
PLEASE LEAVE
MOB BOARDROOM
UNLOCKED

Light Nuclei with Transition Density Amplitudes: An Opportunity to Explore SMEFT/(B)SM Kernels in ^{3}He, ^{4}He, ^{6}Li and Beyond

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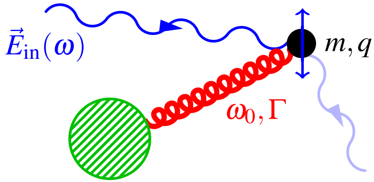
1. Two-Photon Response Explores System Dynamics
2. A New Hope
3. Beyond ^{4}He: TDAs in NCSM with SRG
4. Status, Present Projects, Plans and Aspirations

How do constituents of the nucleon react to external fields?
More efficient & accessible computations on Light Nuclei.
Model-independent predictions/extractions of fundamental (B)SM properties with Theory Uncertainties.

Transition Density Method & ^{3}He Compton: hg/JMcG/AN/DRP: Few-Body Syst. 61 (2020) 61 [2005.12207] [nucl-th]
^{4}He Compton & TDA Digest: Liao/hg/JMcG/AN/DRP: EPJA 60 (2024) 132 [2401.16995] [nucl-th]
^{6}Li Compton & SRG-And-Back: Long/hg/AN/Sun in preparation

(a) Polarisabilities: Stiffness of Charged Constituents in El.- Mag. Fields

Example: induced electric dipole radiation from harmonically bound charge, damping Γ Lorentz/Drude 1900/1905



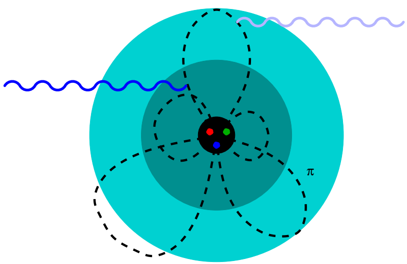
$$\vec{d}_{\text{ind}}(\omega) = \frac{q^2}{m} \underbrace{\frac{1}{\omega_0^2 - \omega^2 - \mathrm{i}\Gamma\omega}}_{=:\, 4\pi\,\alpha_{E1}(\omega)} \vec{E}_{\text{in}}(\omega)$$

$=: 4\pi\,\alpha_{E1}(\omega)$ "displaced volume" $[10^{-4}\ \text{fm}^3]$

Clean, perturbative probe: χiral symmetry of pion-cloud & its breaking, $\Delta(1232)$, spin-constituents.

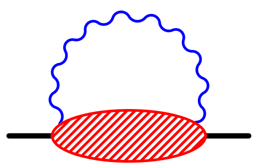
$$2\pi \underbrace{\left[\alpha_{E1}\,\vec{E}^2 + \beta_{M1}\,\vec{B}^2\right]}_{\text{electric, magnetic scalar dipole}}$$

Fundamental hadron properties, like charge, mass, mag. moment, $\langle r_N^2 \rangle \ldots$ PDG



Community Goal: Unified framework, reliable errors for p, d, ^{3}He,...

A2@MAMI, Compton@HIγS, MAXlab Roadmap: IJMPG49 (2022) 010502



Cottingham ΣR explains $M_\gamma^{\text{p}} - M_\gamma^{\text{n}}$ with

p-n difference $\alpha_{E1}^{p-n} = [-1.7 \pm 0.4_{\text{tot}}]$

Hoferichter/Gasser/Leutwyler/Rusetzky 2015

χEFT extracts from p, d data: $[-0.6 \pm 1.2_{\text{tot}}]$

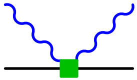
(b) All 1N Contributions to N^{3}LO

Bernard/Kaiser/Meißner 1992-4, Butler/Savage/Springer 1992-3, Hemmert/... 1998
McGovern 2001, hg/Hemmert/Hildebrandt/Pasquini 2003
McGovern/Phillips/hg 2013

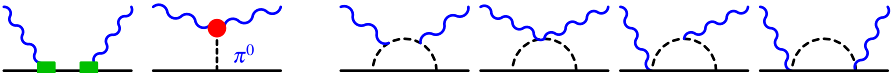
Unified Amplitude: accuracy decreases with ω:

