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Evgeny Epelbaum, Ruhr University Bochum

SMEFT meets ChEFT, TRIUMF, Vancouver, Sep. 29 — Oct. 3, 2025

Are there enhanced 3N forces in chiral EFT?

Dan Hog, EE, Ashot Gasparyan, Jambul Gegelia, Hermann Krebs, in preparation

PHYSICAL REVIEW LETTERS **135,** 022501 (2025)

New Class of Three-Nucleon Forces and Their Implications

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(Received 18 December 2024; revised 10 April 2025; accepted 10 June 2025; published 8 July 2025)

We identify a new class of three-nucleon forces that arises in the low-energy effective theory of nuclear interactions including pions. We estimate their contribution to the energy of neutron and nuclear matter and find that it can be as important as the leading-order three-nucleon forces previously considered in the literature. The magnitude of this force is set by the strength of the coupling of pions to two nucleons and is presently not well constrained by experiments. The implications for nuclei, nuclear matter, and the equation of state of neutron matter are briefly discussed.

erc European Research Council

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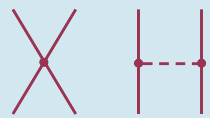
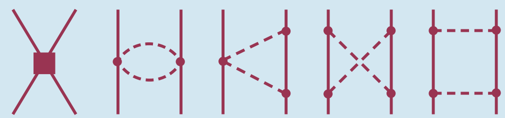
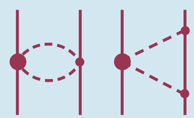
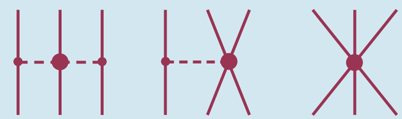
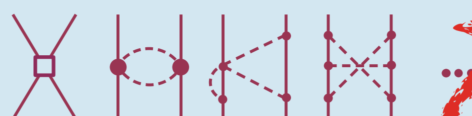
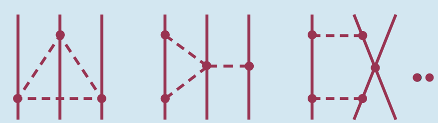
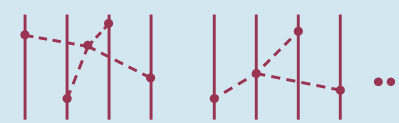
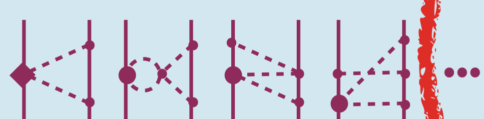
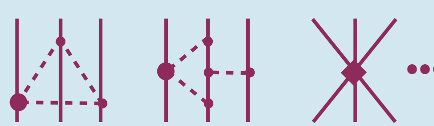
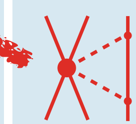
ERATO Exploratory Research for Advanced Technology

Chiral expansion of the nuclear forces (NDA, no Δ's)

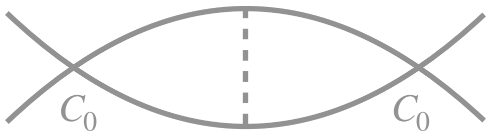
	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)	 Weinberg '90	—	—
NLO (Q^2)		—	—
N²LO (Q^3)	 van Kolck et al. '94 Friar, Coon '94 Kaiser et al. '97 Epelbaum et al. '98	 van Kolck '94; Epelbaum et al. '02	—
N³LO (Q^4)	 Kaiser '00-'02	 Bernard, Epelbaum, Krebs, Meißner '08, '11	 Epelbaum '06, '07
N⁴LO (Q^5)	 Entem, Kaiser, Machleidt, Nosyk '15	 Girlanda et al.'11, Krebs et al. '11,'13	—
N⁵LO (Q^6)	• 1- and 2-loop contributions worked out by Norbert Kaiser • 3-loop contributions unknown	 Cirigliano et al.'25 + many more… (unknown)	?

enhanced

Estimation of the LECs D_2, F_2

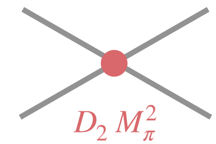
Vincenzo Cirigliano, Maria Dawid, Wouter Dekens, and Sanjay Reddy, PRL 135 (2025) 022501

"Arguments based on the renormalization of the nucleon-nucleon spin-singlet s-wave scattering amplitude [KSW] warrant that the associated LEC, denoted by $\mathbf{D_2}$*, is significantly larger than what is expected in Weinberg's power counting"*



$\longleftarrow$ logarithmic divergence $\propto M_\pi^2$

$\Rightarrow$ include contact term $\longrightarrow$



Kaplan, Savage, Wise, PLB 478 (96) 629

"The above implies that $\mathbf{D_2}$ *is needed at LO in approaches to Chiral EFT that aim to ensure regulator independence."*

Estimation of D_2: $$D_2(\mu) = \underbrace{\frac{g_A^2 m^2}{64\pi^2 F_\pi^2}}_{\sim 0.27} \big(C_0(\mu)\big)^2 \ln\frac{\mu}{\mu_0} \equiv \xi \big(C_0(\mu)\big)^2 \ln\frac{\mu}{\mu_0} \approx \xi \big(C_0(\mu)\big)^2$$

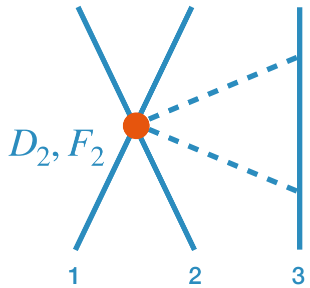
"In Chiral EFT, for typical Λ *employed in calculations,* $|C_0| \sim 1/M_\pi^2 \approx 5\ \mathrm{fm}^2$*, and* $\xi < 0.5$ *predicts* $|D_2| \lesssim 10\ \mathrm{fm}^4$*."*

(In the actual estimations, the range $|D_2| \leq 1/(5F_\pi^4) \simeq 4.2\ \mathrm{fm}^4$ is employed.)

Impact on the EoS of neutron/nuclear matter

Vincenzo Cirigliano, Maria Dawid, Wouter Dekens, and Sanjay Reddy, PRL 135 (2025) 022501

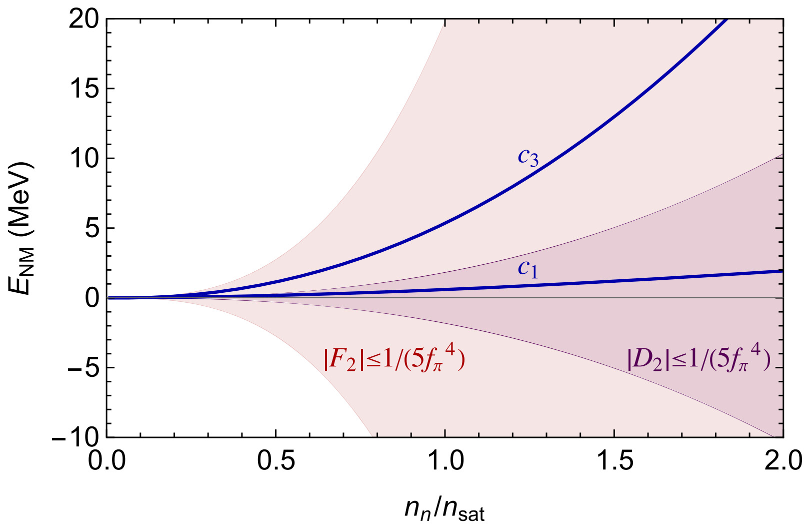
In chiral EFT, $D_2 M_\pi^2$ , comes together with $D_2 M_\pi^2$ , $D_2 M_\pi^2$ , ... $\Rightarrow$ enhanced many-body forces



