TRIUMF

Pinning the subleading three nucleon contact interaction to light nuclei

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TRIUMF

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SMEFT meets ChEFT

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Discovery, accelerated

Outline

- Introduction
  - “Why any/more many-nucleon forces?”
  - The nuclear toolkit
- Leading $(\mathrm{N^2LO})$ and subleading $(\mathrm{N^4LO})$ 3-nucleon contacts
  - Matrix elements and conclusions
  - The question of tuning
  - Effects on ^{3}H
- *Summary & Outlook*

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Some history

All realistic *NN* forces underbind the triton [7], and small differences among them can be traced to nonlocalities.

[*J.L. Friar et al, PRC (1999)*]

Overbound nuclear matter (wrong saturation density)

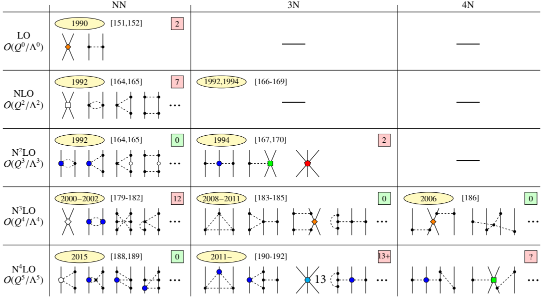
[*A. Akmal et al, Phys. Rev. C **58** (1998)*]

- *Fujita-Miyazawa*: One of the earliest (1957)
- *UIX*: starting from Fujita-Miyazawa
- *Tucson-Melbourne* / *Brazil*

⋮

- Chiral 3NFs: $\mathrm{N^2LO}$ (1994), $\mathrm{N^3LO}$ ($\sim$ 2010), $\mathrm{N^4LO}$ (2011).

N4LO three-nucleon contacts



Adapted from [K. Hebeler, Phys. Rept. (2021)]

3NF LECs

$$\mathbf{N^2LO}\ \ \#2\ :\ \ c_D, c_E$$

$$\mathbf{N^3LO}\ \ \#0$$

$$\mathbf{N^4LO}\ \ \#13\ :\ \ E_i$$

[*Girlanda et al, PRC (2011)*]

Discovery, accelerated

Outstanding problems

- $\mathrm{A_y}$ ***puzzle***: long-standing underestimation of the polarization asymmetry in N-d scattering by $30\ \%$.

  Known since 1990s: lack of spin-orbit 3NFs.

- ***Space-star anomaly***: similar discrepancy in the break-up channel of N-d.

2NFs + modifications offer no resolution

3NFs up to N3LO offer no resolution

[*J. Golak et al, EPJA (2014)*]

[*H. Witala et al, PRC (2021)*]

[*L. Girlanda et al, PRC (2019)*] +

Prospects

- **Scattering exp.**: d-p, p-^{3}He, etc. scattering can provide high quality data.

  [*K. Sekiguchi, FBS (2024)*]

  Fit 15 LECs for 3NF like the 21 LECs for 2NF (at $\mathrm{N^4LO}$).

  This will require controlled regulator effects, etc.

Tools for the nuclear many body problem

Phenomenology

HF, HFB, Fermi liquid theory, etc

- Easier to implement (Simpler models)
- Qualitatively correct
- Uncontrolled approximations: hard to systematically improve

Ab initio

***NCSM**, (VS)IMSRG, QMC, (B)MBPT, (B)CC, etc*

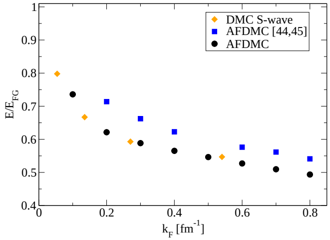
- No extra assumptions: clear connection with nuclear interaction
- Quantitatively correct: systematically improved (uncertainty est.)
- Computationally expensive (only smallish N is feasible)

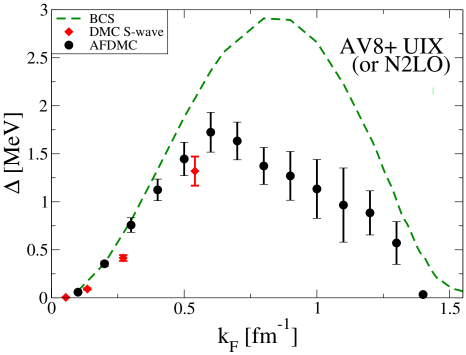
QMC for superfluid neutron matter

Diffusion Monte Carlo:

$$\psi(\tau) = e^{-(H-E_0)\tau}\psi_T \to c_0\psi_0 \;, \tau \to \infty$$

starting from a "physics aware" (i.e., $c_0 \neq 0$) trial state ψ_T





[***GP***, *Diakonos, and Gezerlis, PRC (2020)*]