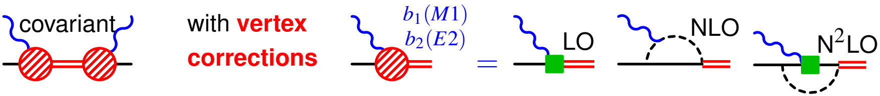
in low régime $\omega \lesssim m_\pi$ at least N^{4}LO $(e^2\delta^4)$: accuracy $\delta^5 \lesssim 2\%$;

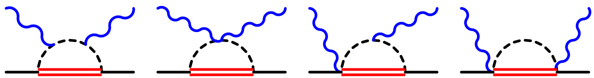
or in high régime $\omega \sim M_\Delta - M_N$ at least NLO $(e^2\delta^0)$: accuracy $\delta^2 \lesssim 20\%$.



Thomson term: $-\dfrac{Z^2\alpha_{EM}}{M}$





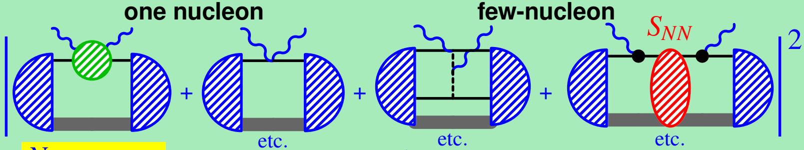


Diagrams	$\omega \lesssim m_\pi$	$\omega \sim M_\Delta - M_N \approx 300$ MeV
Thomson term	$e^2\delta^0$ *LO*	$e^2\delta^0$ ↘NLO
Nucleon / π^0 / pion loops	$e^2\delta^2$ N^{2}LO	$e^2\delta^1$ N^{2}LO
Δ pole with vertex corrections	$e^2\delta^3$ N^{3}LO	$e^2\delta^{-1}$ ↗*LO*
Δ loops	$e^2\delta^3$ N^{3}LO	$e^2\delta^1$ N^{2}LO

Unknowns: short-distance $\delta\alpha, \delta\beta \Longleftrightarrow$ static α_{E1}, β_{M1} (offset) $\Longrightarrow$ ω-dependence *predicted.*

(c) How to Get to the Neutron?

deuteron: hg/.../+Phillips/+McGovern 2004-; **MECs:** Beane/... 1999-2005
^{3}**He:** Shukla/... 2009 +Strandberg/Margaryan/hg/... [1804.01206]
^{4}**He:** hg/.../+Nogga EPJA 60 (2024) 132 [2401.16995]



N structure
$\alpha_{E1}, \beta_{M1}, \gamma_i$

N Born: charge
& mag. moment

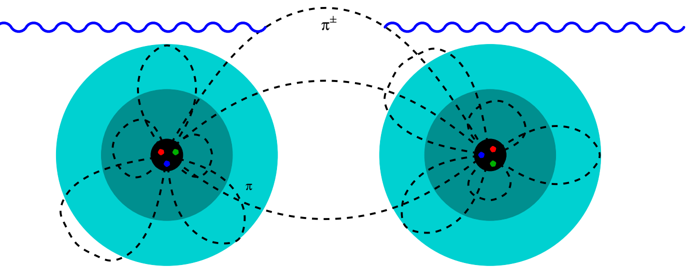
$\pi^{\pm}$ exchange
largest part

coherent rescattering
relevant for $\omega \to 0$

Parameter-free, except N polarisabilities.

Deuteron, ^{4}He, ^{6}Li: sensitive to $\alpha^{p}_{E1} + \alpha^{n}_{E1}, \quad \beta^{p}_{M1} + \beta^{n}_{M1} \implies$ **neutron pols**

^{3}**He**: sensitive to $2\alpha^{p}_{E1} + \alpha^{n}_{E1}, 2\beta^{p}_{M1} + \beta^{n}_{M1} \implies$ **neutron pols**

