega^a\omega^a}{v}$$

Redefined HEFT Lagrangian for $\boxed{g'(h) = -2\mathcal{N}/[v\,\mathcal{F}(h)]}$

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\hat{\mathcal{F}}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a + \mathcal{O}(\omega^4)$$

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

** Alessia Nasoni, Master Thesis 2025; Martellini,Nasoni,SC, forthcoming

- In order to avoid generating interaction terms proportional to $(\partial h)^2$ or $(\omega\partial\omega)\partial h$ in the LO Lagr. $\mathcal{L}_2$, one needs:

$$f_1(h) = -\tfrac{v}{2}\mathcal{F}(h)G_1'(h)\,(1+G_1(h))$$
$$\mathcal{F}'(h)G_1'(h) = -\mathcal{F}(h)G_1(h);$$

solving for $G_1'(h)$, we find

$$f_1(h) = -\tfrac{v}{2}\tilde{\mathcal{N}}\,(1+G_1(h))$$
$$G_1'(h) = \tilde{\mathcal{N}}\mathcal{F}^{-1}(h);$$

Redefined HEFT lagrangian

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\hat{\mathcal{F}}(h)\,\partial_\mu \omega^a \partial^\mu \omega^a + \mathcal{O}(\omega^4)$$

Redefined flare function

$$\hat{\mathcal{F}}(h) = \mathcal{F}(h)\left(1 + g(h)\right)^2$$

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

- For a general normalization $\mathcal{N}$ and solution of $g'(h) = -2\mathcal{N}/[v\,\mathcal{F}(h)]$:

$$g(h) = -\frac{2\mathcal{N}}{v}\int_0^h \frac{ds}{\mathcal{F}(s)} = \mathcal{N}\left(-2\frac{h}{v}+2a\frac{h^2}{v^2}+\frac{2}{3}(b-4a^2)\frac{h^3}{v^3}+\frac{1}{2}(a_3-4ab+8a^3)\frac{h^4}{v^4}+\mathcal{O}(h^5)\right)$$

$$\hat{\mathcal{F}}(h) = \mathcal{F}(h)\Big(1+g(h)\Big)^2$$

- However, for the particular normalization $\mathcal{N} = \frac{a_1}{4} = \frac{a}{2}$:

$$g(h) = -a\frac{h}{v} + a^2\frac{h^2}{v^2} + \frac{1}{3}a(b-4a^2)\frac{h^3}{v^3} + \frac{1}{4}a(a_3-4ab+8a^3)\frac{h^4}{v^4} + \mathcal{O}(h^5)$$

$$\boxed{\hat{\mathcal{F}}(h) = 1 + \hat{a}_2\left(\frac{h}{v}\right)^2 + \hat{a}_3\left(\frac{h}{v}\right)^3 + \hat{a}_4\left(\frac{h}{v}\right)^4 + \mathcal{O}(h^5)}$$

Redefined parameters ($\hat{a}_1 = 0$)

$$\hat{a}_2 = b - a^2$$

$$\hat{a}_3 = a_3 - \frac{4a}{3}\left(b - a^2\right)$$

$$\hat{a}_4 = a_4 - \frac{3}{2} a\, a_3 + \frac{5}{3} a^2 \left(b - a^2\right)$$

Exactly the same effective combinations found

before in the $\omega\omega \rightarrow\ n \times h$ scattering amplitudes !!!

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

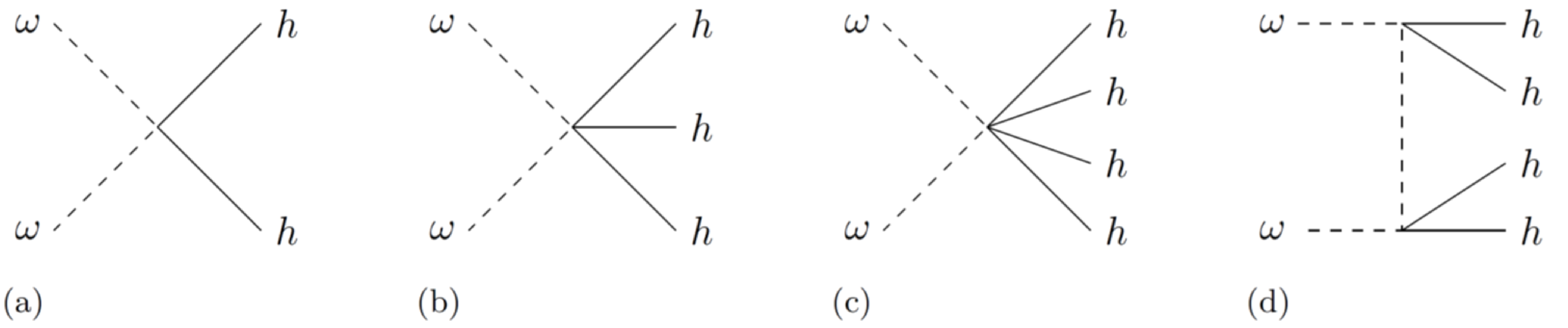


Figure 10. **a)** Only diagram contributing to the process $\omega\omega \to 2h$. **b)** Only diagram contributing to the process $\omega\omega \to 3h$. **c-d)** Only two diagrams contributing to the process $\omega\omega \to 4h$. We have used the simplified Lagrangian (C.6) to generate these amplitudes, so every $\omega\omega h^n$ vertex carries an $\hat{a}_n$ effective coupling. Note that, in addition, one needs to consider all possible permutations for the assignment of the external particles.

* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

2. a.III) $W_L W_L \to n\, h$ VBS

some basic pheno

SMEFT theory: $\omega\omega \to 2h, 3h, 4h \ldots$ suppression

- SMEFT⇔HEFT relations for the <u>relevant combinations</u>:

$$\boxed{\begin{aligned} \hat{a}_2 &= d + 2d^2(1+\rho) + \mathcal{O}(d^3) \\ \hat{a}_3 &= \frac{4}{3}d^2(1+\rho) + \mathcal{O}(d^3) \\ \hat{a}_4 &= \frac{1}{3}d^2(1+\rho) + \mathcal{O}(d^3) \end{aligned}}$$

$$d = \frac{2v^2 c_{H\Box}^{(6)}}{\Lambda^2} \quad , \quad \rho = \frac{c_{H\Box}^{(8)}}{2(c_{H\Box}^{(6)})^2}$$

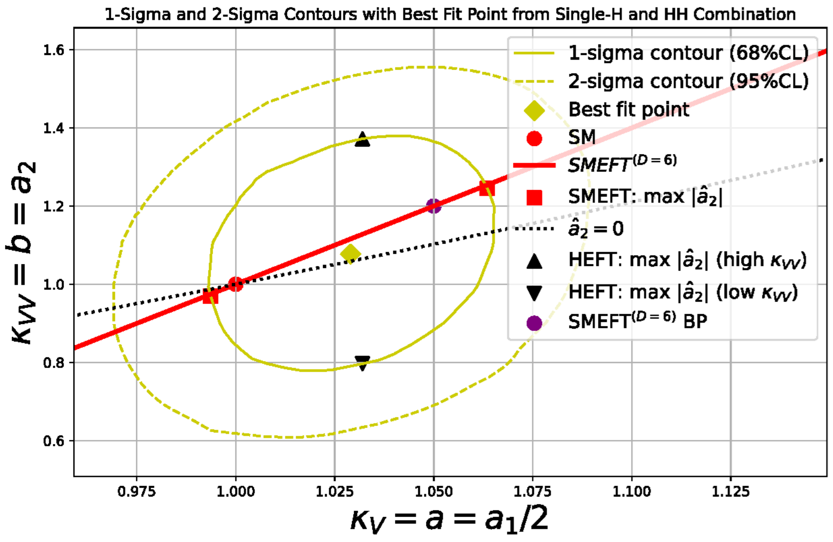
- Multi-Higgs fine-tuned suppression in SMEFT