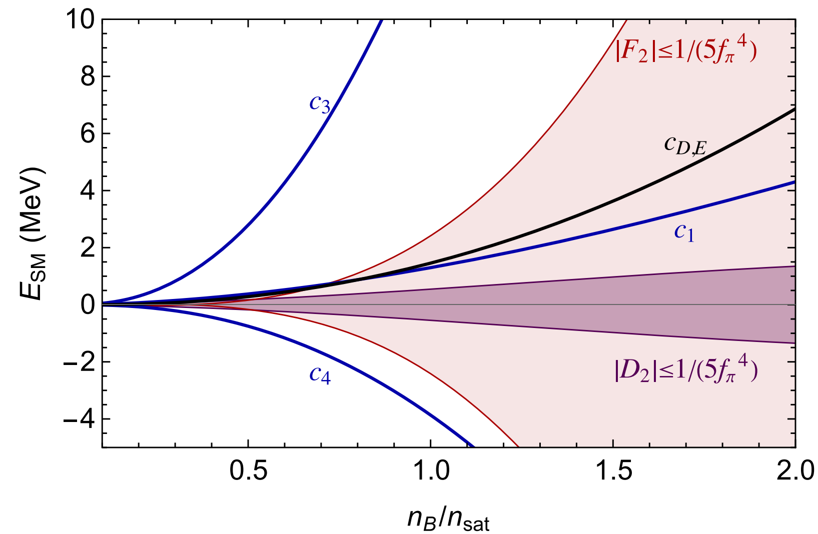
$$V = (1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2)\left[v_{D_2}(q_3) + v_{F_2}(q_3)\right] + \text{permutations}$$

where
$$v_{D_2}(q) = \frac{3 g_A^2 D_2 M_\pi^2}{256 \pi F_\pi^4}(2M_\pi^2 + q^2)\, A(q),$$
$$v_{F_2}(q) = \frac{3 g_A^2 F_2}{512 \pi F_\pi^4}(2M_\pi^2 + q^2)^2\, \underbrace{A(q)}_{A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}}$$

Neutron matter



Symmetric nuclear matter



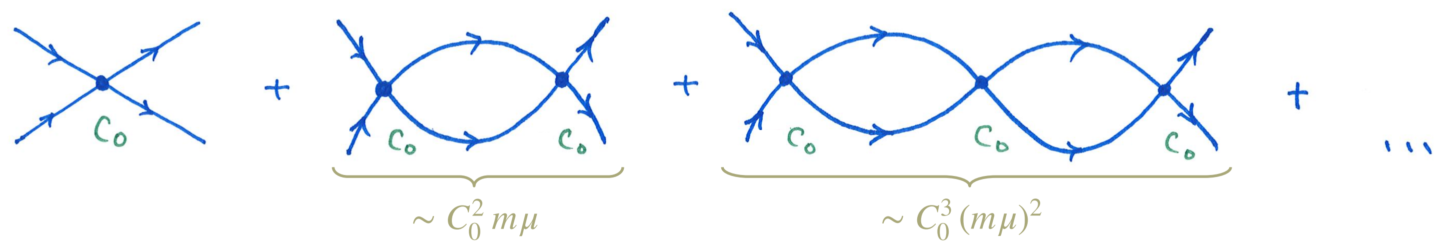
Do RG arguments warrant the need to promote the D_2, F_2 terms?

LECs in nuclear EFTs

The NN system is known to be „fine tuned“:

$$T(k) = -\frac{4\pi}{m}\frac{1}{k\cot\delta - ik} = -\frac{4\pi}{m}\frac{1}{-\frac{1}{a}+\frac{1}{2}rk^2+\ldots - ik} = \begin{cases} \mathcal{O}(1) & \text{for } \; a\sim r\sim M_\pi^{-1} \\ \mathcal{O}(Q^{-1}) & \text{for } \; a\to\infty \end{cases}$$

This fine tuning must be implemented in nuclear EFTs (not a prediction).

$T^{(-1)} =$ 

Power counting schemes (assuming $a \sim \mathcal{O}(Q^{-1})$):

- Kaplan-Savage-Wise (KSW) Kaplan, Savage, Wise, PLB 424 (98); NPB 534 (98)

  All renorm. scales soft $(\mu_i \sim Q)$, enhanced LECs: $C_0 = \mathcal{O}(Q^{-1})$, $C_2 \sim \mathcal{O}(Q^{-2})$, ...

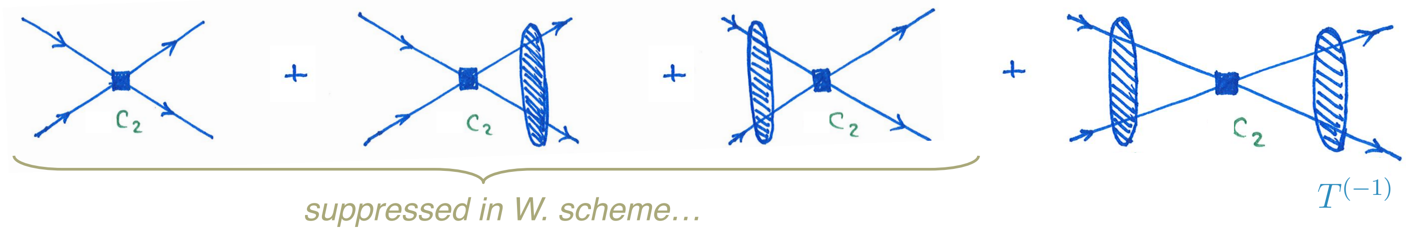
- Weinberg (W) Weinberg, PLB 251 (90); NPB 363 (91)

  In the formulation of EE, Gegelia, Meißner, NPB 925 (17), can be realized by using the renorm. conditions:

  $\mu_{\text{linear}} \sim \Lambda_b$, $\mu_{\text{all other}} \sim Q$, natural-sized (NDA) LECs: $C_n = \mathcal{O}(1)$

LECs in nuclear EFTs

$T^{(0)} =$



E.g., calculating the amplitude in π-less EFT to NLO and matching to the ERE, one finds using $\mu_{\text{linear}} \equiv \mu, \;\; \mu_{\text{all other}} = 0$:

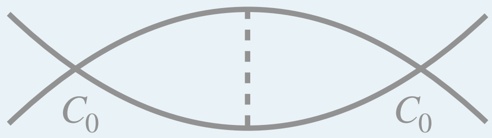
$$C_0^r(\mu) = \frac{4\pi}{m}\frac{1}{a^{-1} - \mu}, \qquad C_2^r(\mu) = \frac{\pi}{m}\frac{r}{(a^{-1} - \mu)^2}$$

BTW, renormalization conditions leading to self-consistent π-less/halo EFT for even stronger fine-tuned systems ($l > 0$) have also been worked out EE, Gegelia, Huesmann, Meißner, FBS 62 (21)

- Finite-cutoff chiral EFT Bochum-Bonn, Idaho, NLEFT, Norfolk, Chalmers, St. Louis, …
  - — choose $\Lambda \sim \Lambda_b$, in practice $\Lambda \in [400, 550]$ MeV
  - — *implicit* renormalization: express bare $C_i(\Lambda)$ through observables (fit to data)
  - — conjectured to be equivalent to the W choice or renorm. conditions EE, Gegelia, Meißner, NPB 925 (17)
  - — renormalizability proven to NLO for 2N using the BPHZ scheme Gasparyan, EE, PRC 105 (22); 107 (23)
  - — *a-posteriori* consistency checks: Naturalness of the LECs, residual Λ-dependence, …
  - — underwent/undergoing extensive statistical consistency tests (BUQEYE) Millican et al., 2508.17558

Back to the argument by *Cirigliano et al.*

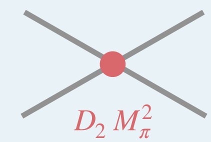
But what about



??