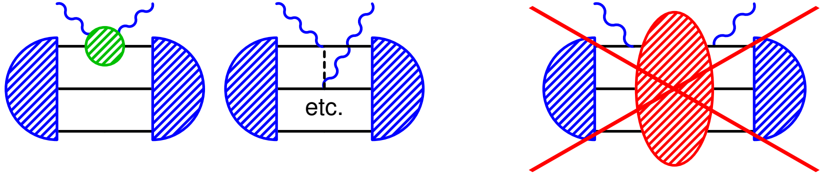


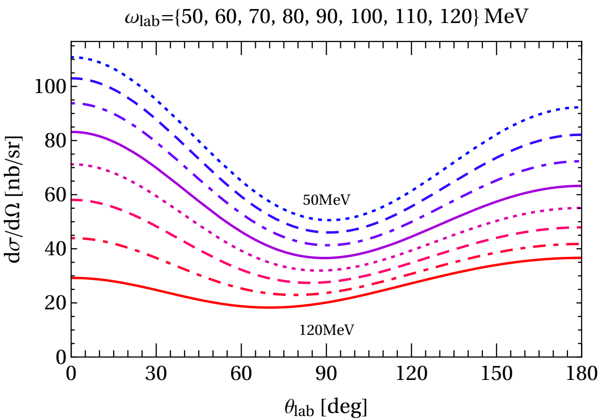
Model-independently subtract binding effects.

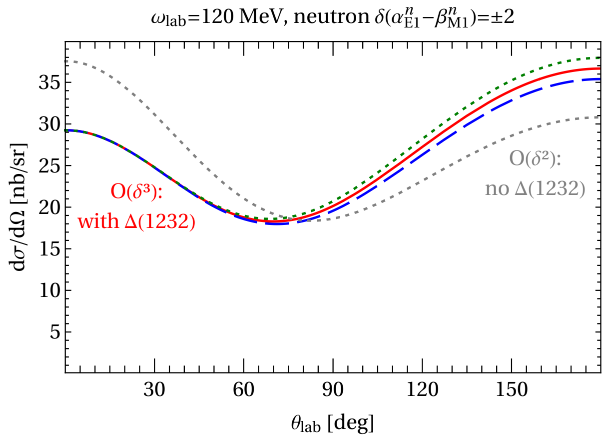
Chirally consistent 1N & few-N: potentials, wave functions, currents, π-exchange.

Test charged-pion component of NN force.

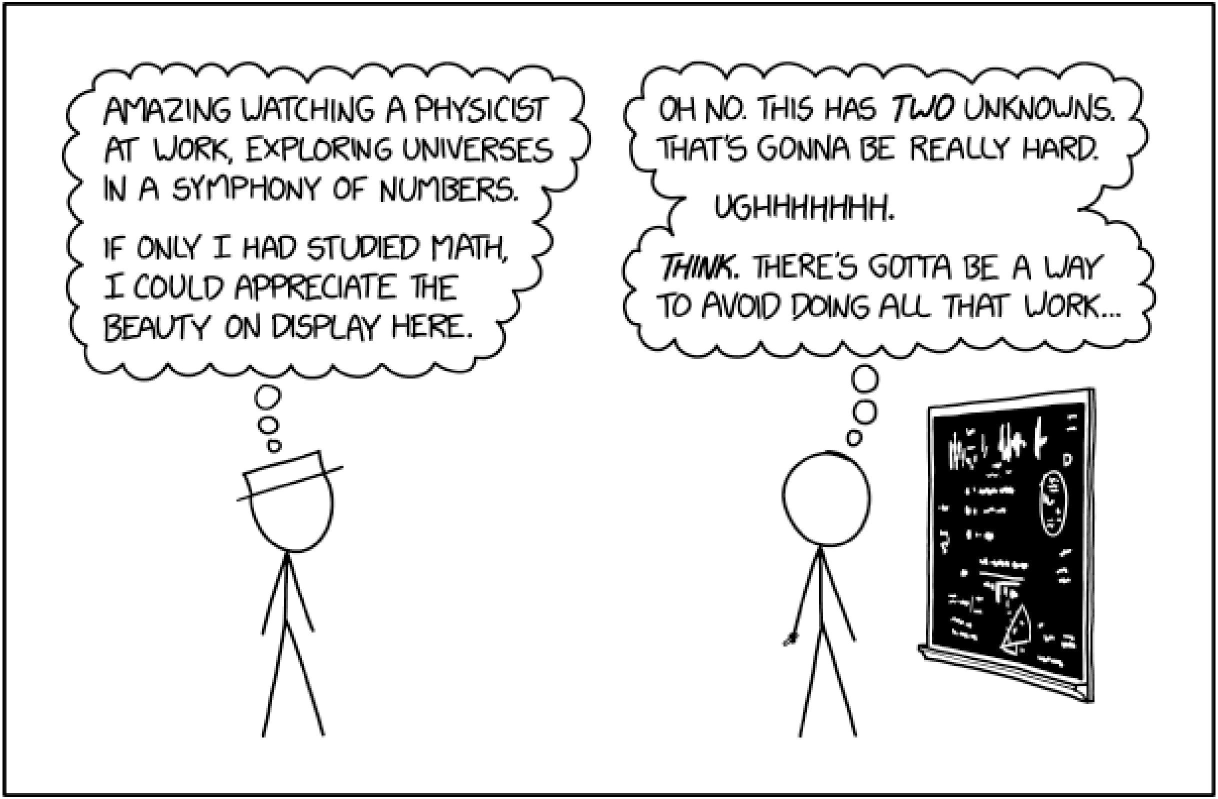
(d) The Hard Way: ^{3}He Theory Took A PhD