[***GP***, *Gezerlis, Condens. Matter (2021)*]

[*Gandolfi,* ***GP****, Carlson, Gezerlis, Schmidt, Universe (2022)*]

Discovery, accelerated

Outline

- Introduction
  - "Why any/more many-nucleon forces?"
  - Subleading three-nucleon forces at N3LO and N4LO
- Leading $(\mathrm{N}^2\mathrm{LO})$ and subleading $(\mathrm{N}^4\mathrm{LO})$ 3-nucleon contacts
  - Partial wave decomposition
  - Implementation in the No-Core Shell Model
  - Effects on ^{3}H
- *Summary & Outlook*

Leading ($\mathrm{N}^2\mathrm{LO}$) three-nucleon contacts

There are 2 contacts: E (pure contact) and D (1-π exch. +contact.)

$$V_E = -\sum_{jk} E\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$V_D = -\sum_{ijk} \frac{g_A}{8f_\pi^2} D \frac{\boldsymbol{\sigma_j} \cdot \mathbf{q}_j}{\mathbf{q}_j^2 + M_\pi^2} (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)(\boldsymbol{\sigma}_i \cdot \mathbf{q}_j)$$

Tuned to binding energy ^{3}H and ^{4}He, or properties of light nucl., or GT ME of ^{3}H, or a_{nd} etc.

[*Nogga, et al, PRC (2006)*] [*Navràtil, et al, PRL (2007)*] [*Gazit, et al, PRL (2009)*] [*Epelbaum, et al, PRC (2002)*]

There are also c_1,c_3, and c_4 (2-π exch.)

$$V_{1,3,4} = \sum_{ijk} \frac{1}{2} \left(\frac{g_A}{2f_\pi}\right) \frac{(\boldsymbol{\sigma}_i \cdot \mathbf{q}_i)(\boldsymbol{\sigma}_j \cdot \mathbf{q}_j)}{(\mathbf{q}_i^2 + M_\pi^2)(\mathbf{q}_j^2 + M_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta$$

where

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta}\left[-\frac{4c_1 M_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2}\mathbf{q}_i \cdot \mathbf{q}_j\right] + \sum_\gamma \frac{c_4}{f_\pi^2} \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \boldsymbol{\sigma}_k \left[\mathbf{q}_i \times \mathbf{q}_j\right]$$

(c_1,c_3, and c_4 enter the subleading 2-π exch. in the 2-nucleon potential.)

Subleading ($\mathrm{N}^4\mathrm{LO}$) three-nucleon contacts

Thirteen new contacts at $\mathrm{N}^4\mathrm{LO}$: ($\mathbf{q}_i = \mathbf{p}_i - \mathbf{p}_i'$):

$$O_1 = \sum_{i\neq j\neq k} \left(-q_i^2\right) \,, \quad O_2 = \sum_{i\neq j\neq k} \left(-q_i^2\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \,, \quad O_3 = \sum_{i\neq j\neq k} \left(-q_i^2\right) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \,,$$

$$O_4 = \sum_{i\neq j\neq k} \left(-q_i^2\right) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$O_5 = \sum_{i\neq j\neq k} \left(-3\mathbf{q}_i \cdot \boldsymbol{\sigma}_i \mathbf{q}_i \cdot \boldsymbol{\sigma}_j + q_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right) \,, \quad O_6 = \sum_{i\neq j\neq k} \left(-3\mathbf{q}_i \cdot \boldsymbol{\sigma}_i \mathbf{q}_i \cdot \boldsymbol{\sigma}_j + k_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$O_7 = \sum_{i\neq j\neq k} \left(-\frac{i}{4}\mathbf{q}_i\right) \times (\mathbf{p}_i - \mathbf{p}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \,, \quad O_8 = \sum_{i\neq j\neq k} \left(-\frac{i}{4}\mathbf{q}_i\right) \times (\mathbf{p}_i - \mathbf{p}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$O_9 = \sum_{i\neq j\neq k} (-\mathbf{q}_i \cdot \boldsymbol{\sigma}_i \mathbf{q}_j \cdot \boldsymbol{\sigma}_j) \,, \quad O_{10} = \sum_{i\neq j\neq k} (-\mathbf{q}_i \cdot \boldsymbol{\sigma}_i \mathbf{q}_j \cdot \boldsymbol{\sigma}_j) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$O_{11} = \sum_{i\neq j\neq k} (-\mathbf{q}_i \cdot \boldsymbol{\sigma}_j \mathbf{q}_j \cdot \boldsymbol{\sigma}_i) \,, \quad O_{12} = \sum_{i\neq j\neq k} (-\mathbf{q}_i \cdot \boldsymbol{\sigma}_j \mathbf{q}_j \cdot \boldsymbol{\sigma}_i) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$O_{13} = \sum_{i\neq j\neq k} (-\mathbf{q}_i \cdot \boldsymbol{\sigma}_j \mathbf{q}_j \cdot \boldsymbol{\sigma}_i) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k \,.$$