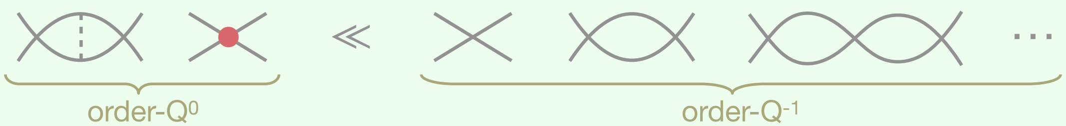
(the contact part of the OPEP is absorbed into C_0)

- Yes, the diagram is logarithmically divergent, and removing this divergence requires the introduction of the D_2-counter term

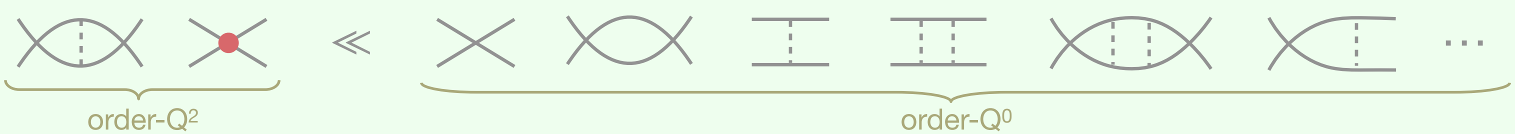


- However, for both KSW and W, these diagrams yield sub-leading contributions:

KSW:



W:



No inconsistency of the W approach! (but W's original justification for iterating V using m_N is indeed formally inconsistent...)

The OPEP:

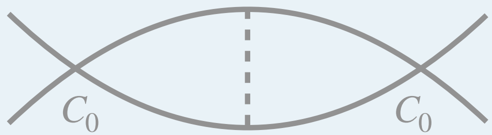
$$\vec{\sigma}_1 \cdot \hat{q}\vec{\sigma}_2 \cdot \hat{q} - \frac{1}{3}\vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$-\frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2} = -\frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left(\frac{q^2 S_{12}(\hat{q})}{q^2 + M_\pi^2} - \frac{1}{3}\vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{M_\pi^2}{q^2 + M_\pi^2} + \xcancel{\text{contact}} \right)$$

Notice: $\langle \text{spin-0} | S_{12} | \text{spin-0} \rangle = 0, \quad \langle \text{spin-1} | S_{12} | \text{spin-1} \rangle \neq 0 \quad \text{but} \quad \langle {}^3\text{S}_1 | S_{12} | {}^3\text{S}_1 \rangle = 0$

Back to the argument by *Cirigliano et al.*

But what about



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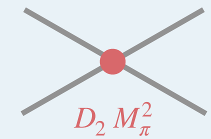
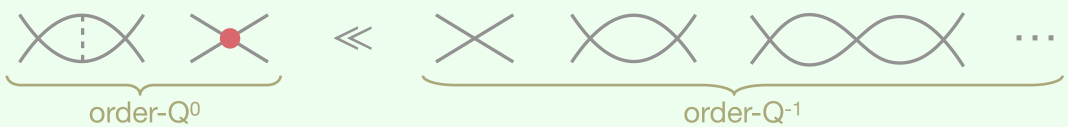
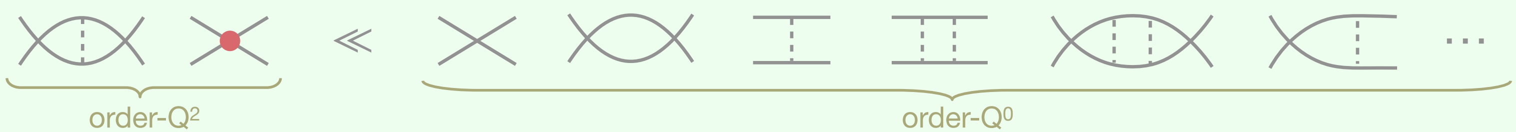


- However, for both KSW and W, these diagrams yield sub-leading contributions:

KSW:



W:



No inconsistency of the W approach! (but W's original justification for iterating V using m_N is indeed formally inconsistent…)

- Demanding ***exact*** ***μ*-independence** at each expansion order is not necessary from the EFT point of view and would disqualify, e.g., well-established extensions of HB ChPT including

  — infrared regularized baryon ChPT Becher, Leutwyler, EPJC 9 (99)

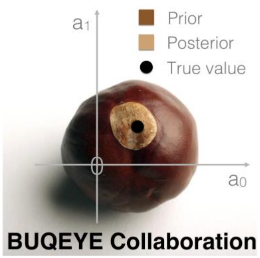
  — EOMS formulation of the covariant BChPT Gegelia, Japaridze, PRD 60 (99); Fuchs et al., PRD 68 (03)

  — ChPT with explicit Δ-isobar DoF (SSE) Hemmert, Holstein, Kambor, J. Phys. G 24 (98)

Estimation of the LECs D_2, F_2

We follow the approach of *Cirigliano et al.* and estimate D_2 (and F_2) by the coefficient of the Log:

$$|D_2| \;\sim\; \frac{g_A^2 m^2}{64\pi^2 F_\pi^2}\, C_0^2$$



„Statistically consistent distributions for Λ_b across orders are only found for the SMS potential at a regulator scale of 450 or 500 MeV."

Millican, Furnstahl, Melendez, Phillips, arXiv: 2508.17558

$\Rightarrow\;\; |D_2| \;\sim\; 0.026,\, 0.81\ \mathrm{fm}^4$ using the SMS N^{4}LO$^+$ potentials with $\Lambda = 450,\, 500\ \mathrm{MeV}$

Alternatively, one can look at the values of the order-Q^2 momentum-dependent NN LECs C_i.
For the 28 LECs, we have $\max\big(|C_i|\big) \;=\; 0.65\ \mathrm{fm}^4$.

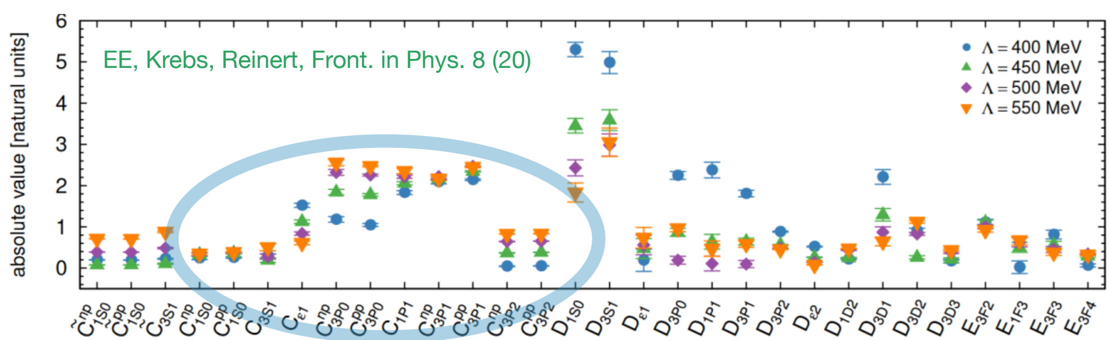
We thus estimate

$$|D_2|, |F_2| \;\leq\; 1\ \mathrm{fm}^4$$

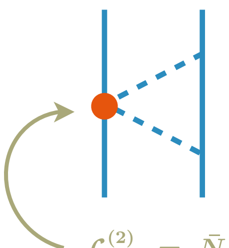
to be compared with

$$|D_2|, |F_2| \;\leq\; 4.2\ \mathrm{fm}^4$$

[Cirigliano et al., PRL 135 (25)]