Observables available as *Mathematica* notebook.


AMAZING WATCHING A PHYSICIST AT WORK, EXPLORING UNIVERSES IN A SYMPHONY OF NUMBERS.
IF ONLY I HAD STUDIED MATH, I COULD APPRECIATE THE BEAUTY ON DISPLAY HERE.
OH NO. THIS HAS *TWO* UNKNOWNS. THAT'S GONNA BE REALLY HARD.
UGHHHHHHH.
THINK. THERE'S GOTTA BE A WAY TO AVOID DOING ALL THAT WORK...

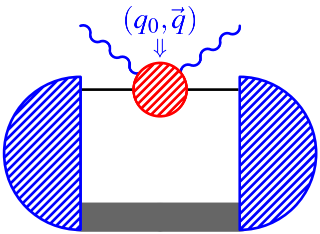
xkcd 2019

2. A New Hope

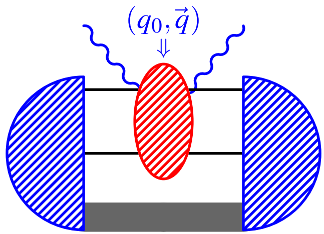
hg/McGovern/Phillips/Nogga: FewB Sys 61 (2020) 61 [2005.12207]
Alex Long: PhD Project 2022-26
cf. Shell Model Onebody Densities Burrows/Elster/...2017-

(a) Transition-Density Amplitude (TDA) Method: Idea

applied to Dark Matter in d, ^{3}He, ^{4}He:
de Vries/Köber/Nogga/Shain [2310.11343]

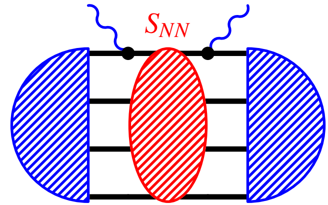


$A-1$ **spectator nucleons**
1 **Active Nucleon:**
one-body density



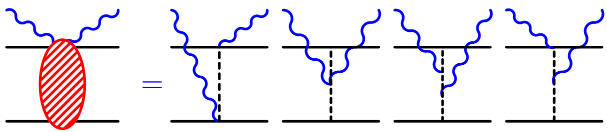
$A-2$ **spectator nucleons**
2 **Active Nucleons:**
two-body density

only depends on quantum numbers of actives and mom. transfer



no spectators
All Nucleons active:
rescattering
important as $\omega \lesssim \frac{1}{R}$

$\mathcal{O}(e^2\delta^2)$ 2N kernel:
Compton



Beane/...1999-2005

hg/McGovern/Phillips/Nogga: FewB Sys 61 (2020) 61 [2005.12207]
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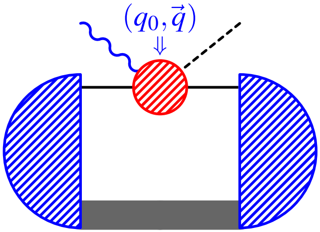