[*Girlanda et al, PRC* ***84*** *(2011)*]

N^{4}LO three-nucleon contacts: Regulators

With a regulator

$$O \quad \rightarrow \quad F(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \Lambda) O F(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}'_3; \Lambda)$$

Choices

$$\text{non-local regulator}: \quad F(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \Lambda) = \exp\left[-\frac{1}{4}\left(\pi_1^2 + \pi_2^2\right)^2/\Lambda^4\right]$$

$$\text{local regulator}: \quad F(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \Lambda) = \prod_i \exp\left(-q_i^4/\Lambda^4\right)$$

$$\vdots$$

$\mathrm{N^4LO}$ three-nucleon contacts

And in coordinate space

$$
\begin{aligned}
V_{\text{cont}}^{(\text{N4LO})} = \sum_{i\neq j\neq k} \Bigg\{ & (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)\, S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k)\, (\mathbf{L}\cdot\mathbf{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
& + (E_9 + E_{10} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k)\, \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \\
& + (E_{11} + E_{12} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)\, \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \Bigg\}
\end{aligned}
$$

where

$$
Z_0(r) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} F(\mathbf{k}^2;\Lambda)
$$

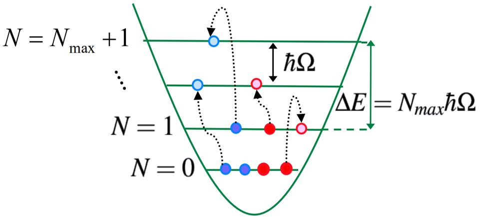
No Core Shell Model

Solution via a systematic expansion on A-body harmonic oscillator states.

Characterized by the HO frequency ($\hbar\Omega$, energy spacing of excitations) and number of excitations ($N_{\rm max}$).

[*Navratil et al, Phys.Rev.C* ***61*** *(2000)*]

[*Barrett et al. PPNP* ***69*** *(2013)*]



$$\left|\Psi_A^{J^\pi T}\right\rangle = \sum_{N=0}^{N_{\rm max}} \sum_n c_{Nn}^{J^\pi T} \left|\Phi_{Nn}^{J^\pi T}\right\rangle$$

The basis for PW decomposition

The Jj-coupled three-body partial wave basis: [*W. Glöckle (1983)* +]

$$|N\alpha JT\rangle = |N[(ls)j(\mathcal{L}\mathcal{S})\mathcal{J}]JT(t\tau)\rangle$$

$$|N\alpha JT\rangle = |N[(l_{ij}s_{ij})j_{ij}(\mathcal{L}_k\mathcal{S}_k)\mathcal{J}_k]JT(t_{ij}\tau_k)\rangle \ , \quad \text{antisymm. in } (ij)$$

Channels specified by $J, T, \mathcal{P} = (-1)^{l+\mathcal{L}}$ and $\alpha = (l, s, j\mathcal{L}, \mathcal{J}, t)$.

N4LO 3NF: Partial Waves

$$O_1 = \sum_{i \neq j \neq k} \left(-k_i^2\right) \ , \quad O_2 = \sum_{i \neq j \neq k} \left(-k_i^2\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \ ,$$

$$V_{1,2} = \sum_{i \neq j \neq k} \left(E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\right) \left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik})$$