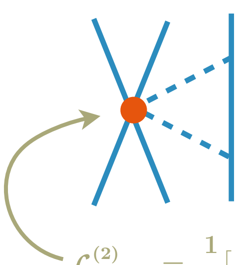
The TPE two-nucleon force



$$V_{2N}^{(Q^3)} = -\frac{3g_A^2}{16\pi F_\pi^4}(2M_\pi^2+q^2)\big[4c_1M_\pi^2 - c_3(2M_\pi^2+q^2)\big]A(q)+\ldots$$

Kaiser et al., NPA 625 (97); EE et al., NPA 637 (98)

$$\mathcal{L}_{\pi N}^{(2)} = \bar{N}(c_1\langle\chi_+\rangle + c_3 u\cdot u + \ldots)N$$



$$V_{3N}^{(Q^6)} = \frac{3g_A^2}{512\pi F_\pi^4}(2M_\pi^2+q^2)\big[2D_2M_\pi^2 + F_2(2M_\pi^2+q^2)\big]A(q)+\ldots$$

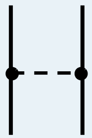
Cirigliano et al., PRL 135 (25)

$$\mathcal{L}_{\pi NN}^{(2)} = \frac{1}{4}\big[-D_2\langle\chi_+\rangle + F_2\langle u\cdot u-(v\cdot u)^2\rangle\big]\,(N^TP_iN)^\dagger(N^TP_iN)$$

Thus, it’s instructive to first look at the well-understood long-range NN force…

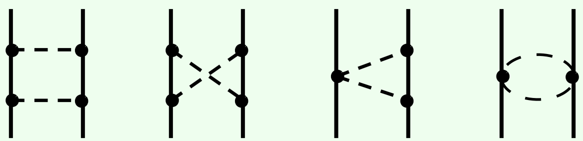
Pion exchange NN potential

Leading order (Q^0)



$$V_{1\pi}^{(0)} = -\frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{q^2+M_\pi^2}$$

Next-to-leading order (Q^2)

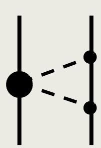


$$L(q) = \frac{\sqrt{q^2+4M_\pi^2}}{q}\ln\frac{\sqrt{q^2+4M_\pi^2}+q}{2M_\pi}$$

$$V_{2\pi}^{(2)}(q) = -\frac{1}{384\pi^2F_\pi^4}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[4M_\pi^2(5g_A^4-4g_A^2-1)+q^2(23g_A^4-10g_A^2-1)+\frac{48g_A^4M_\pi^4}{4M_\pi^2+q^2}\right]L(q)$$
$$-\frac{3g_A^4}{64\pi^2F_\pi^4}\left(\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}-\vec{\sigma}_1\cdot\vec{\sigma}_2\,q^2\right)L(q)$$

Next-to-next-to-leading order (Q^3)

strong numerical enhancement

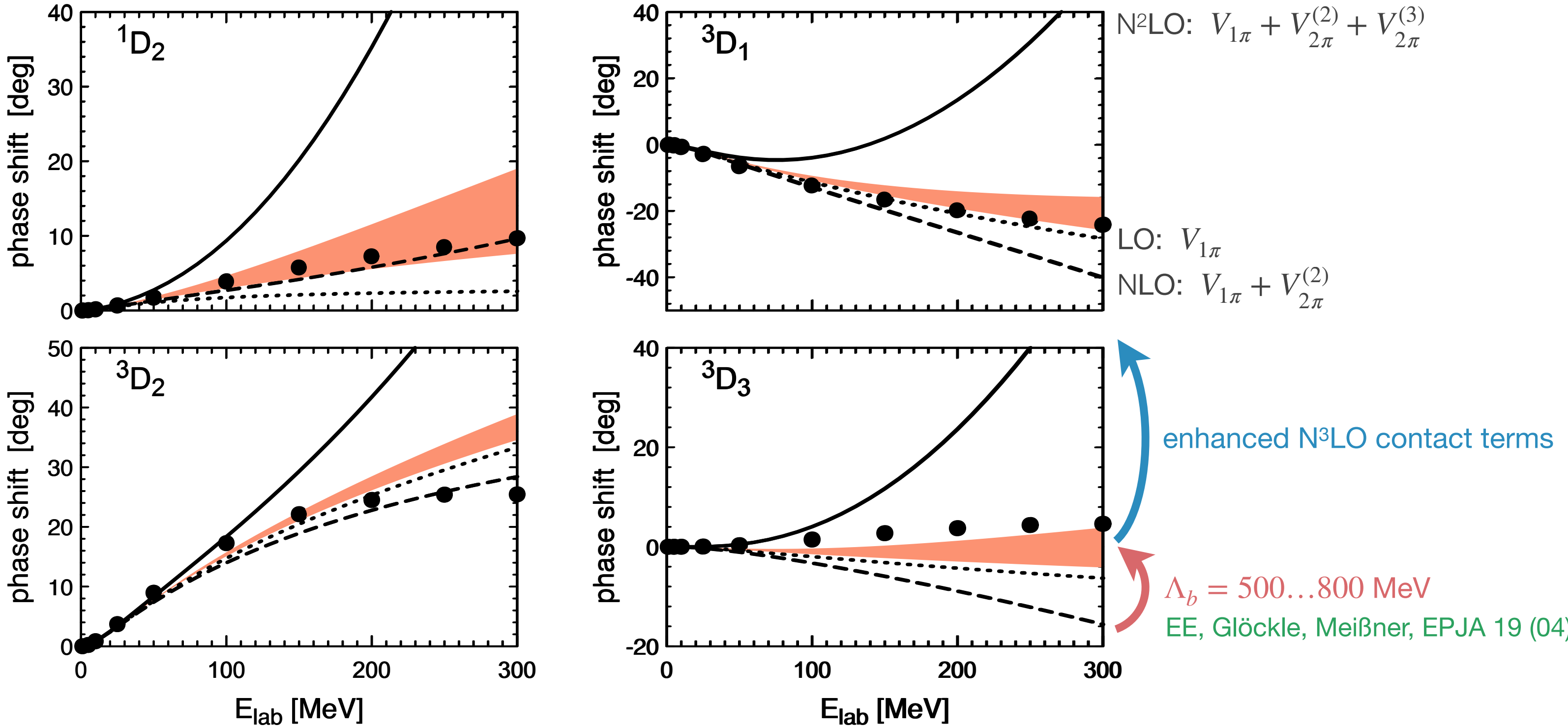


$$V_{2\pi}^{(3)} = -\frac{3g_A^2}{16\pi F_\pi^4}\left[2M_\pi^2(2c_1-c_3)-c_3q^2\right](2M_\pi^2+q^2)A(q)$$
$$-\frac{g_A^2c_4}{32\pi F_\pi^4}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}-\vec{\sigma}_1\cdot\vec{\sigma}_2\,q^2\right)(4M_\pi^2+q^2)\,A(q)$$

$$A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}$$

The two-pion exchange NN potential

Testing chiral EFT predictions in peripheral NN scattering Kaiser, Brockmann, Weise, NPA 625 (97)