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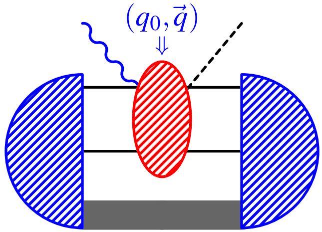

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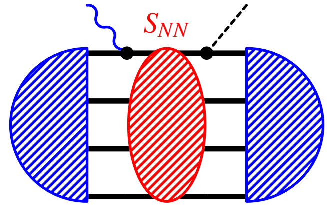
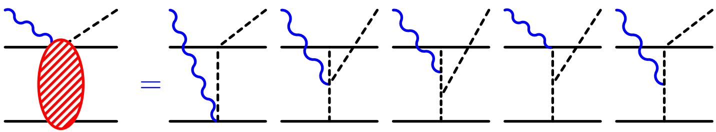


no spectators
All Nucleons active:
rescattering
important as $\omega \lesssim \frac{1}{R}$

$\mathcal{O}(e^2\delta^2)$ 2N kernel:
π **production**



Beane/...1995-97

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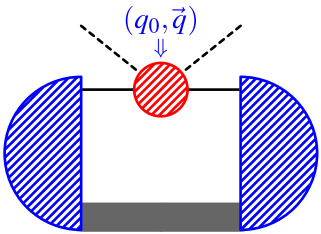

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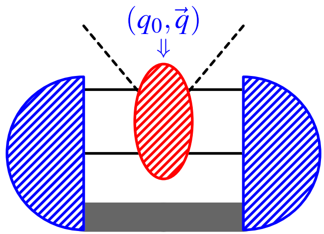
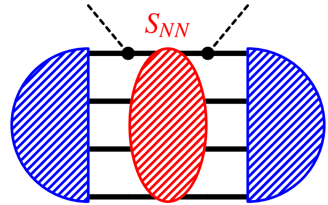


$A-2$ spectator nucleons
2 Active Nucleons:
two-body density

only depends on quantum numbers of actives and mom. transfer



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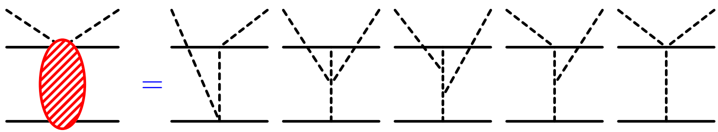
$\mathcal{O}(e^2\delta^2)$ 2N kernel:
π scattering



Weinberg 1992
Beane/. . . 1998

hg/McGovern/Phillips/Nogga: FewB Sys 61 (2020) 61 [2005.12207]
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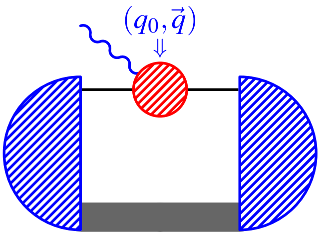

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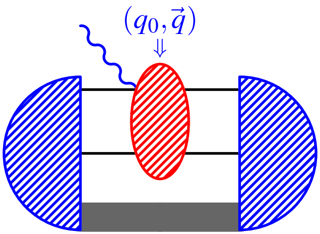
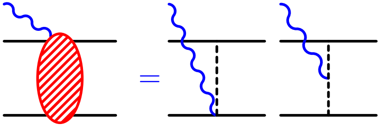


$A-2$ spectator nucleons
2 Active Nucleons:
two-body density

only depends on quantum numbers of actives and mom. transfer

$\mathcal{O}(e^2\delta^2)$ 2N kernel:
form factor



checked for d, ^{3}He, ^{4}He in
de Vries/Köber/Nogga/Shain
[2310.11343]

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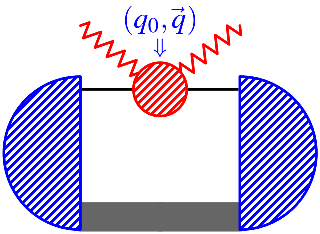