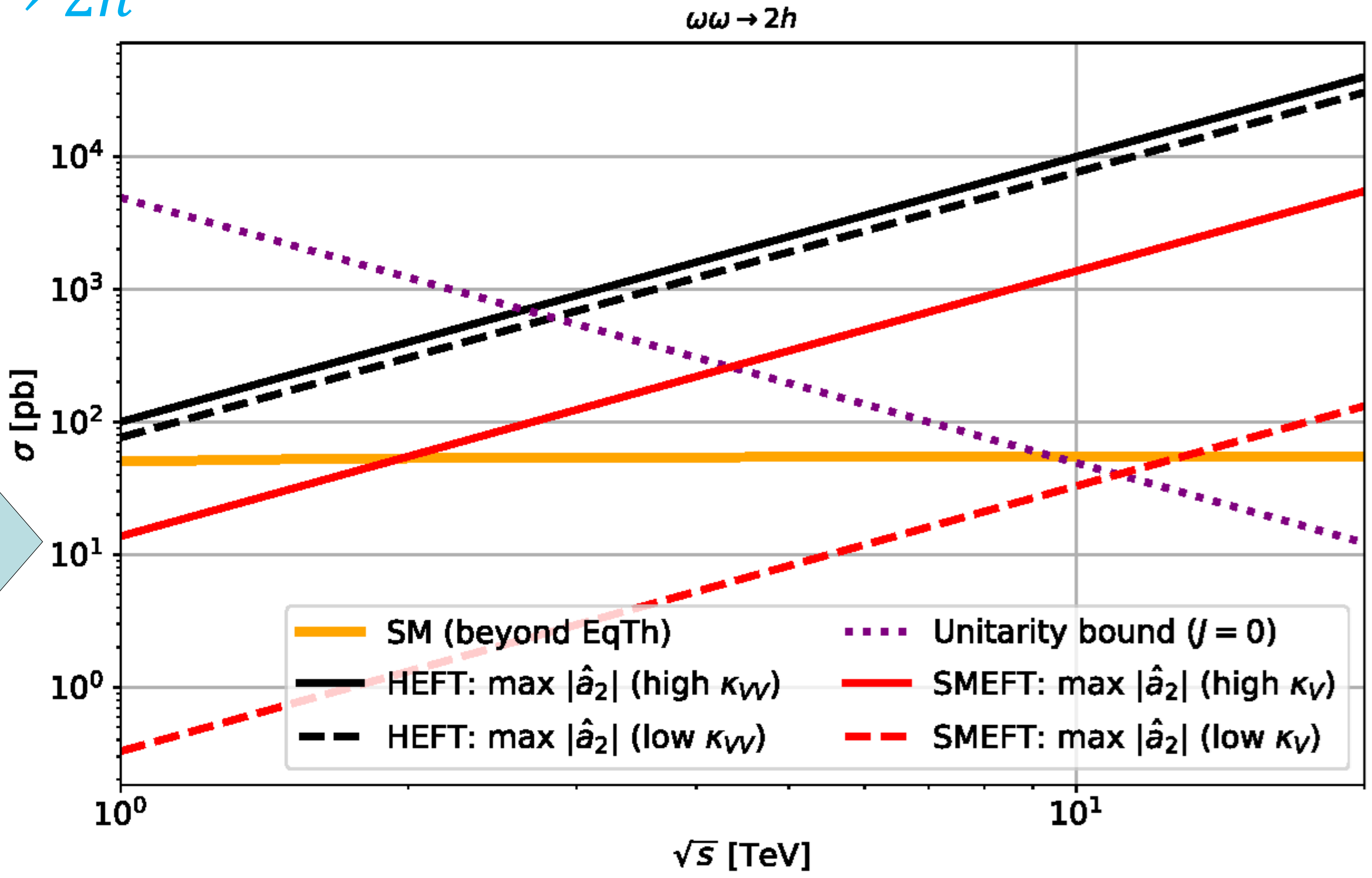
* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

SMEFT pheno: $\omega\omega \to 2h, 3h, 4h \ldots$ suppression

- **An illustrative example:** $\omega\omega \to 2h$

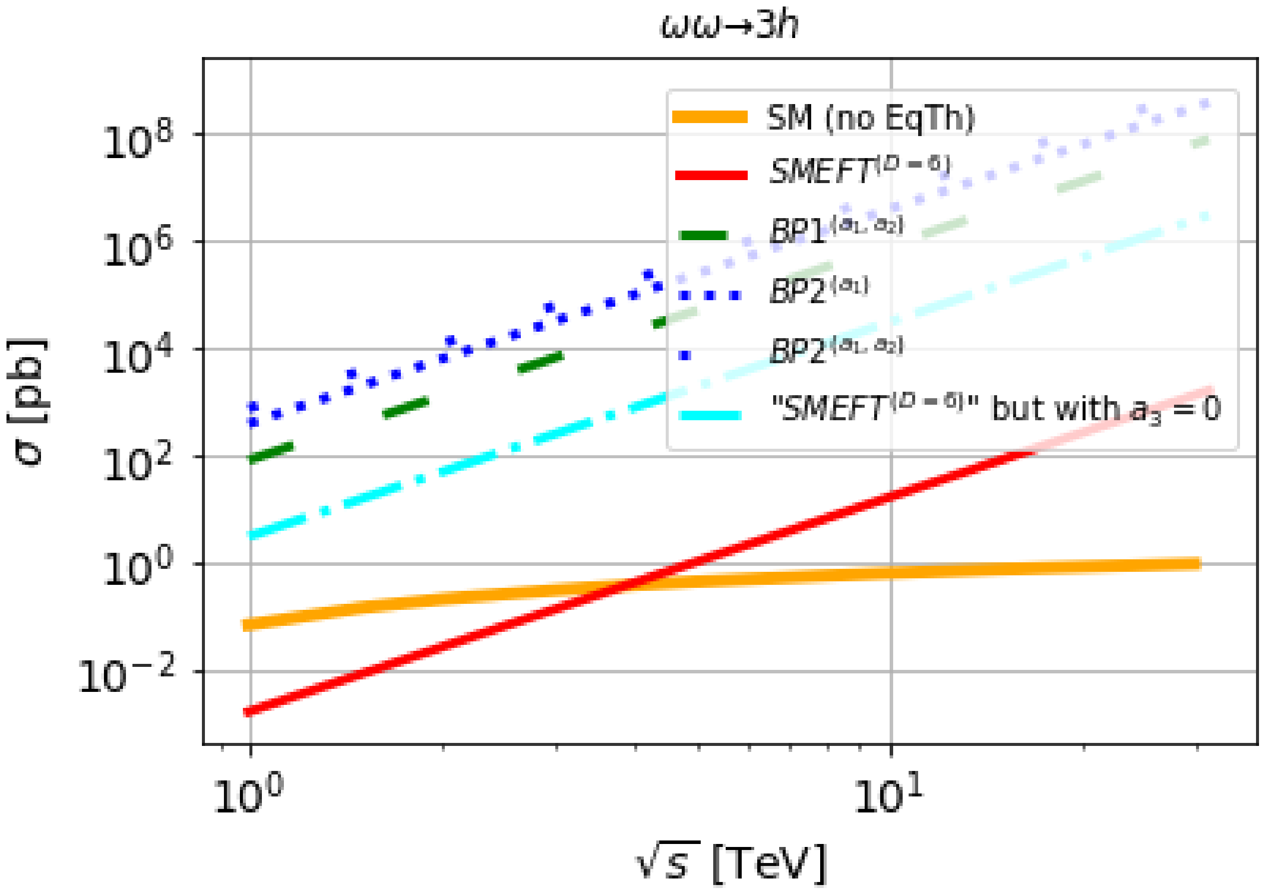
Data from CMS-PAS-HIG-23-006:





SMEFT pheno: $\omega\omega \to 2h, 3h, 4h \ldots$ suppression

- **Another illustrative example:** $\omega\omega \to 3h$



* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

Scanning of the $\omega\omega \to 3h$ cross section predictions for $\sqrt{s} = 1$ TeV:

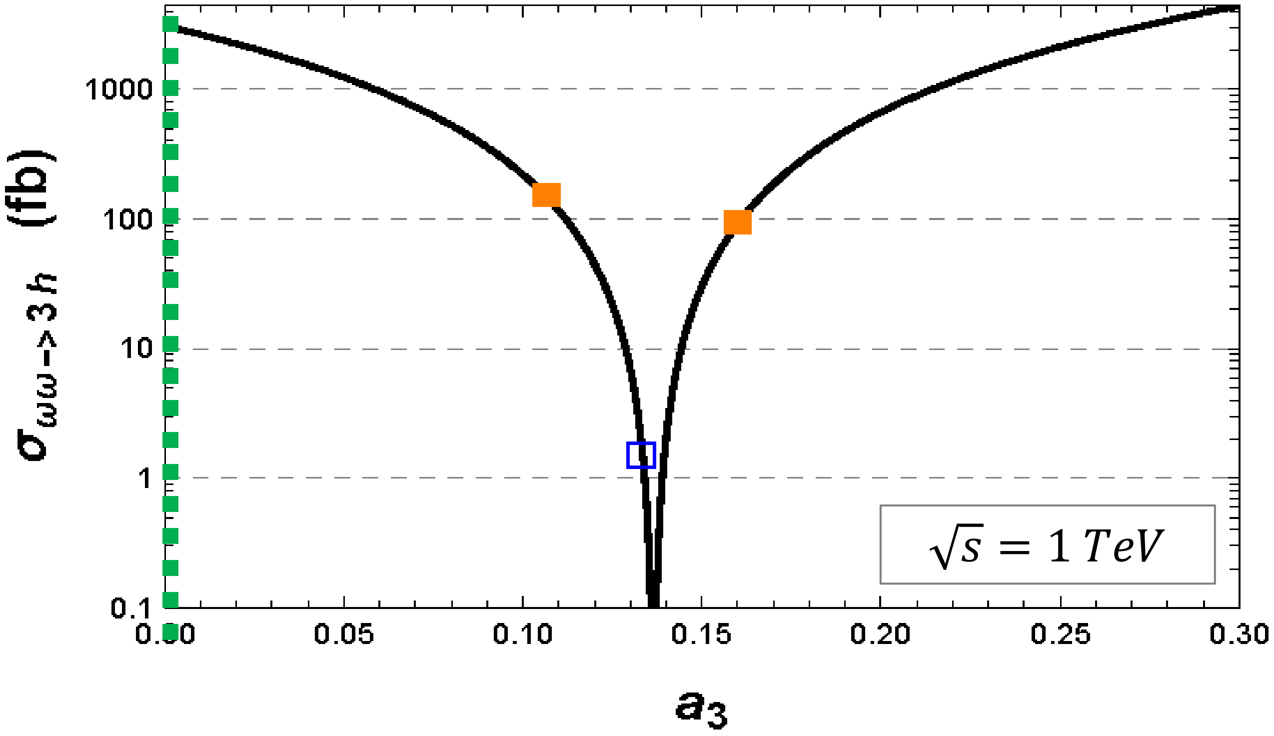
For SMEFT starting @ D=6:
$$\Delta b = 4\Delta a \quad *$$

- Empty Blue square - SMEFT$^{(D=6)}$ -BP $(c_{(6)} = 1, \;\; v^2/\Lambda^2 \; = 0.05)$:

$$a = a_1/2 = a^{SMEFT(D=6)} = 1.05, \qquad b = a_2 = a_2^{SMEFT\,(D=6)} = 1.2, \qquad a_3 = a_3^{SMEFT\,(D=6)} = 0.1\hat{3}$$

- Full Orange square - HEFT:

$$a = a_1/2 = a^{SMEFT(D=6)} = 1.05, \qquad b = a_2 = a_2^{SMEFT\,(D=6)} = 1.2, \qquad a_3 = a_3^{SMEFT\,(D=6)} \times (1 \pm 20\%)$$



$$\sigma_{\omega\omega\to 3h}^{SMEFT} \sim \frac{1}{\Lambda^8}$$

- Analogous to previous $a_2 = b = \kappa_{2V}$ scannings for LHC and FCC analyses:

(x) Englert,Naskar,Sutherland, JHEP 11 (2023) 158

* G. Buchalla,Cata,Celis,Krause, NPB 917 (2017) 209-233 (e.g., for Real Singlet Ext),
* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

Scanning of the $\omega\omega \to 3h$ cross section predictions for $\sqrt{s} = 1$ TeV:

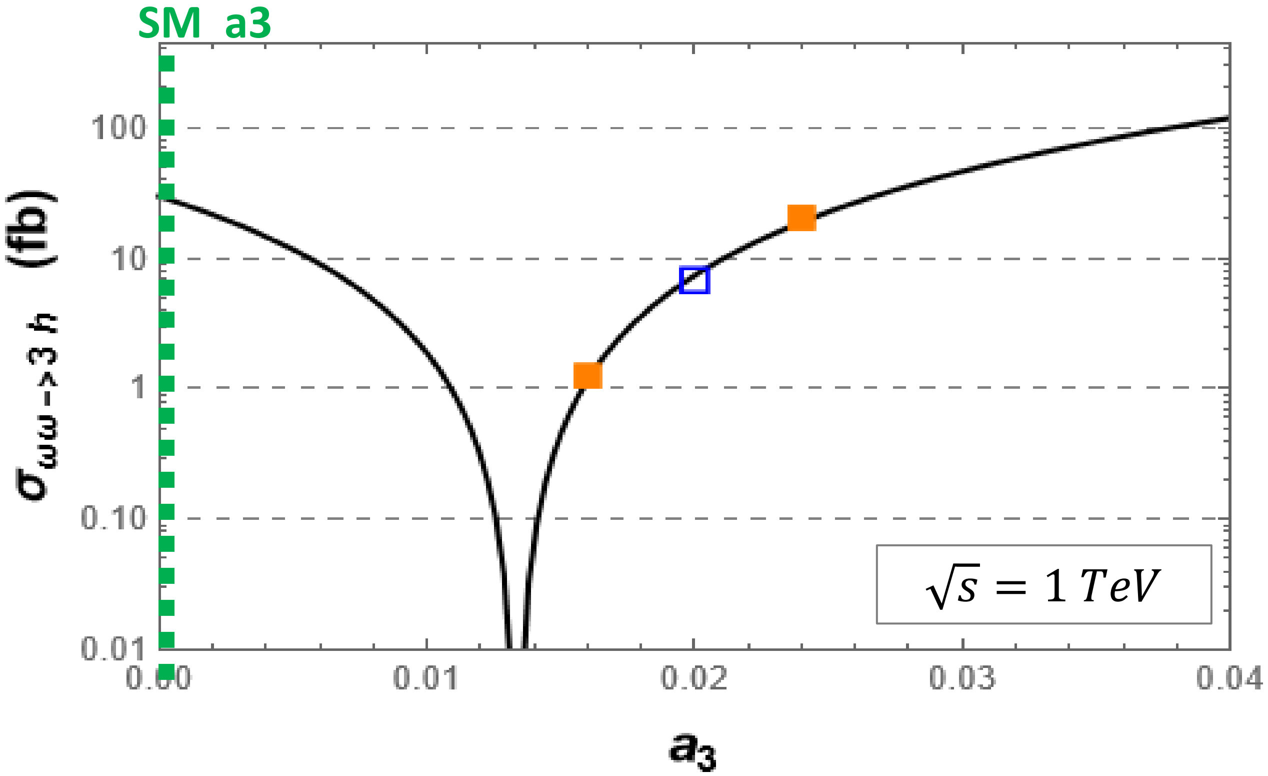
For SMEFT starting @ D=8:
$\boldsymbol{\Delta b = 6\Delta a}$ *

- Empty Blue square - SMEFT$^{(D=8)}$ -BP $(c_{(6)} = 0,\ c_{(8)} = 1,\ v^2/\Lambda^2 = 0.05)$:

$$a = a_1/2 = a^{SMEFT(D=8)} = 1.05, \qquad \mathrm{b} = a_2 = a_2^{SMEFT\,(D=8)} = 1.3, \qquad a_3 = a_3^{SMEFT\,(D=8)} = 0.4$$

- Full Orange square - HEFT:

$$a = a_1/2 = a^{SMEFT(D=8)} = 1.05, \qquad \mathrm{b} = a_2 = a_2^{SMEFT\,(D=8)} = 1.3, \qquad a_3 = a_3^{SMEFT\,(D=8)} \times (1 \pm 20\%)$$



$$\sigma^{SMEFT}_{\omega\omega\to 3h} \sim \frac{1}{\Lambda^8}$$

- Analogous to previous $a_2 = b = \kappa_{2V}$ scannings for LHC and FCC analyses:

(x) Englert,Naskar,Sutherland, JHEP 11 (2023) 158

* Dawson,Fontes,Quezada-Calonge,Sanz-Cillero, PRD 108 (2023) 5, 055034 (e.g., for 2HDM),
* Delgado,Gómez-Ambrosio,Martínez-Martín,Salas-Bernárdez,SC, JHEP 03 (2024) 037

2. b) Other applications:

$$W_L W_L \to Z_L Z_L\, h \quad \text{(tree)},$$

$$W_L W_L \to hhh \quad \text{(1-loop)},$$

etc.