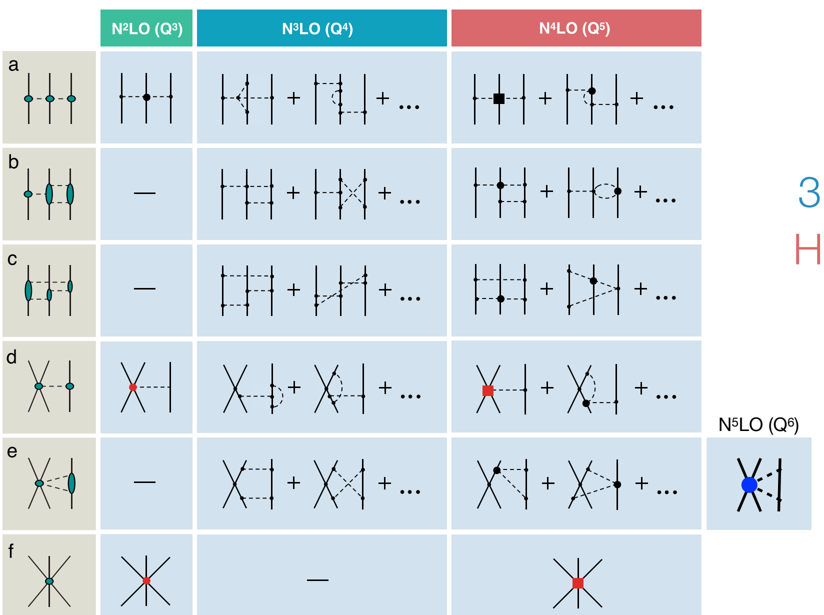


Triple enhancement at N²LO: Factor of π, large numerical factor + large c_i's… Breakdown of χEFT?

$$V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} \mu d\mu \, \frac{\rho(\mu)}{q^2+\mu^2} + \ldots = \frac{2}{\pi}\int_{2M_\pi}^{\Lambda_b} \mu d\mu \, \frac{\rho(\mu)}{q^2+\mu^2} + \underbrace{\frac{2}{\pi}\int_{\Lambda_b}^{\infty} \mu d\mu \, \frac{\rho(\mu)}{q^2+\mu^2} + \ldots}_{\textit{purely short-range contributions}}$$

⇒ the ***long-range*** part of the TPEP enhancement improves agreement with the data

Numerical estimations and comparison with lower orders



3NF contributions have been worked out to N⁴LO.

How does the size of the „new type“ 3NF compare to that of lower-order contributions?

Cirigliano, Dawid, Dekens, Reddy, PRL 135 (25)

— Ishikawa, Robilotta, PRC 76 (07)

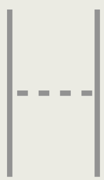
— Bernard, EE, Krebs, Meißner, PRC 77 (08); PRC 84 (11); Krebs, Gasparyan, EE, PRC 85 (12); PRC 87 (13); EE, Gasparyan, Krebs, Shat, EPJA 51 (15)

— Girlanda, Kievsky, Viviani, PRC 84 (11); PRC 102 (20)

Induced short-range 3NF contributions

Idea: Look at the short-range 3NF contributions induced by the parameter-free π-exchange 3NFs

Consider, for example, the one-pion exchange NN potential:



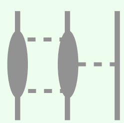
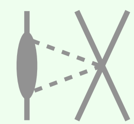
$$V_{1\pi}(\vec q) = -\frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{\vec\sigma_1\cdot\vec q\,\vec\sigma_2\cdot\vec q}{q^2+M_\pi^2}$$

$$\vec\sigma_1\cdot\hat r\,\vec\sigma_2\cdot\hat r-\frac{1}{3}\vec\sigma_1\cdot\vec\sigma_2$$

$$V_{1\pi}(\vec r) = \frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,M_\pi^2\,\frac{e^{-M_\pi r}}{4\pi r}\left[S_{12}(\hat r)\left(1+\frac{3}{M_\pi r}+\frac{3}{(M_\pi r)^2}\right)+\frac{1}{3}\vec\sigma_1\cdot\vec\sigma_2\right]$$
$$-\frac{g_A^2}{12F_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,\vec\sigma_1\cdot\vec\sigma_2\,\delta(\vec r)$$

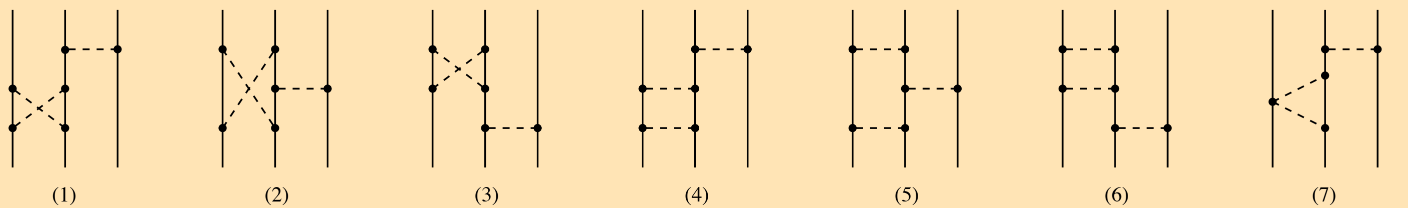
The induced short-range terms in p-space can be read out by writing $\vec\sigma_1\cdot\vec q\vec\sigma_2\cdot\vec q$ in terms of $S_{12}(\hat q)$:

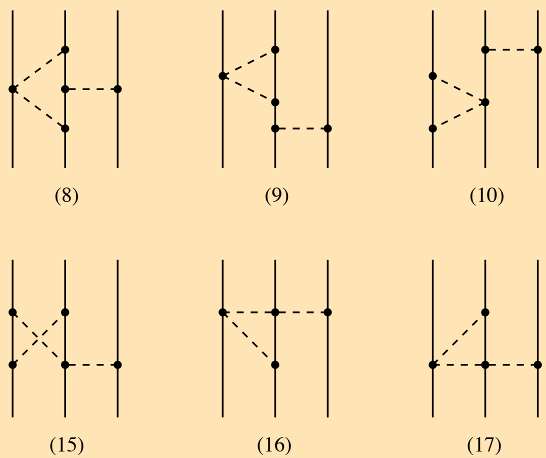
$$V_{1\pi}(\vec q) = -\frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(\frac{q^2S_{12}(\hat q)}{q^2+M_\pi^2}-\frac{1}{3}\vec\sigma_1\cdot\vec\sigma_2\frac{M_\pi^2}{q^2+M_\pi^2}+\frac{1}{3}\vec\sigma_1\cdot\vec\sigma_2\right)$$

Similarly,  induces short-range contributions of the type 

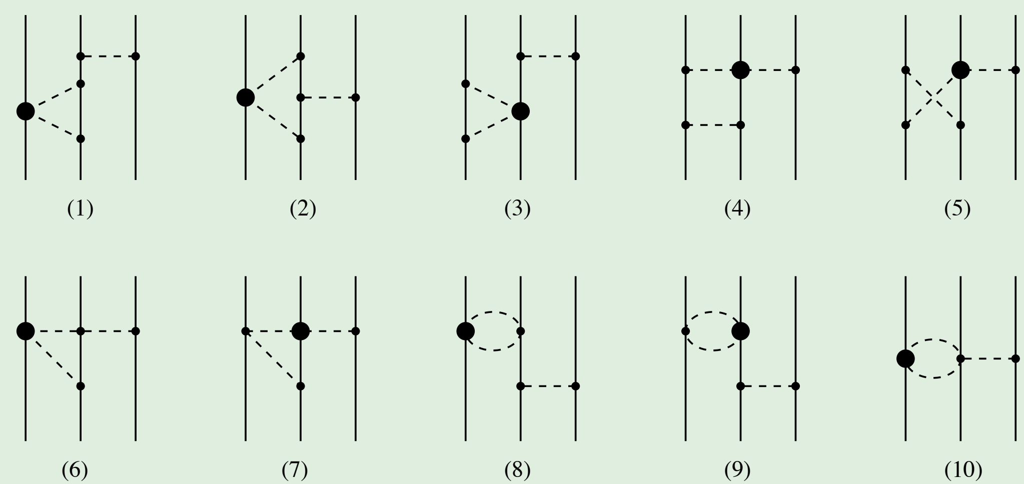
Induced short-range 3NF contributions

Bernard, EE, Krebs, Meißner, PRC 77 (08)