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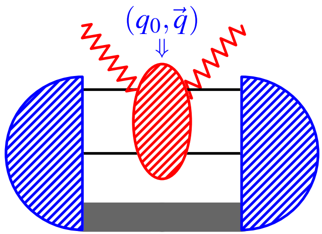

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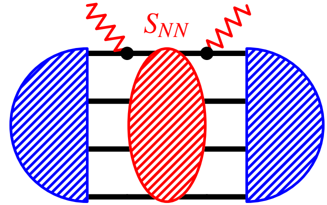

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All Nucleons active:
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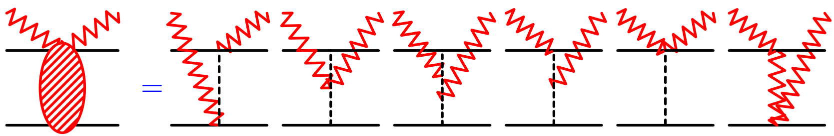
$\mathcal{O}(e^2\delta^2)$ 2N kernel:
(B)SM/Dark Matter



etc.

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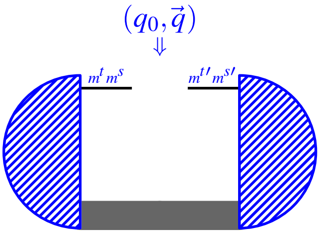

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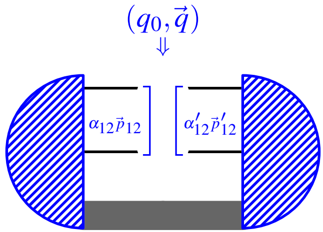

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***n*-Body Transition Density Amplitude:**

n nucleons with intrinsic momenta
and specific quantum numbers α_{12}
– absorb momentum transfer $(q_0, \vec{q})$;
– re-arrange quantum numbers to α'_{12};
– get absorbed back into nucleus.

Idea: Split calculation into

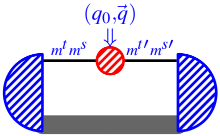
kernel: interaction with n active nucleons	|	**structure: $A-n$ spectators**
recycle same reaction for different nuclei	|	**recycle same nucleus for different reactions**
Compton on ^{3}He, ^{3}H, ^{4}He, ^{6}Li, . . .	|	**^{4}He Compton, π prod., FFs, dark matter,. . .**

χEFT hierarchy of few-body interactions: onebody, twobody $\gg$ threebody $\gg$ fourbody. . .

Computationally highly efficient: well-developed, sophisticated numerical few-body techniques.

(b) In Equations. . . : Probe Of Nucleus with In/Out Spin M/M'

$$A^{M'M}(\vec{q}) = \binom{A}{1} \langle M'|\mathcal{O}_{\rm 1N}(\vec{q})|M\rangle + \binom{A}{2} \langle M'|\mathcal{O}_{\rm 2N}(\vec{q})|M\rangle + \underbrace{\binom{A}{3} \langle M'|\mathcal{O}_{\rm 3N}(\vec{q})|M\rangle + \quad \dots}_{\text{suppressed at } q\sim m_\pi \text{ by } \chi\text{EFT hierarchy}}$$

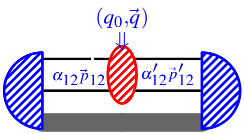


Hit active one-nucleon spin m^s, isospin m_t: sum with Onebody TDA instantaneous.

$$\langle M'|\mathcal{O}_{\rm 1N}(\vec{q})|M\rangle = \sum_{m^s m^{s\prime};m^t m^{t\prime}} \mathcal{O}_{\rm 1N}^{m^{s\prime}m^s;m^{t\prime}m^t}(\vec{q}) \quad \rho^{M'M}_{{\rm 1N}\, m^{s\prime}m^s;m^{t\prime}m^t}(\vec{q})$$