- Why not further exploiting these Lagrangian simplifications?

  - Other tree-level processes: (A. Nasoni, Master Thesis 2025)

    New (a_1, a_2) relevant combinations

  - Other 1-loop computations (S. Martellini, Master Thesis 2025)

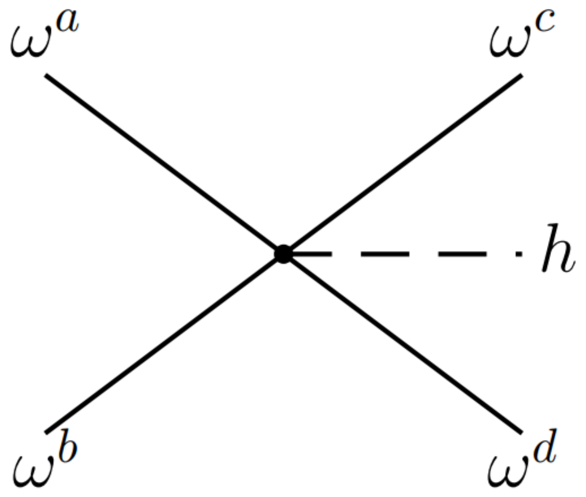
    Corrections to complicated processes

$\omega\omega \to zzh$ [LO = tree $O(p^2)$]

- Consider a more general set of field redefinitions:

$$\begin{cases} \omega^a \to \tilde{\omega}^a(\omega, h) \equiv \omega^a + G_1(h)\,\omega^a + G_2(h)\dfrac{\omega^2}{v^2}\omega^a, \\ h \to \tilde{h}(\omega, h) \equiv h + f_1(h)\dfrac{\omega^2}{v} + f_2(h)\dfrac{\omega^4}{v^3}. \end{cases}$$

- An adequate choice of $f_1(h), f_2(h), G_1(h), G_2(h)$ reduces the LO computation to 1 diagram:



* Alessia Nasoni, **Master Thesis 2025**; Martellini,Nasoni,SC, forthcoming

- Amplitude:

$$iA(\omega^a\omega^b \to \omega^c\omega^d h) =$$
$$= iA(s,t,u;s_h,t_h,u_h)\delta_{ab}\delta_{cd} + iA(t,s,u;t_h,s_h,u_h)\delta_{ac}\delta_{bd}+$$
$$+ iA(u,t,s;u_h,t_h,s_h)\delta_{ad}\delta_{bc},$$

with the $O(p^2)$ determination $$A(s,s_h) = -\frac{s+s_h}{2v^3}\left[a_1 + a_1a_2 - \frac{a_1^3}{2}\right]$$

- LO cross section:

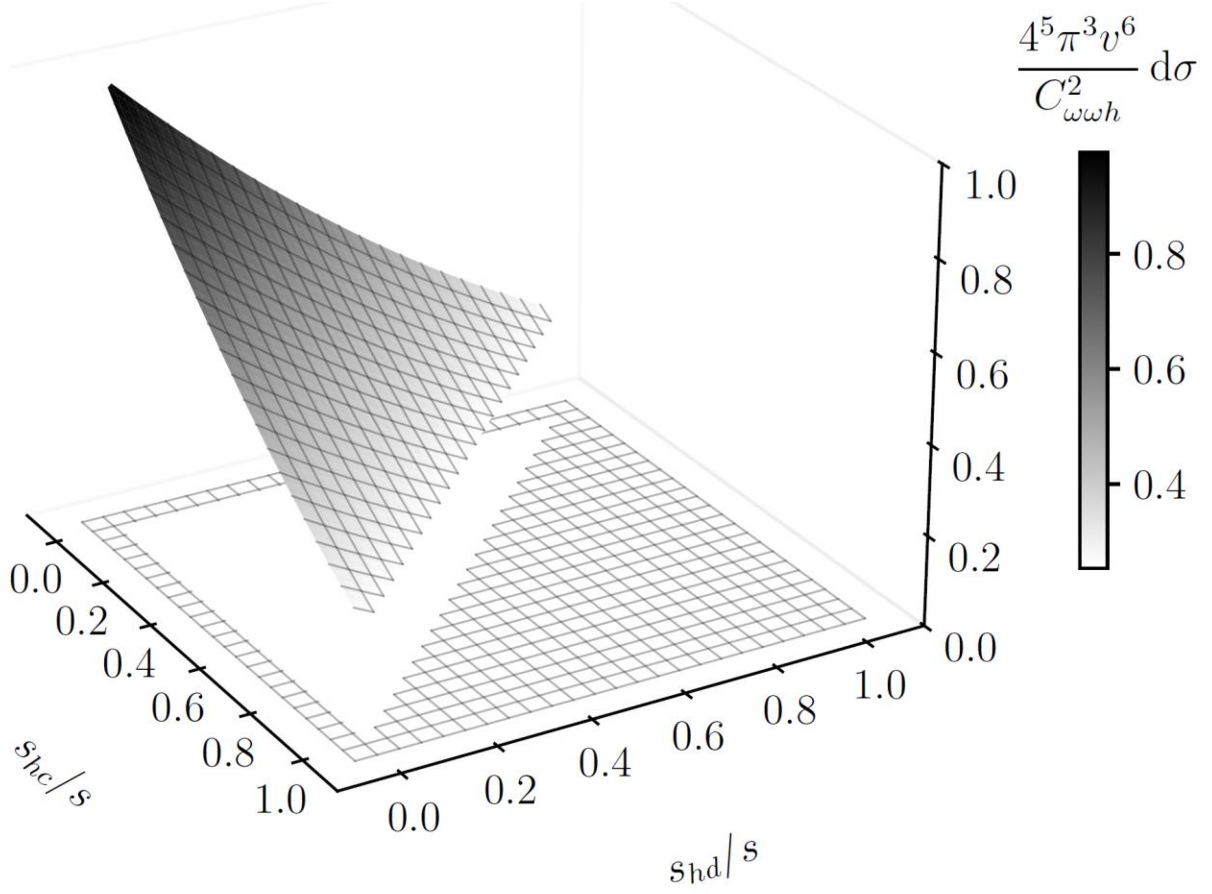
$$\sigma(s;a_1,\,a_2) = \frac{11s^2}{6144\pi^3 v^6}\left(a_1 + a_1a_2 - \frac{a_1^3}{2}\right)^2$$