Similar to c_E. Partial wave decomposition

$$\begin{aligned}
E_1 W_1 + E_2 W_2 = \delta_{ss'} & \left[E_1 \delta_{tt'} + E_2 \hat{t}\hat{t}' (-1)^{t+t'+T+\frac{1}{2}} \begin{Bmatrix} t & t' & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} t & t' & 1 \\ \frac{1}{2} & \frac{1}{2} & T \end{Bmatrix}\right] \times \\
& \times \hat{j}\hat{j}' \hat{\mathcal{J}}\hat{\mathcal{J}}' \hat{l}' \hat{\mathcal{L}}' (-1)^{J-\frac{1}{2}\mathcal{J}'-\mathcal{J}+l+\mathcal{L}+s} \times \\
& \times \sum_X (-1)^X \hat{X}^2 \begin{Bmatrix} l & s & j \\ j' & X & l' \end{Bmatrix} \begin{Bmatrix} X & j & j' \\ J & \mathcal{J}' & \mathcal{J} \end{Bmatrix} \begin{Bmatrix} \mathcal{L}' & 1/2 & \mathcal{J}' \\ \mathcal{J} & X & \mathcal{L} \end{Bmatrix} C^{l'Xl}_{000} C^{\mathcal{L}'X\mathcal{L}}_{000} \times \\
& \times \int d\xi_{12} \xi_1^2 \xi_2^2 R_{nl}(\xi_1, b) R_{\mathcal{N}\mathcal{L}}(\xi_2, b) R_{n'l'}(\xi_1, b) R_{\mathcal{N}'\mathcal{L}'}(\xi_2, b) \\
& \times Z_0^{(2)}\left(\sqrt{2}\xi_1; \Lambda\right) Z_{0,X}\left(\sqrt{\frac{1}{2}}\xi_1, \sqrt{\frac{3}{2}}\xi_2; \Lambda\right) .
\end{aligned}$$

N4LO 3NF: Partial Waves

$$O_3 + O_5 = \tilde{O}_5 = \sum_{i\neq j\neq k} \left(-3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i \mathbf{k}_i\cdot\boldsymbol{\sigma}_j\right)\ , \quad O_4 + O_6 = \tilde{O}_6 = \sum_{i\neq j\neq k} \left(-3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i \mathbf{k}_i\cdot\boldsymbol{\sigma}_j\right)\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\ .$$

Similar to c_D. Partial wave decomposition

$$E_5\tilde{W}_5 + E_6\tilde{W}_6 = -36\delta_{ss'}\left[E_5\delta_{tt'} + E_6 3\hat{t}\hat{t}'(-1)^{t+t'+T+\frac{1}{2}}\begin{Bmatrix} t & t' & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{Bmatrix}\begin{Bmatrix} t & t' & 1 \\ \frac{1}{2} & \frac{1}{2} & T\end{Bmatrix}\right]\hat{j}\hat{j}'\hat{\mathcal{J}}\hat{\mathcal{J}}'\hat{s}\hat{s}'$$
$$\times(-1)^{J-\mathcal{J}+s+j'}\begin{Bmatrix} s & s' & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{Bmatrix}\hat{l}'\hat{\mathcal{L}}'\sum_{K=0,2}\hat{K}(-1)^{K/2}\sum_{VX}(-1)^{V}\hat{V}\hat{X}^2 C^{11K}_{000}C^{Vl'l}_{000}C^{XKV}_{000}C^{X\mathcal{L}'\mathcal{L}}_{000}$$
$$\times\sum_{Z}\hat{Z}^2\begin{Bmatrix} l & s & j \\ l' & s' & j' \\ V & 1 & Z\end{Bmatrix}\begin{Bmatrix} \mathcal{L} & \frac{1}{2} & \mathcal{J} \\ \mathcal{L}' & \frac{1}{2} & \mathcal{J}' \\ X & 1 & Z\end{Bmatrix}\begin{Bmatrix} j & j' & Z \\ \mathcal{J}' & \mathcal{J} & J\end{Bmatrix}\begin{Bmatrix} V & 1 & Z \\ 1 & X & K\end{Bmatrix}$$
$$\times\int d\xi_1 2\xi_1^2\xi_2^2 R_{nl}(\xi_1,b)R_{\mathcal{N}\mathcal{L}}(\xi_2,b)R_{n'l'}(\xi_1,b)R_{\mathcal{N}'\mathcal{L}'}(\xi_2,b)f_K\left(\sqrt{2}\xi_1;\Lambda\right)Z_{0,X}\left(\sqrt{\frac{1}{2}}\xi_1,\sqrt{\frac{3}{2}}\xi_2;\Lambda\right)$$

N4LO 3NF: Partial Waves

$$O_9 = \sum_{i\neq j\neq k} \left(-\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j\right) \;, \quad O_{10} = \sum_{i\neq j\neq k} \left(-\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