Hit active $2\mathrm{N}$ system of in/out spin/total/orbital angular momentum $(s_{12}\in\{0;1\}, m^s_{12}; j_{12}, m_{12}; l_{12}, m_{12}-m^s_{12})$,
isospin $(t_{12}\in\{0;1\}, m^t_{12})$, relative momentum $\vec{p}_{12}$

$$\langle M'|\mathcal{O}_{\rm 2N}(\vec{q})|M\rangle = \sum_{\substack{\alpha_{12}=(s_{12};j_{12}m_{12};l_{12};t_{12}m^t_{12})\\ \alpha'_{12}=(s'_{12};j'_{12}m'_{12};l'_{12};t'_{12}m^{t\prime}_{12})}} \int \frac{\mathrm{d}p'_{12}\, p'^2_{12}}{(2\pi)^3} \quad \mathrm{d}p_{12}\, p^2_{12} \qquad \rho^{M'M}_{{\rm 2N}\,\alpha'_{12}\alpha_{12}}(p'_{12},p_{12};\vec{q})$$
$$\times \sum_{m^{12\prime}_s m^s_{12}} \qquad \mathrm{CG}^{(l'_{12}s'_{12})j'_{12}}_{(m'_{12}-m^{s\prime}_{12},m^{s\prime}_{12})m'_{12}} \quad \mathrm{CG}^{(l_{12}s_{12})j_{12}}_{(m_{12}-m^s_{12},m^s_{12})m_{12}}$$
$$\underbrace{\times \langle l'_{12}(m'_{12}-m^{s\prime}_{12})|\ \mathcal{O}_{\rm 2N}^{\alpha'_{12}m^{s\prime}_{12};\alpha_{12}m^s_{12}}(\vec{p}'_{12},\vec{p}_{12};\vec{q})\ |l_{12}(m_{12}-m^s_{12})\rangle}_{\text{orbital angular momentum projection of } \mathcal{O}_{\rm 2N} \text{ done inside code}}$$



sum+integrate with Twobody TDA on fine momentum grid, $j_{12}\le 2$: ~ 15 minutes on i7-12700, not parallelised.

TDA Production Cost: $13\mathrm{CPUhrs}$ on JURECA (FZ Jülich) ($j_{12}\le 5$; accuracy $\ll 1\%$)

TDA Storage Cost: onebody: kB; twobody:$15\mathrm{MB}$ per q^2 for ^{6}Li as `.h5.gz` (Hierarchical Data HDF5).

Density Repository via `python` script at **`pypi.org/project/nucdens/`** (clickable link, in progress)

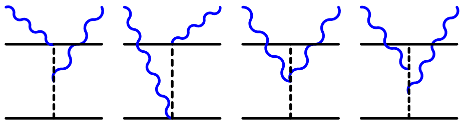
2N/3N Interactions Used: χEFT-inspired potentials χSMS[400-550]2NN^{4}LO +3NN^{3}LO Reinert/. . . 2018; Maris 2021;
Idaho N^{3}LO +3NFa/b Entem/Machleidt 2003; for checks (hard-core) AV18+UIX Wiringa/. . . 1995,. . .

^{3}He: CPU time reduced from days to hours; extensive checks; same result as traditional.

$\Longrightarrow$ Compute to higher numerical accuracy (integration mesh, j_{12},...): $\approx 1\%$ change in ^{3}He.

$A \geq 6$ “inconceivable” with traditional method.