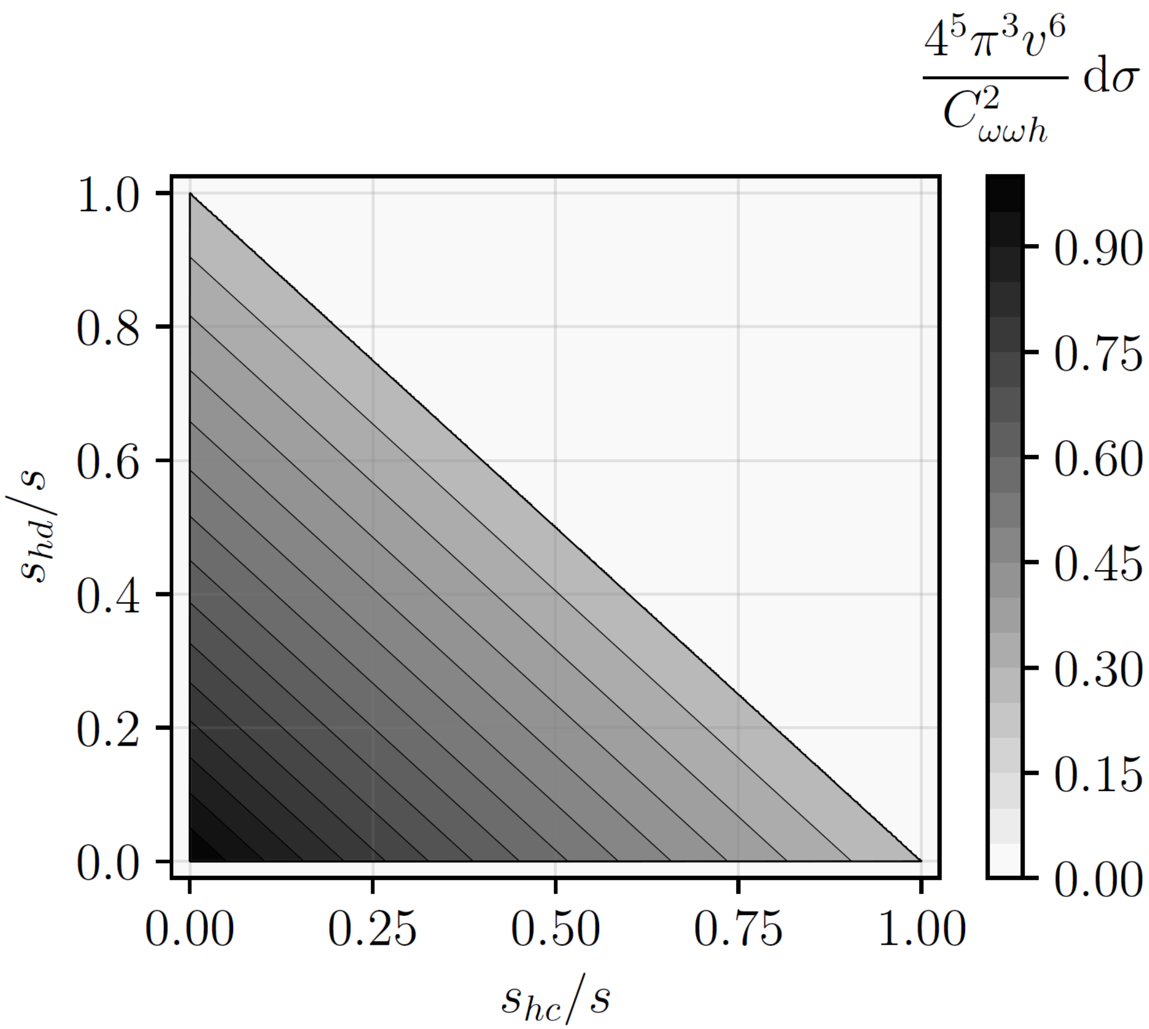
with a similar strong SMEFT suppression $$\sigma_{\mathrm{SMEFT}} = \frac{11s^2}{1536\pi^3v^6}\left(4d^4\left(1+\rho\right)^2 + \mathcal{O}(d^5)\right)$$

$$d = \frac{2v^2 c_{H\Box}^{(6)}}{\Lambda^2}, \qquad \rho = \frac{c_{H\Box}^{(8)}}{2\left(c_{H\Box}^{(6)}\right)^2}$$

* Alessia Nasoni, **Master Thesis 2025**; Martellini,Nasoni,SC, forthcoming

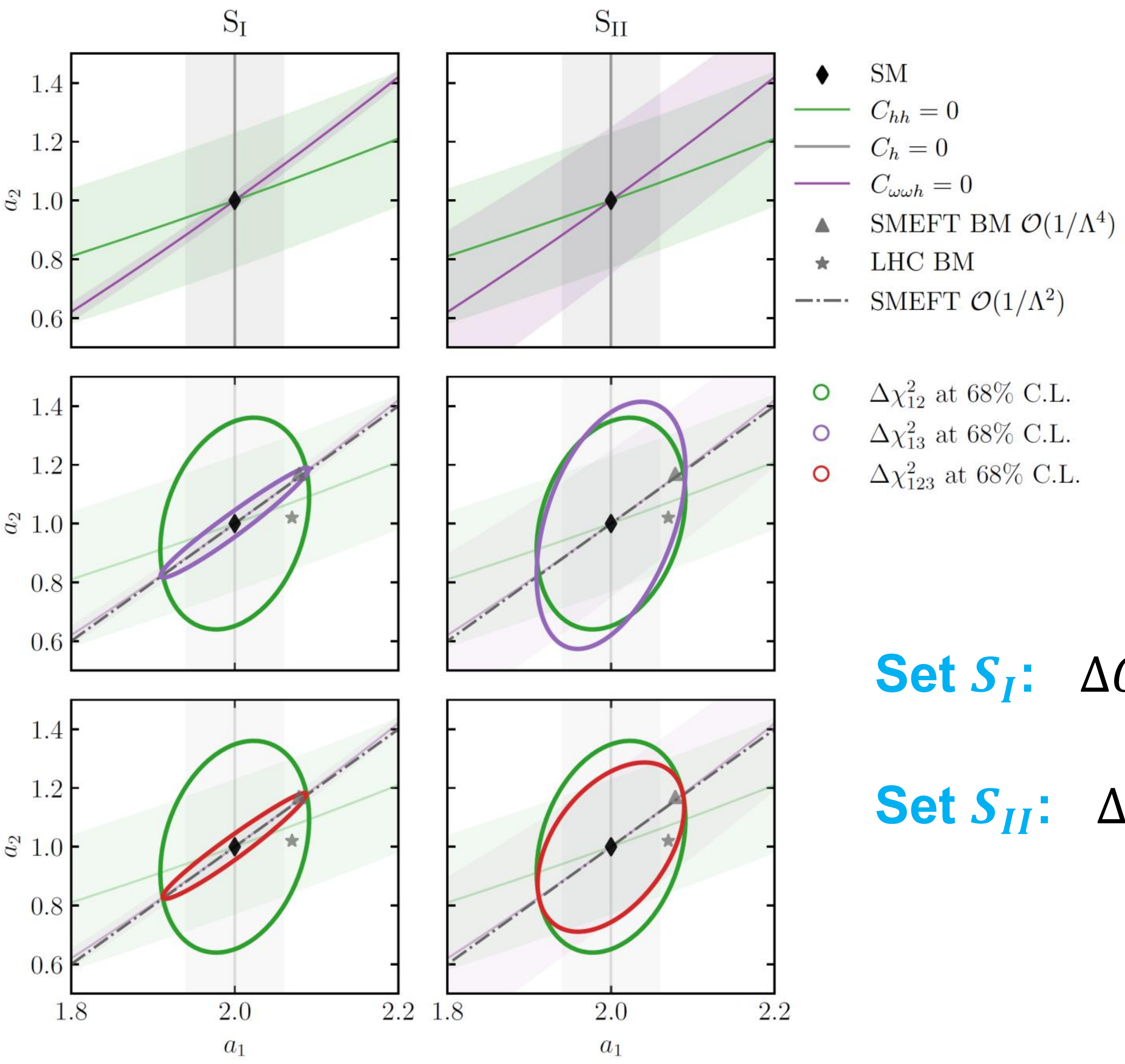
- Dalitz-plot structure for $W_L^+ W_L^- \to Z_L Z_L h$ (a very specific shape)





$$\omega^a(p_1) + \omega^b(p_2) \to \omega^c(p_3) + \omega^d(p_4) + h(p_h)$$

* Alessia Nasoni, **Master Thesis 2025**; Martellini,Nasoni,SC, forthcoming



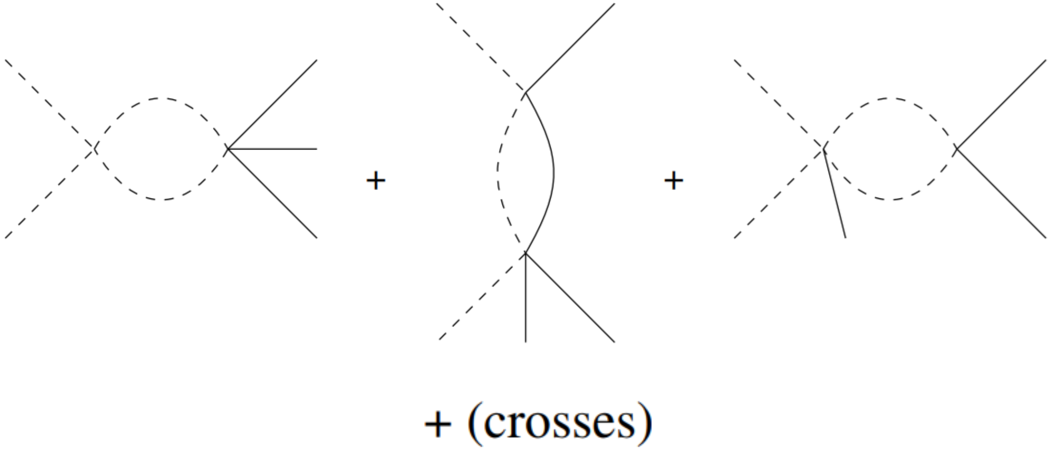
Set S_I: $\Delta C_{ww\to wwh} = 0.06 \approx \Delta a_1^{exp}$ (optimistic)