Similar to c_1. Partial wave decomposition

$$E_9 W_9 + E_{10} W_{10} = -6\left[E_5 \delta_{tt'} + E_6 \left\langle \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \right\rangle\right] (-1)^{s+j'+J-\mathcal{J}} \hat{l}' \hat{\mathcal{L}}' \hat{s} \hat{s}' \hat{j} \hat{j}' \hat{\mathcal{J}} \hat{\mathcal{J}}' \begin{Bmatrix} 1 & s & s' \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix}$$

$$\times \sum_{XVRY} \sum_{K_3=0}^{1} (-1)^Y \hat{X}^2 \hat{V} \hat{R} \hat{Y} \widehat{1-K_3} \left[\binom{3}{2K_3}\right]^{\frac{1}{2}} C^{XK_3Y}_{000} C^{X1-K_3R}_{000} C^{Y1V}_{000} C^{l'Vl}_{000} C^{\mathcal{L}'R\mathcal{L}}_{000} \times$$

$$\times \begin{Bmatrix} Y & X & K_3 \\ 1-K_3 & 1 & R \end{Bmatrix} \begin{Bmatrix} Y & j' & j \\ J & \mathcal{J} & \mathcal{J}' \end{Bmatrix} \begin{Bmatrix} j & l & s \\ j' & l' & s' \\ Y & V & 1 \end{Bmatrix} \begin{Bmatrix} \mathcal{J} & \mathcal{L} & \frac{1}{2} \\ \mathcal{J}' & \mathcal{L}' & \frac{1}{2} \\ Y & R & 1 \end{Bmatrix}$$

$$\times \int d\xi_{12} \xi_1^2 \xi_2^2 R_{nl}(\xi_1) R_{\mathcal{N}\mathcal{L}}(\xi_2) R_{n'l'}(\xi_1) R_{\mathcal{N}'\mathcal{L}'}(\xi_2) f_1\left(\sqrt{2}\xi_1\right) \left(\sqrt{\frac{1}{2}}\xi_1\right)^{K_3} \left(\sqrt{\frac{3}{2}}\xi_2\right)^{1-K_3}$$

$$\times f_{1,X}\left(\sqrt{\frac{1}{2}}\xi_1, \sqrt{\frac{3}{2}}\xi_2; \Lambda\right)$$

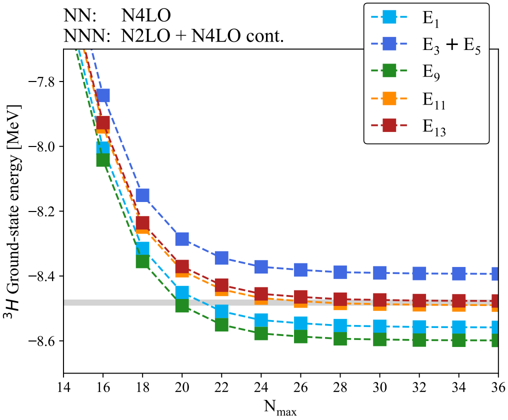
N4LO 3NF: Partial Waves

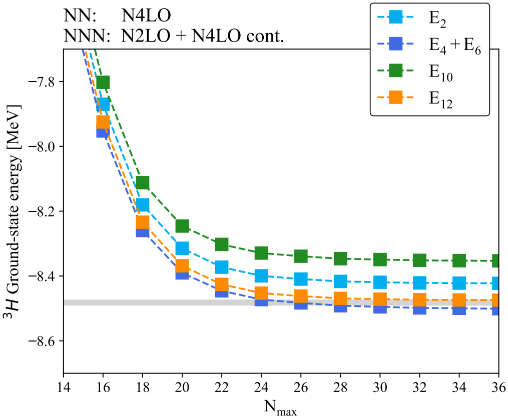
$$O_{11} = \sum_{i \neq j \neq k} \left(-\mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i\right) \,, \quad O_{12} = \sum_{i \neq j \neq k} \left(-\mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \,, \quad O_{13} = \sum_{i \neq j \neq k} \left(-\mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k$$