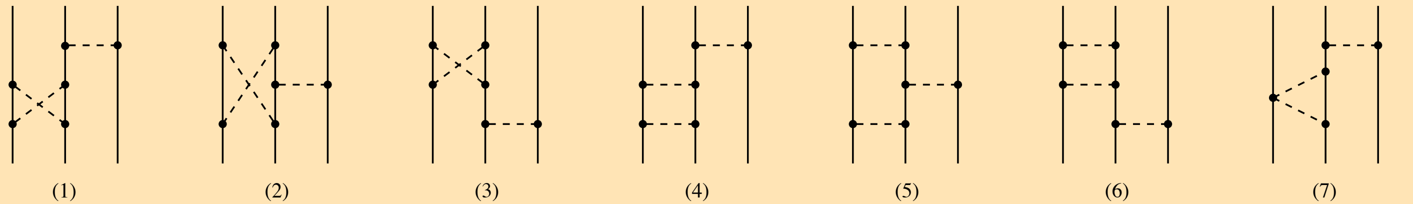
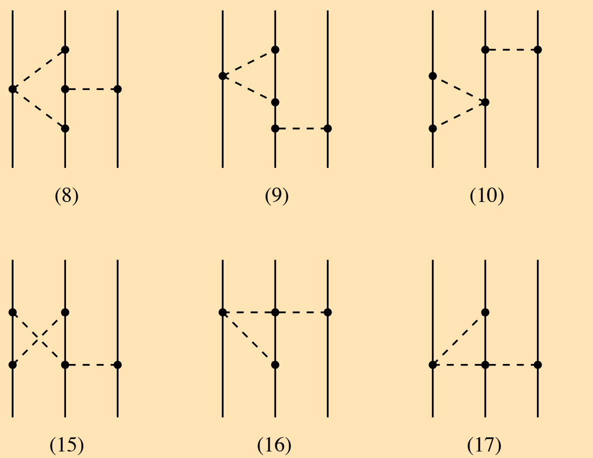
Krebs, Gasparyan, EE, PRC 87 (13)



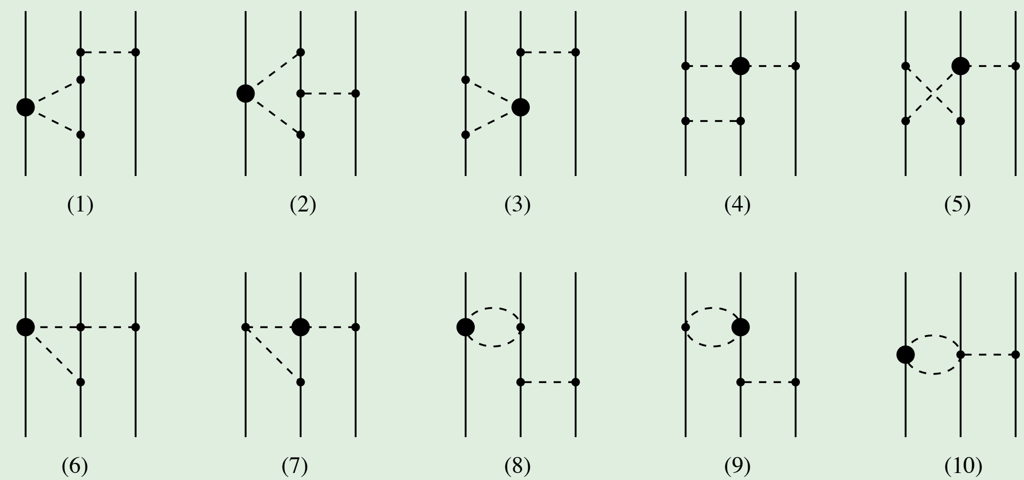
$$
\begin{aligned}
V_{2\pi\text{-}1\pi} \;=\; \frac{\vec{\sigma}_3 \cdot \vec{q}_3}{q_3^2 + M_\pi^2} \Big[& \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \; [\vec{\sigma}_2 \cdot \vec{q}_1 \; \vec{q}_1 \cdot \vec{q}_3 \; F_1(q_1) + \vec{\sigma}_2 \cdot \vec{q}_1 \; F_2(q_1) + \vec{\sigma}_2 \cdot \vec{q}_3 \; F_3(q_1)] + \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \; [\vec{\sigma}_1 \cdot \vec{q}_1 \; \vec{q}_1 \cdot \vec{q}_3 \; F_4(q_1) \\
& + \vec{\sigma}_1 \cdot \vec{q}_3 \; F_5(q_1) + \vec{\sigma}_2 \cdot \vec{q}_1 \; \vec{q}_1 \cdot \vec{q}_3 \; F_6(q_1) + \vec{\sigma}_2 \cdot \vec{q}_1 \; F_7(q_1) + \vec{\sigma}_2 \cdot \vec{q}_3 \; \vec{q}_1 \cdot \vec{q}_3 \; F_8(q_1) + \vec{\sigma}_2 \cdot \vec{q}_3 \; F_9(q_1)] \\
& + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, [\vec{\sigma}_1 \times \vec{\sigma}_2 \cdot \vec{q}_1 \; (\vec{q}_1 \cdot \vec{q}_3 \; F_{10}(q_1) + F_{11}(q_1)) + \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_1 \; \vec{q}_1 \cdot \vec{\sigma}_2 \; F_{12}(q_1)] \Big]
\end{aligned}
$$

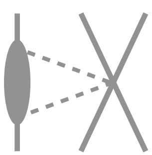
Induced short-range 3NF contributions

Bernard, EE, Krebs, Meißner, PRC 77 (08)

Krebs, Gasparyan, EE, PRC 87 (13)





The induced contributions can be written as:

$$V_{\text{induced}} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_2 \cdot \vec{\sigma}_3 \, v_1(q_1) + \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3 \, v_2(q_1) + \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_2 \cdot \vec{\sigma}_3 \, v_3(q_1) + \text{permutations}$$

Induced short-range 3NF contributions

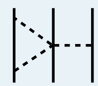
N³LO (Q^4):

$$\boxed{v_1^{(4)}(q) = -\frac{g_A^4}{768\pi F_\pi^6}\left[(8g_A^2-4)M_\pi^2 + (3g_A^2-1)q^2\right]A(q)}$$

$$v_2^{(4)}(q) = \frac{g_A^6}{384\pi F_\pi^6} q^2 A(q)$$

enhancement by a factor of π

$$\boxed{v_3^{(4)}(q) = \frac{g_A^4}{384\pi F_\pi^6}(2M_\pi^2 + q^2)A(q)}$$



Clearly, the convergence is not ideal…

E.g., $\displaystyle \frac{v_{D_2}^{(6)}(q)}{v_3^{(4)}(q)} = \frac{9}{2g_A^2} D_2 F_\pi^2 M_\pi^2 \sim 0.3$

instead of ~ 0.1.