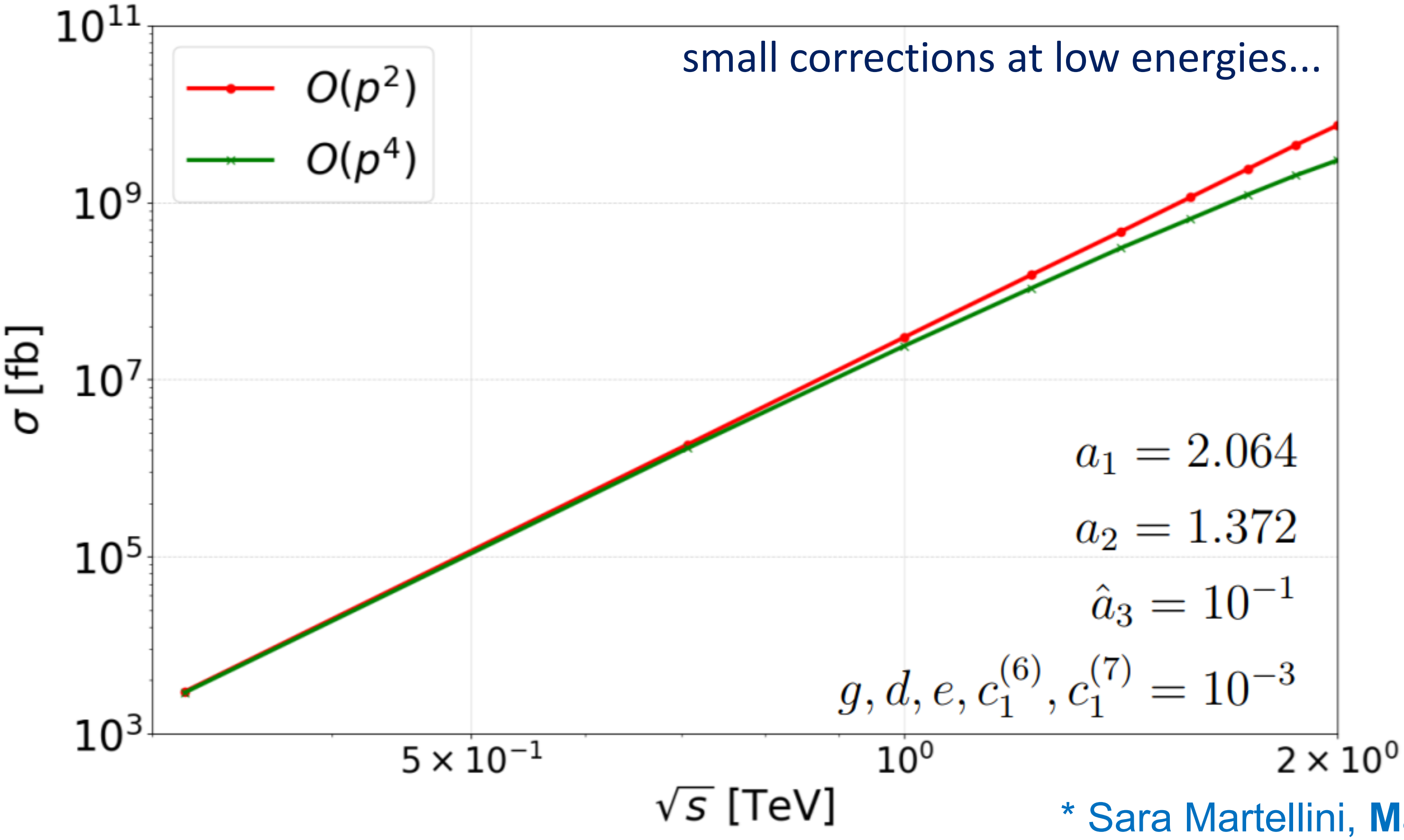
Set S_{II}: $\Delta C_{ww\to wwh} = 0.5 \approx 2\, \Delta a_2^{exp}$ (pessimistic)

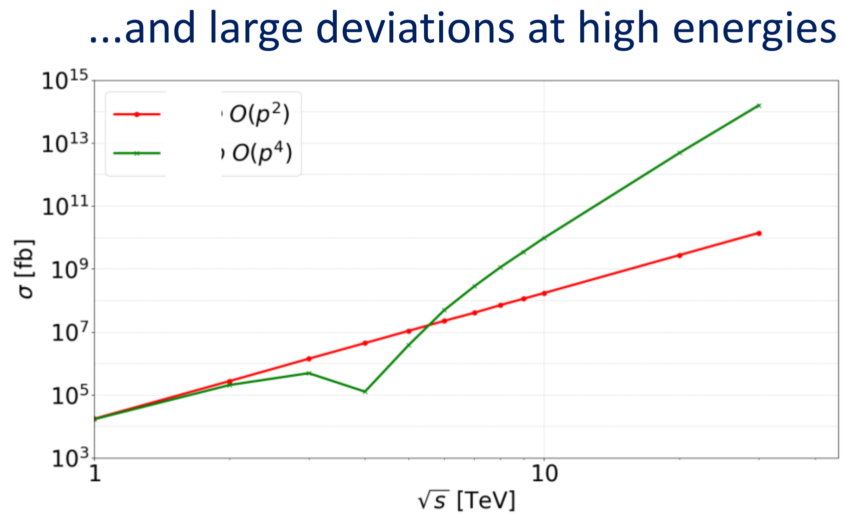
* Alessia Nasoni, **Master Thesis 2025**; Martellini,Nasoni,SC, forthcoming

$\omega\omega \to hhh$ [LO+NLO = $O(p^2)$+$O(p^4)$ tree + $O(p^4)$ loop

- All 1-loop diagrams for $W_L^+ W_L^- \to hhh$ encoded by these 3 topologies:







* Sara Martellini, **Master Thesis 2025**; Martellini,Nasoni,SC, forthcoming

2. c) Other applications:

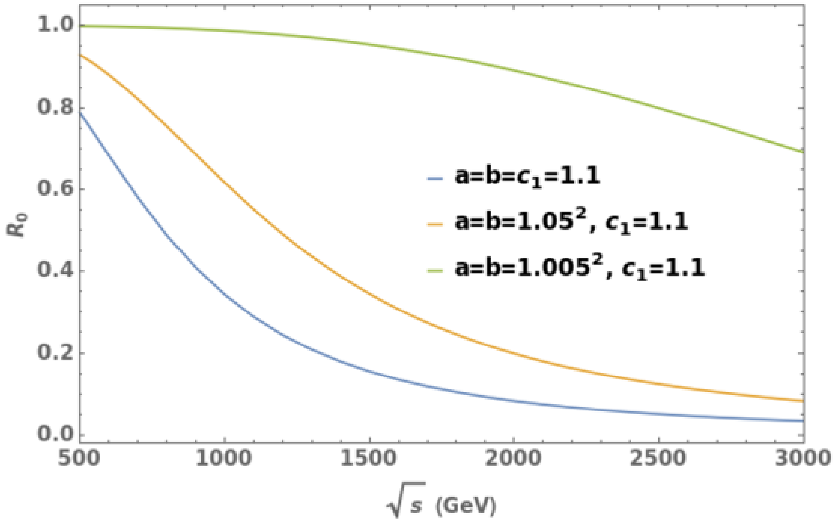
$$W_L W_L \rightarrow fermions$$

Orientative comparison of the F & B loop importance
(not summed F+B cross section)

$$R_0 = \frac{|a_0^F|}{|a_0^F|+|a_0^B|}$$

Fermion corrections are relevant for values close to the SM

In this region of the parameter space the GB approximation breaks down



R_0 ratio for different values of a, b and c_1

* Quezada-Calonge (PhD 2024); Dobado,Quezada-Calonge,Sanz-Cillero; in preparation

3. EW Chiral Lagr. and heavy BSM states:

V and A mass bounds from (S,T)

- The Lagrangian reads:

$$\Delta\mathcal{L}_{\rm RT} = \frac{v^2}{4}\left(1+\frac{2\kappa_W}{v}h\right)\langle\, u_\mu u^\mu\,\rangle_2$$
$$+\,\langle\, V^1_{3\,\mu\nu}\left(\frac{F_V}{2\sqrt{2}}f_+^{\mu\nu}+\frac{iG_V}{2\sqrt{2}}[u^\mu,u^\nu]+\frac{\widetilde{F}_V}{2\sqrt{2}}f_-^{\mu\nu}+\frac{\widetilde{\lambda}_1^{hV}}{\sqrt{2}}\left[(\partial^\mu h)u^\nu-(\partial^\nu h)u^\mu\right]+C_0^{V_3^1}J_T^{\mu\nu}\right)\rangle_2$$
$$+\,\langle\, A^1_{3\,\mu\nu}\left(\frac{F_A}{2\sqrt{2}}f_-^{\mu\nu}+\frac{\lambda_1^{hA}}{\sqrt{2}}\left[(\partial^\mu h)u^\nu-(\partial^\nu h)u^\mu\right]+\frac{\widetilde{F}_A}{2\sqrt{2}}f_+^{\mu\nu}+\frac{i\widetilde{G}_A}{2\sqrt{2}}[u^\mu,u^\nu]+\widetilde{C}_0^{A_3^1}J_T^{\mu\nu}\right)\rangle_2\,.$$

- Including resonance masses, we have 12 resonance parameters. This number can be reduced by using short-distance information, but contrary to QCD, we ignore the underlying theory (BSM).