Partial wave decomposition

$$\begin{aligned}
E_{11}W_{11} + E_{12}W_{12} + E_{13}W_{13} &= -6\left[E_{11}\delta_{tt'} + E_{12}\left\langle \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \right\rangle + E_{13}\left\langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \right\rangle\right] \\
&\times \begin{Bmatrix} 1 & s & s' \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \hat{l}'\hat{\mathcal{L}}'\hat{s}\hat{s}'\hat{j}\hat{j}'\hat{\mathcal{J}}\hat{\mathcal{J}}'(-1)^{s+j'+J-\mathcal{J}} \sum_{VR} \hat{V}\hat{R} C^{l'Vl}_{000} C^{\mathcal{L}'R\mathcal{L}}_{000} \sum_{Y} \hat{Y} C^{Y1V}_{000} \times \\
&\times \sum_{G} (-1)^{G} \hat{G}^2 \begin{Bmatrix} 1 & V & G \\ 1 & R & Y \end{Bmatrix} \begin{Bmatrix} G & j' & j \\ J & \mathcal{J} & \mathcal{J}' \end{Bmatrix} \begin{Bmatrix} j & l & s \\ j' & l' & s' \\ G & V & 1 \end{Bmatrix} \begin{Bmatrix} \mathcal{J} & \mathcal{L} & \frac{1}{2} \\ \mathcal{J}' & \mathcal{L}' & \frac{1}{2} \\ G & R & 1 \end{Bmatrix} \times \\
&\times \sum_{K_3=0}^{1} \left[\binom{3}{2K_3}\right]^{\frac{1}{2}} \widehat{1-K_3} \sum_{X} \hat{X}^2 C^{XK_3Y}_{000} C^{X1-K_3R}_{000} \begin{Bmatrix} Y & X & K_3 \\ 1-K_3 & 1 & R \end{Bmatrix} \times \\
&\times \int d\xi_{12}\xi_1^2\xi_2^2 R_{nl}\left(\xi_1\right) R_{\mathcal{N}\mathcal{L}}\left(\xi_2\right) R_{n'l'}\left(\xi_1\right) R_{\mathcal{N}'\mathcal{L}'}\left(\xi_2\right) f_1\left(\sqrt{2}\xi_1\right) \left(\sqrt{\frac{1}{2}}\xi_1\right)^{K_3} \left(\sqrt{\frac{3}{2}}\xi_2\right)^{1-K_3} \\
&\times f_{1,X}\left(\sqrt{\frac{1}{2}}\xi_1, \sqrt{\frac{3}{2}}\xi_2; \Lambda\right)
\end{aligned}$$

N4LO three-nucleon contacts at ^{3}H





*[**GP** and Navratil, in preparation (2025)].*

$\mathrm{N}^4\mathrm{LO}$ three-nucleon contacts

We won't (over)fit $\mathbf{13}$ new contacts.

We'll take a subset from an isospin basis (separating by system relevance).

$$\sum_{i=1}^{13} E_i O_i = \sum_{T=1/2}^{3/2} \sum_i h_i^{(T)} D_i^{(T)}$$
$$D_i^{(T)} = \sum_j c_{ij}^{(T)} O_j$$

where $h_i^{(T)}$ are new LECs.

Two isospin channels: $D_i^{(1/2)}, D_i^{(3/2)}$, defined from the solutions

$$P_T D_i^{(T)} P_T = D_i^{(T)} \tag{1}$$

where P_T are isospin projection operators.

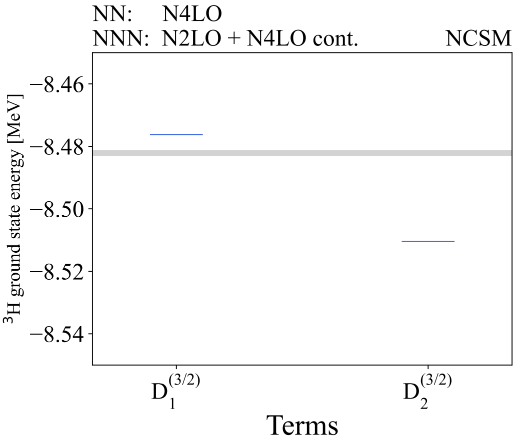
N4LO three-nucleon contacts

$$O_1,\ O_2,\ O_3,\ O_4,\ O_5,\ O_6,\ O_7,\ O_8,\ O_9,\ O_{10},\ O_{11},\ O_{12},\ O_{13} \quad \longrightarrow$$