The strongest enhancement at N⁴LO:

$$\frac{v_3^{(5)}(0)}{v_1^{(4)}(0)} = \frac{8(6c_1 - c_2 - 5c_3)M_\pi}{(2g_A^2 - 1)\pi} \sim 2.6$$

N⁴LO (Q^5):

$$v_1^{(5)}(q) = -\frac{g_A^2 c_4}{144\pi^2 F_\pi^6}\frac{L(q)}{q^2+4M_\pi^2}\left[4(4g_A-1)M_\pi^4 + (17g_A^2-5)M_\pi^2 q^2 + (4g_A^2-1)q^4\right]$$

$$v_2^{(5)}(q) = \frac{g_A^4 c_4}{48\pi^2 F_\pi^6} q^2 L(q)$$

enhancement by large numerical coefficients + large c_i's

$$\boxed{v_3^{(5)}(q) = \frac{g_A^4}{384\pi^2 F_\pi^6}\frac{L(q)}{q^2+4M_\pi^2}\left[32c_1 M_\pi^2(3M_\pi^2+q^2) - c_2(16M_\pi^4 + 16M_\pi^2 q^2 + 3q^4) - c_3(80M_\pi^4 + 68M_\pi^2 q^2 + 13q^4)\right]}$$

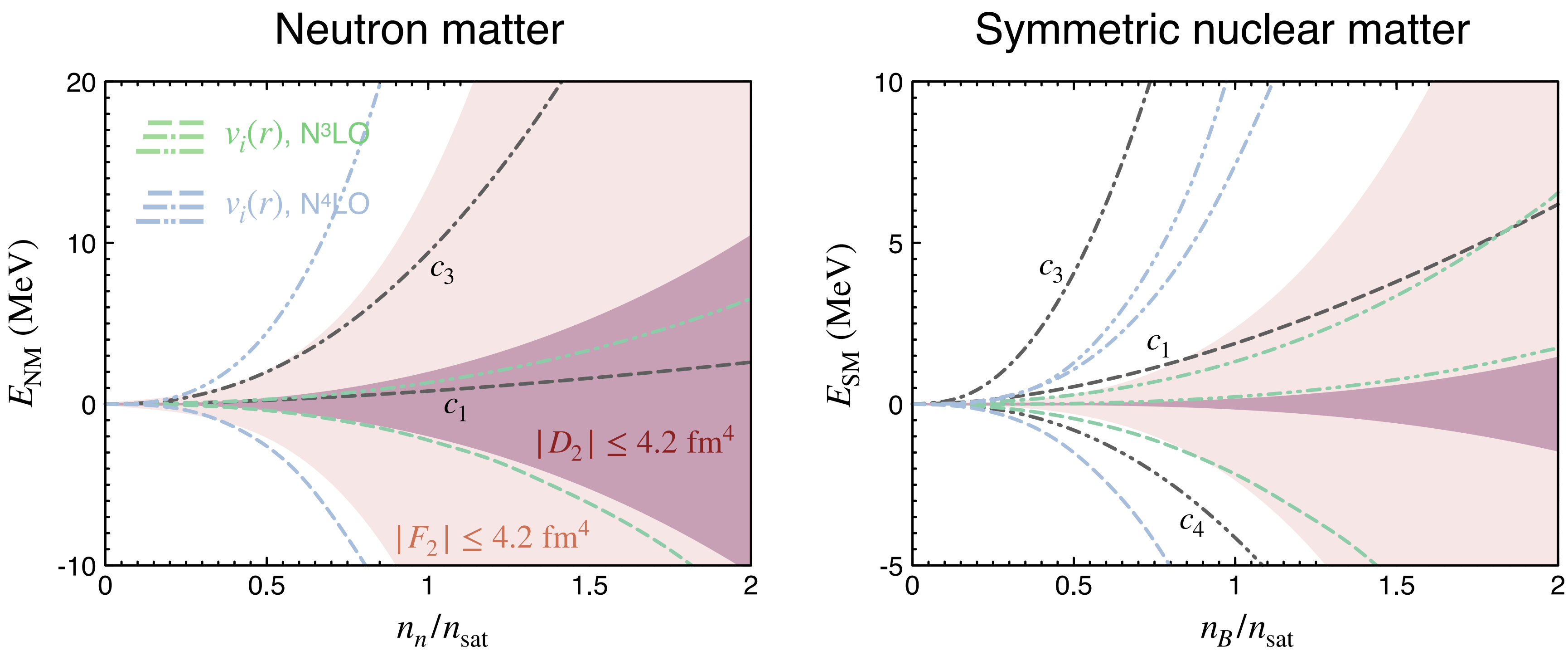
N⁵LO (Q^6):

$$V^{(6)}_{\text{Cirigliano et al.}} = (1 - \vec\sigma_2\cdot\vec\sigma_3)\left[v_{D_2}^{(6)}(q_1) + v_{F_2}^{(6)}(q_1)\right] + \text{permutations}$$

$$\boxed{v_{D_2}^{(6)}(q) = \frac{3g_A^2 D_2 M_\pi^2}{256\pi F_\pi^4}(2M_\pi^2+q^2)\,A(q),}\qquad v_{F_2}^{(6)}(q) = \frac{3g_A^2 F_2}{512\pi F_\pi^4}(2M_\pi^2+q^2)^2\,A(q)$$

enhancement by a factor of π

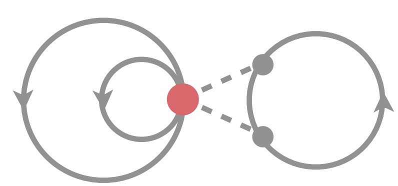
Neutron and symmetric nuclear matter



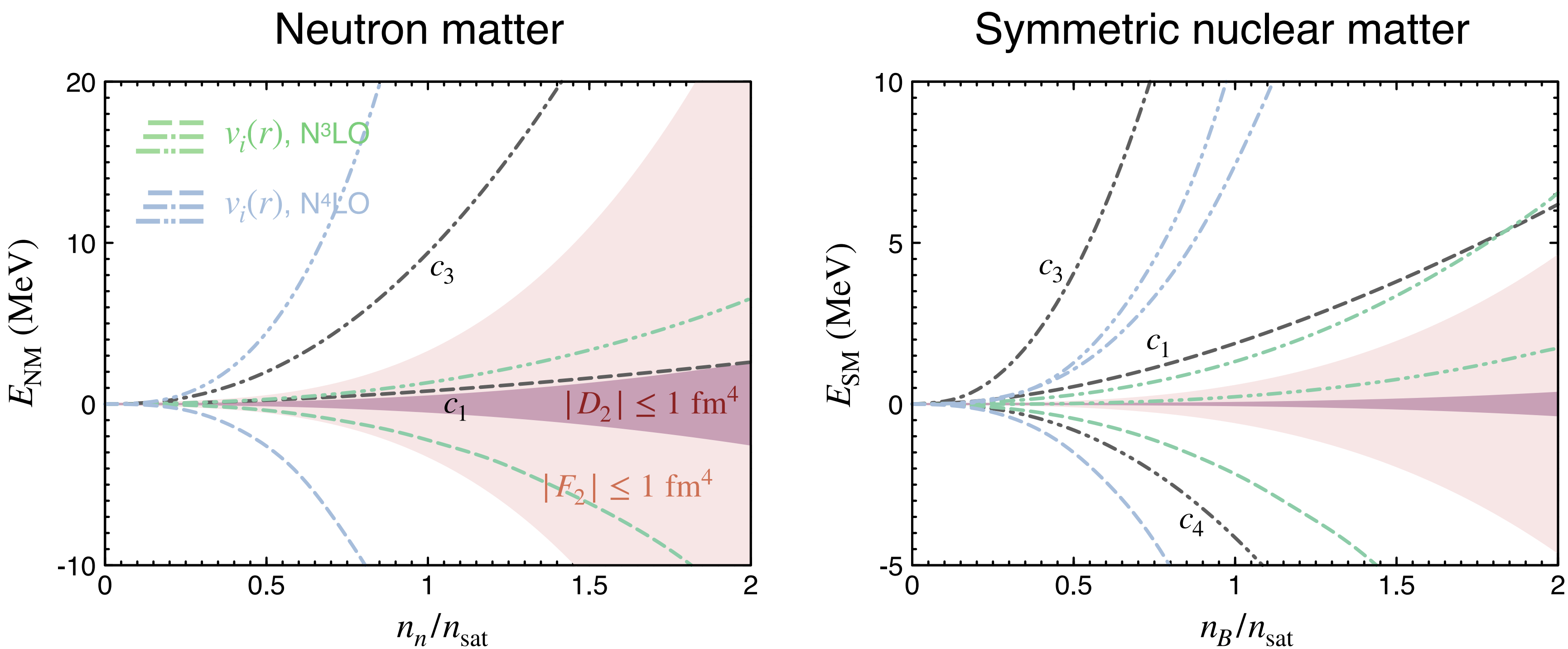
We start with reproducing the results of Cirigliano et al., extended by the induced 2π-contact contributions v_1, v_2, v_3 at N3LO, N4LO, i.e.:

— using Cirigliano et al. values for D_2, F_2

— without imposing a regulator

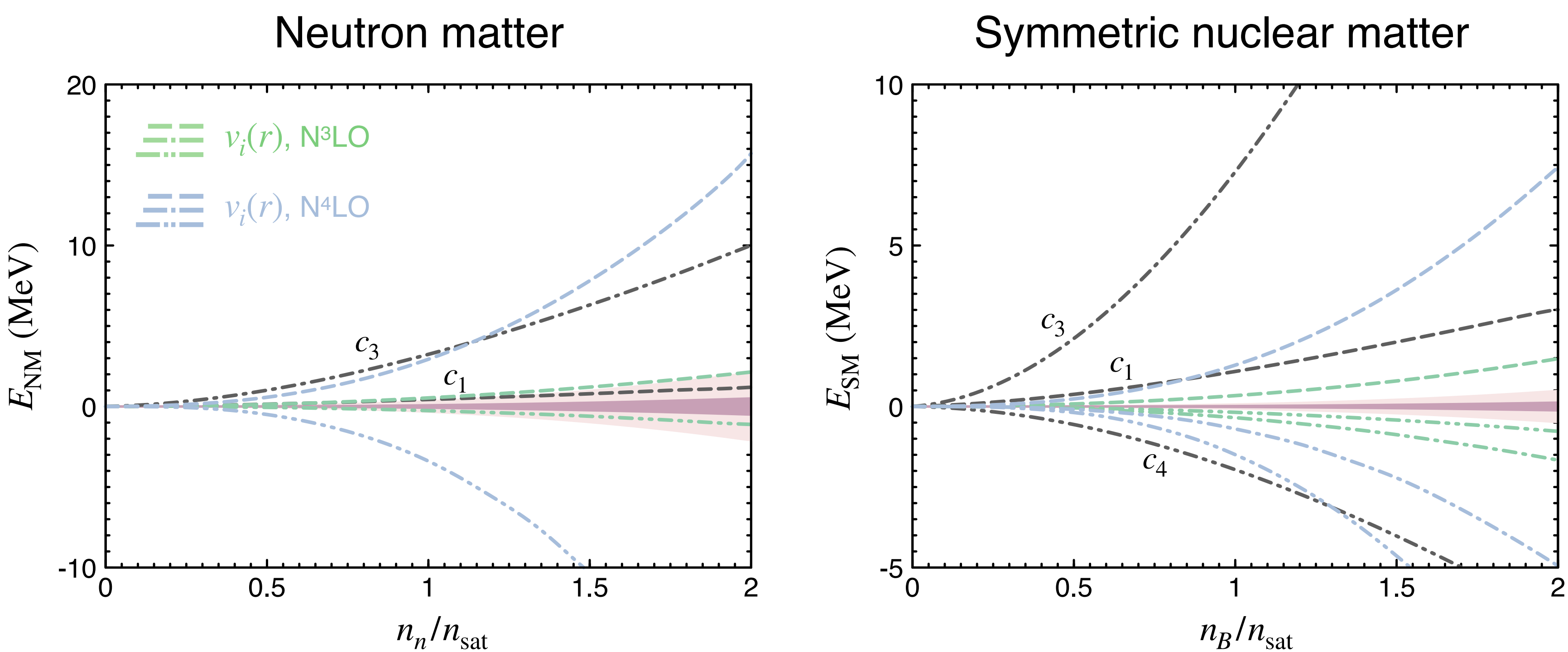
$\langle V_{3N} \rangle_{\text{Fermi gas}} =$  $+ \ldots$

Neutron and symmetric nuclear matter



Using realistic values for the LECs D_2, F_2 and Dimensional Regularization

Neutron and symmetric nuclear matter



Now, realistic values for the LECs D_2, F_2 + SMS regulator with $\Lambda = 500$ MeV:

N2LO (c_i's): $$\frac{1}{(q_1^2+M_\pi^2)(q_3^2+M_\pi^2)} \;\to\; \frac{e^{-\frac{q_1^2+M_\pi^2}{\Lambda^2}}\, e^{-\frac{q_3^2+M_\pi^2}{\Lambda^2}}}{(q_1^2+M_\pi^2)(q_3^2+M_\pi^2)}$$

N3LO: $$v_i(q) \;\to\; -\frac{2q^2}{\pi}\int_{2M_\pi}^{\infty}\frac{d\mu}{\mu}\,\frac{\rho_i(\mu)}{q^2+\mu^2}\,e^{-\frac{q^2+M_\pi^2}{2\Lambda^2}}$$

N4,5LO: $$v_i(q) \;\to\; \frac{2q^4}{\pi}\int_{2M_\pi}^{\infty}\frac{d\mu}{\mu^3}\,\frac{\rho_i(\mu)}{q^2+\mu^2}\,e^{-\frac{q^2+M_\pi^2}{2\Lambda^2}}$$

Remember: these are still only rough order-of-magnitude estimations…

Summary and conclusions

- No justification for promoting the D_2, F_2 3NFs to N^{3}LO from RG arguments.

- The scaling and estimation of D_2, F_2 by Cirigliano et al. correspond to the KSW power counting. In this scheme, short-range forces (including the D_2, F_2-ones) are indeed enhanced relative to long-range ones. But known to converge poorly…

- Assuming natural-sized D_2, F_2, their (regularized) contributions to the EoS appear to be rather small $\Rightarrow$ no reason to worry about these particular N^{5}LO diagrams.

- F_2 receives contributions from the Δ and may be potentially enhanced. A systematic way to account for the enhancement is ChEFT with explicit Δ DoF. The leading-loop (N^{3}LO) Δ-contributions to the 2π 3NF are derived in Krebs, Gasparyan, EE, PRC 98 (18).

- The convergence of the chiral expansion for the long-range 3NFs is known to be far from ideal Krebs, Gasparyan, EE, PRC 85 (12), 87 (13); EE et al., EPJA 51 (15) — very similar to the long-range NN force. The parameter-free ChEFT predictions for the 2π-exchange NN potential have been unambiguously verified in NN scattering EE et al., EPJA 51 (15); PRL 115 (15); Reinert et al., EPJA 54 (18).