  - Vanishing form factors at high energies allow us to determine $\left(G_V\,,\widetilde{G}_A\,,\lambda_1^{hA}\,,\widetilde{\lambda}_1^{hV}\right)$ in terms of the remaining parameters:

$$\frac{G_V}{F_A}=-\frac{\widetilde{G}_A}{\widetilde{F}_V}=\frac{\lambda_1^{hA}v}{\kappa_W F_V}=-\frac{\widetilde{\lambda}_1^{hV}v}{\kappa_W\widetilde{F}_A}=\frac{v^2}{F_VF_A-\widetilde{F}_V\widetilde{F}_A}\,.$$

  - Weinberg sum rules (WSRs) at LO and at NLO.

    - 1st WSR. Vanishing of the 1/s term of $\Pi_{VV}(s)-\Pi_{AA}(s)$: $\left(F_V^2-\widetilde{F}_V^2\right)-\left(F_A^2-\widetilde{F}_A^2\right)=v^2$

    - 2nd WSR. Vanishing of the 1/s² term of $\Pi_{VV}(s)-\Pi_{AA}(s)$: $\left(F_V^2-\widetilde{F}_V^2\right)M_V^2-\left(F_A^2-\widetilde{F}_A^2\right)M_A^2=0$

    - 1st WSR + LHC diboson production imply that contributions from fermionic cuts, terms with , are negligible.

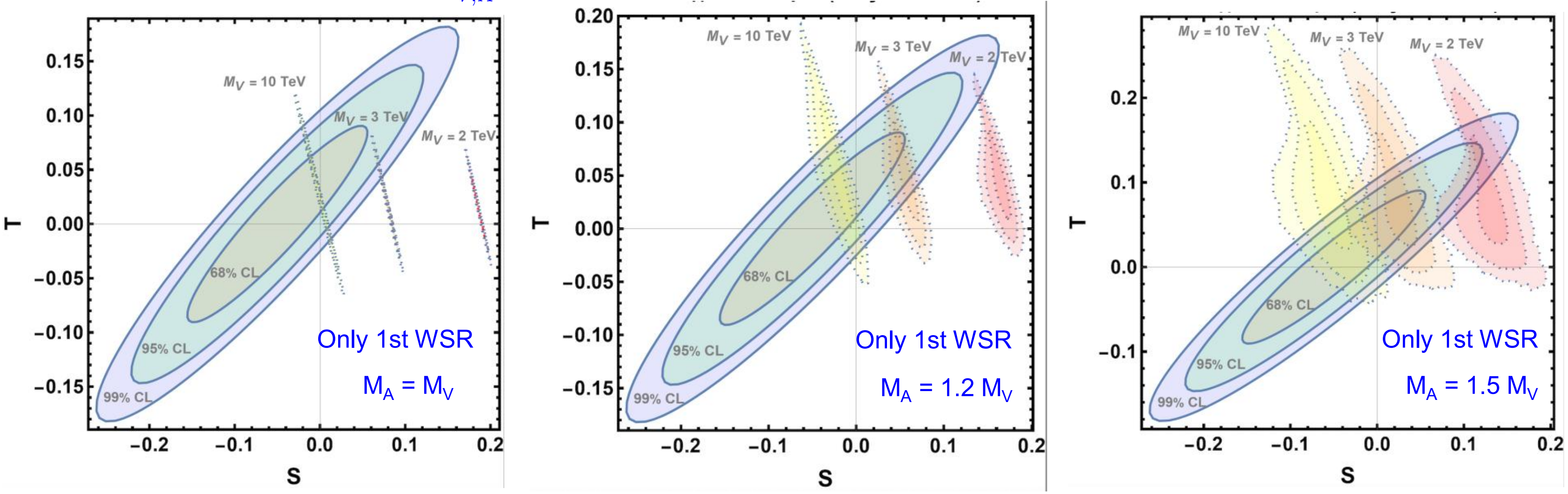
(thanks to I. Rosell)

ii) NLO results

Results in terms of only M_V, M_A (only 1st WSR), κ_W and $\frac{\widetilde{F}_{V,A}}{F_{V,A}}$.

S = -0.05 ± 0.07 * T = 0.00 ± 0.06*

κ_W=1.023 ± 0.026 * $\frac{\widetilde{F}_{V,A}}{F_{V,A}} = 0.00 \pm 0.33$



$M_A > M_V \gtrsim 2$ TeV (95% CL)

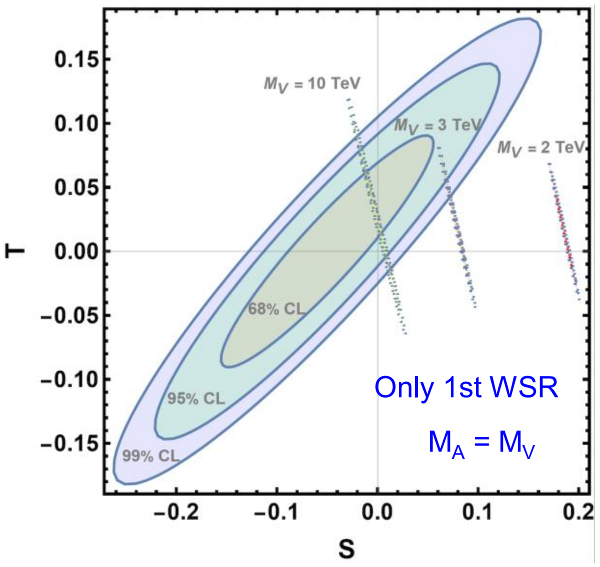
* PDG '24

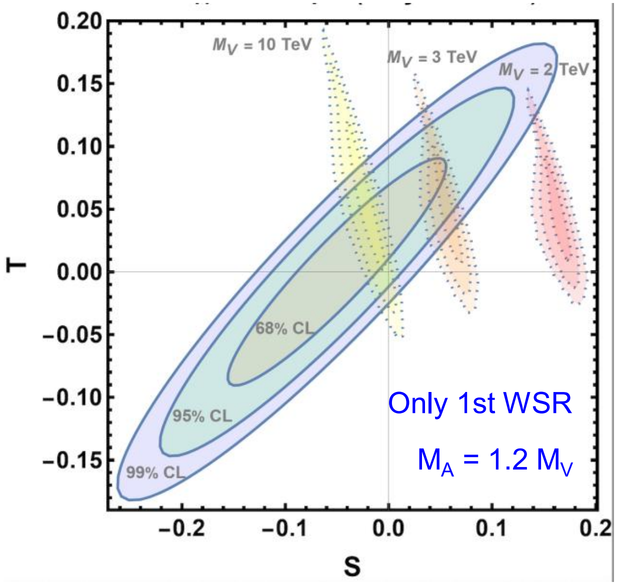
(thanks to I. Rosell)