$$D_1^{(1/2)} = -O_1 + O_2$$
$$D_2^{(1/2)} = -O_7 + O_8$$
$$D_i^{(1/2)} = O_i \ , \quad i = 3,4,5,6$$
$$D_7^{(1/2)} = \frac{1}{2} O_1 + O_7 + O_9$$
$$D_8^{(1/2)} = \frac{1}{2} O_1 - O_7 + O_{10}$$
$$D_9^{(1/2)} = \frac{1}{2} O_1 - O_7 + O_{11}$$
$$D_{10}^{(1/2)} = \frac{1}{2} O_1 - O_7 + O_{12}$$
$$D_{11}^{(1/2)} = \frac{1}{2} O_1 - O_7 + O_{13}$$
$$D_1^{(3/2)} = O_2 - \frac{1}{3}(O_4 + O_6) - (O_3 + O_5) - \frac{3}{2} O_9 - \frac{1}{2} O_{10} + O_{13}$$
$$D_2^{(3/2)} = -2O_2 + 2(O_3 + O_5) + \frac{2}{3}(O_4 + O_6) + 3O_9 + O_{10} + 3O_{11} + O_{12}$$

Originally derived in [*Alessia Nasoni, BSc Thesis*]

Triton ground state



*[**GP** and Navratil, in preparation (2025)].*

$T = 3/2$ terms shouldn't contribute at a $T = 1/2$ ground state

Regulator effects

Typically **one** of the **three** possible terms is taken (rest related by permutations)

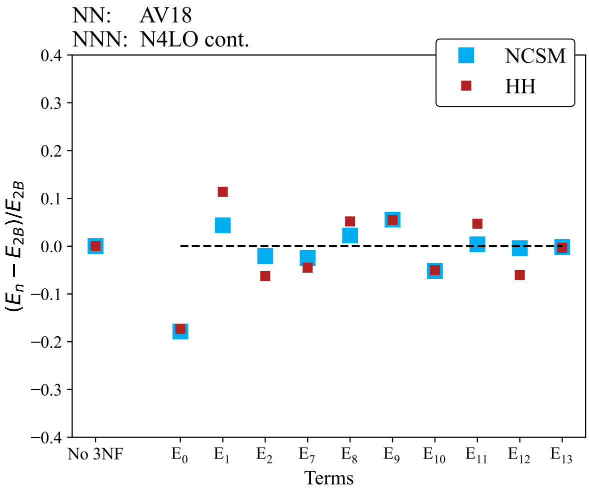
$$\sum_{i\neq j\neq k} W_{ijk} = W_1 + W_2 + W_3$$

For a properly antisymmetrized many-body wavefunction, they are equal so

$$\langle N\alpha JT| \sum_{i\neq j\neq k} W_{ijk} \,|N'\alpha' JT\rangle = 3\langle N\alpha JT|\, W_2 \,|N'\alpha' JT\rangle$$

But the local regulator breaks this.

Comparison with HH

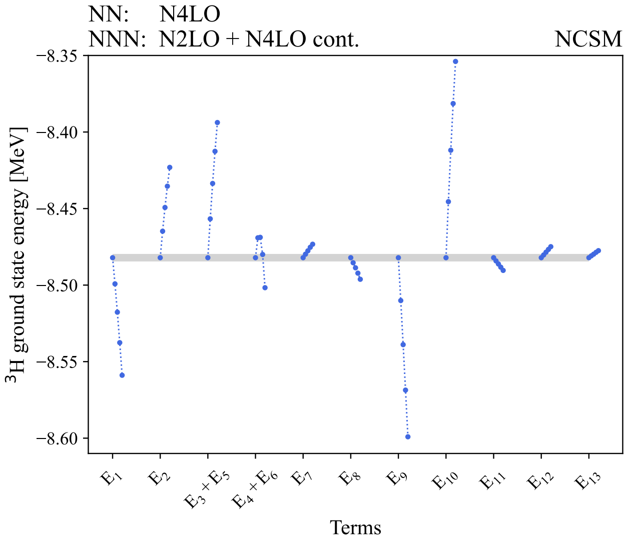


With only a 2NF: varying degree of agreement due to regulator effects.

■ *from Luca Girlanda*

*[**GP** and Navratil, in preparation (2025)]*.

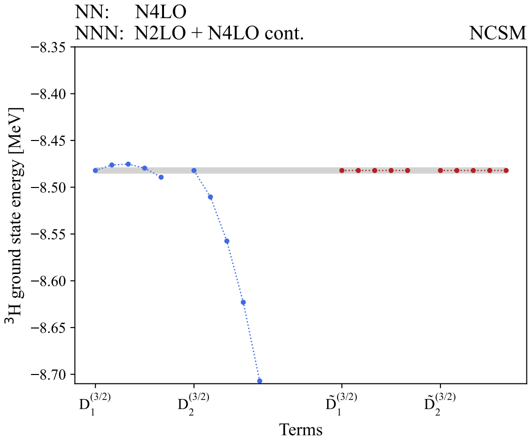
Varying the LECS



Varying the dimensionless LECs:

$$c_{E_i} = \frac{E_i}{F_\pi^4 \Lambda_\chi^3} = 0.25,\ 0.5,\ 0.75,\ 1.0$$

$\mathrm{N^4LO}$ $T = 3/2$ 3NF contacts



Restore symmetry:

$$\tilde{D}_i^{(3/2)} = P_{3/2} D_i^{(3/2)} P_{3/2}$$

$$P_{3/2} = \frac{1}{2} + \frac{1}{6}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 + \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)$$

All comb. with a local regulator

$$O_1 = \left[\sum_{i<j=2,3} w^{(1)}_{i,j}\right]$$

$$O_2 = \left[w^{(1)}_{2,2} + 2w^{(1)}_{2,3} + w^{(1)}_{3,3}\right] t_{2,3} + w^{(1)}_{2,2} t_{1,3} + w^{(1)}_{3,3} t_{1,2}$$

$$O_5 = \left[\sum_{a=2,3}\sum_{i,j=2,3} w^{(2)}_{i,j;1,a} + \sum_{a=2,3} w^{(2)}_{a,a;a,1} + w^{(2)}_{2,2;2,3} + w^{(2)}_{3,3;3,2}\right]$$

$$x, y = 2, 3$$

$$i, j = 1, 2, 3$$

$$O_6 = \left[\sum_{a=2,3}\sum_{i,j=2,3} w^{(2)}_{i,j;1,a} t_{1,a} + \sum_{a=2,3} w^{(2)}_{a,a;a,1} t_{1,a}\right] + \left[w^{(2)}_{2,2;2,3} + w^{(2)}_{3,3;3,2}\right] t_{2,3}$$

$$\boxed{w^{(1)}_{x,y} = \mathbf{q}_x \cdot \mathbf{q}_y}$$

$$\boxed{w^{(2)}_{x,y;i,j} = (\mathbf{q}_x \cdot \boldsymbol{\sigma}_i)(\mathbf{q}_y \cdot \boldsymbol{\sigma}_j)}$$

$$O_9 = w^{(2)}_{2,3;2,3} - \left(w^{(2)}_{2,2;1,2} + w^{(2)}_{3,2,;1,2} + w^{(2)}_{2,3;1,3} + w^{(2)}_{3,3;1,3}\right)$$

$$O_{10} = w^{(2)}_{2,3;2,3} t_{2,3} - \left(w^{(2)}_{2,2;1,2} t_{1,2} + w^{(2)}_{3,2;1,2} t_{1,2} + w^{(2)}_{2,3;1,3} t_{1,3} + w^{(2)}_{3,3;1,3} t_{1,3}]\right)$$

$$O_{11} = w^{(2)}_{2,3;3,2} - \left(w^{(2)}_{2,2;2,1} + w^{(2)}_{3,2,;2,1} + w^{(2)}_{2,3;3,1} + w^{(2)}_{3,3;3,1}\right)$$

$$O_{12} = w^{(2)}_{2,3;3,2} t_{2,3} - \left(w^{(2)}_{2,2;2,1} t_{1,2} + w^{(2)}_{3,2,;2,1} t_{1,2} + w^{(2)}_{2,3;3,1} t_{1,3} + w^{(2)}_{3,3;3,1} t_{1,3}\right)$$

$$O_{13} = w^{(2)}_{2,3;3,2} t_{1,2} - \left(w^{(2)}_{2,2;2,1} t_{1,3} + w^{(2)}_{3,2,;2,1} t_{1,3} + w^{(2)}_{2,3;3,1} t_{1,2} + w^{(2)}_{3,3;3,1} t_{1,2}\right)$$

To be continued . . .

Summary & Next steps

Summary

- On our way towards a consistent implementation of the subleading 3NF contacts at the $\mathrm{N^4LO}$.
- Given available data, $T = 3/2$ terms can soon be fitted.
- Various ways to define local $T = 3/2$ contributions can be tested.

Some Next steps

1. Explore their importance for neutron rich-er nuclei
2. Neutron drops
3. Explore other subleading 3NFs (see Evgeny's talk)

TRIUMF

Thank you.

Discovery, accelerated