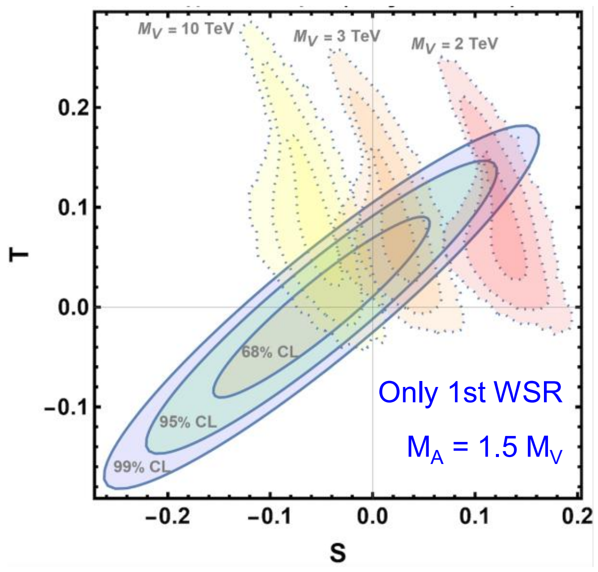
ii) NLO results

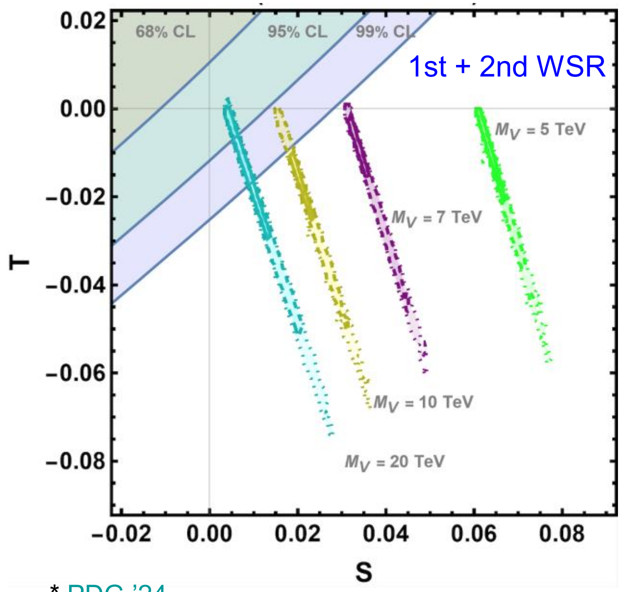
Results in terms of only M_V, M_A (only 1st WSR), κ_W and $\frac{\widetilde{F}_{V,A}}{F_{V,A}}$.

S = -0.05 ± 0.07 * T = 0.00 ± 0.06*
κ_W=1.023 ± 0.026 * $\frac{\widetilde{F}_{V,A}}{F_{V,A}} = 0.00 \pm 0.33$









$M_A > M_V \gtrsim 10$ TeV (95% CL)

(thanks to I. Rosell)

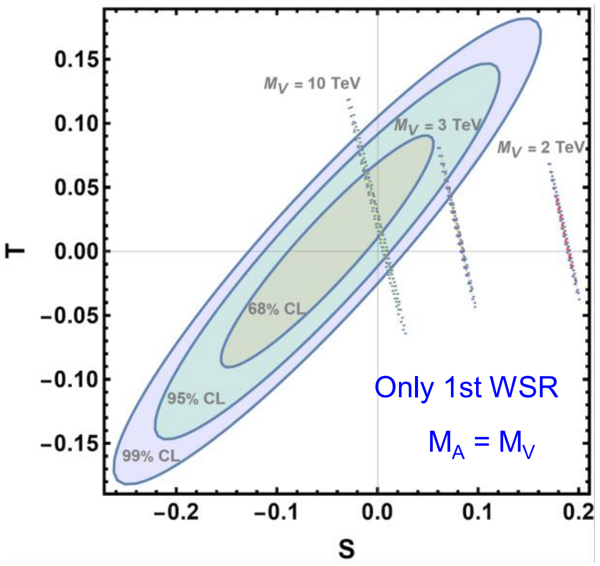
* PDG '24

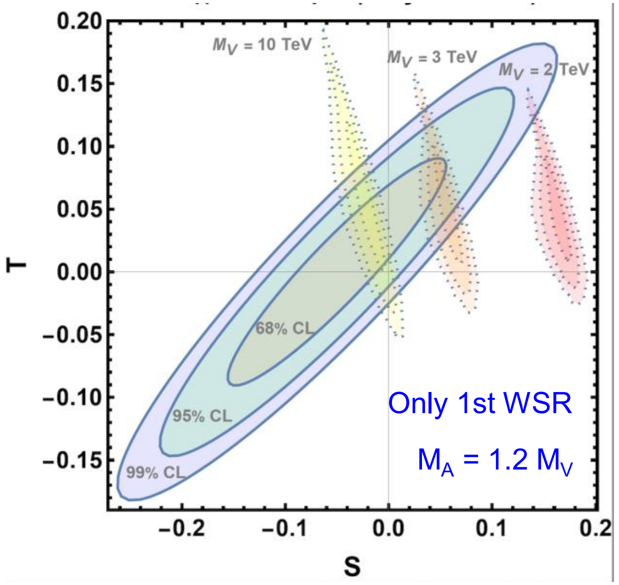
ii) NLO results

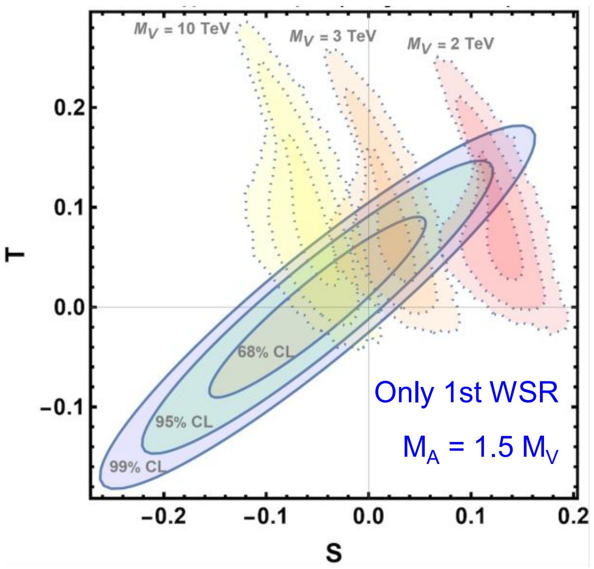
Results in terms of only M_V, M_A (only 1st WSR), κ_W and $\frac{\widetilde{F}_{V,A}}{F_{V,A}}$.

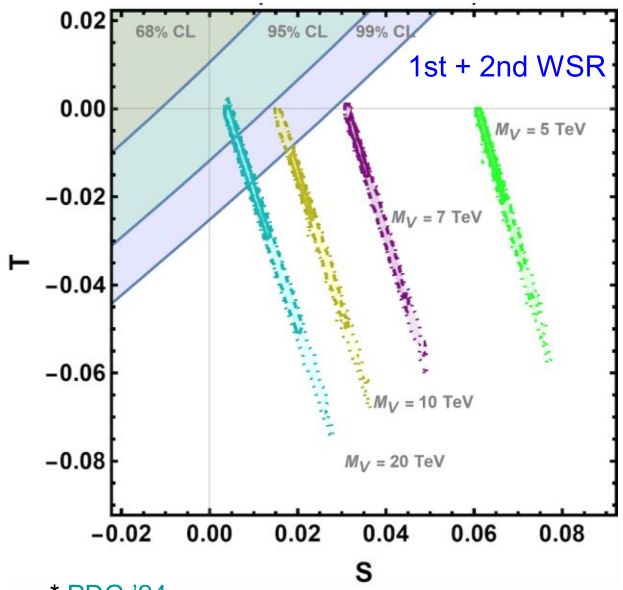
$S = -0.05 \pm 0.07$ * $\quad T = 0.00 \pm 0.06$*

$\kappa_W = 1.023 \pm 0.026$ * $\quad \frac{\widetilde{F}_{V,A}}{F_{V,A}} = 0.00 \pm 0.33$









$M_A > M_V \gtrsim 2$ TeV (only the 1st WSR, 95% CL)

$M_A > M_V \gtrsim 10$ TeV (1st and 2nd WSR, 95% CL)

* PDG '24

(thanks to I. Rosell)

Conclusions

- **Compact structure for $W_L W_L \to n \times h$ scattering:**

  [Ruled by very particular combinations, $\hat{\kappa}_{2V} \equiv \hat{a}_2$ for $2h$, $\hat{a}_3$ for $3h$, etc.]

- **Field redefinitions helps us to understand this simplicity**

- **Strong multi-Higgs suppression in SMEFT** wrt to HEFT

  –Even for small O(10%) deviations in SMEFT $a_{1,2,3,4\ldots}$–

  –Much larger XS in pure HEFT–