twobody Compton matrix elements



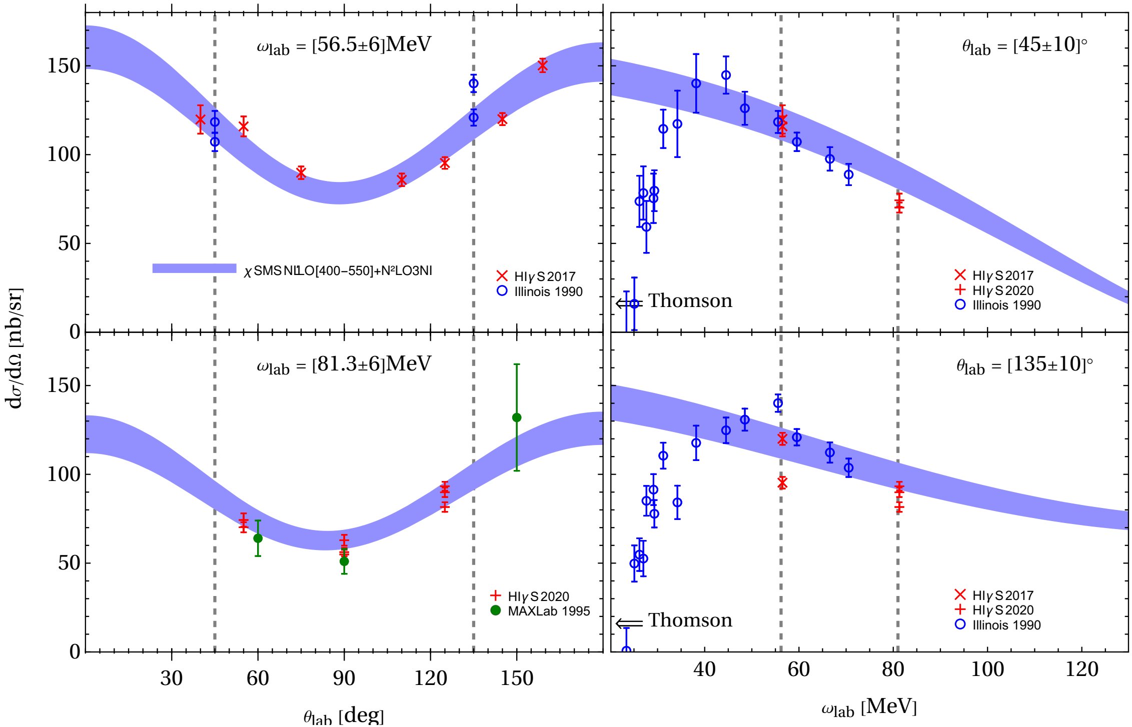
	$\omega = 50$ MeV, $\theta = 30°$				$\omega = 120$ MeV, $\theta = 165°$			
	Idaho N^{3}LO+3NFb		AV18+UIX		Idaho N^{3}LO+3NFb		AV18+UIX	
$\{M', M; \lambda', \lambda\}$	value [fm^3]	rel.dev.	value [fm^3]	rel.dev.	value [fm^3]	rel.dev.	value [fm^3]	rel.dev.
$\{\frac{1}{2}, \frac{1}{2}; 1, 1\}$	$-.07132$	0.1%	$-.09343$	0.2%	$-.00149$	0.0%	$-.00188$	0.2%
$\{\frac{1}{2}, \frac{1}{2}; -1, 1\}$	$-.00543$	0.3%	$-.00702$	0.3%	$-.10220$	0.8%	$-.12570$	0.8%
$\{\frac{1}{2}, \frac{1}{2}; 1, -1\}$	$-.00543$	0.3%	$-.00702$	0.3%	$-.10220$	0.8%	$-.12570$	0.8%
$\{\frac{1}{2}, \frac{1}{2}; -1, -1\}$	$-.07132$	0.1%	$-.09343$	0.2%	$-.00149$	0.0%	$-.00188$	0.2%

low-q^2 mostly probes diagonal TDA elements

high-q^2 mostly probes off-diagonal TDA elements

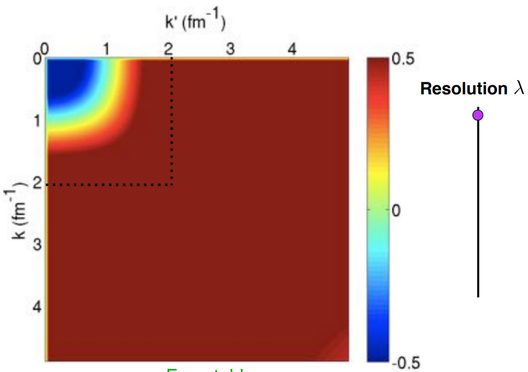
(d) ^{4}He: Confronting Data

$\mathcal{O}(e^2\delta^3)$ with established α_{E1}, β_{M1}; theory uncertainties: vary χSMS potential (dominant), numerics (tiny).



(a) TDAs with Similarity Renormalisation Group Transformation SRG: Szpigel/Perry 2000

Unitarity transformation $U\,V(p,p')\,U^\dagger$: Physics unchanged; reduce components $p \gtrsim \overline{\Lambda}_\chi$ χEFT breakdown scale.



Furnstahl

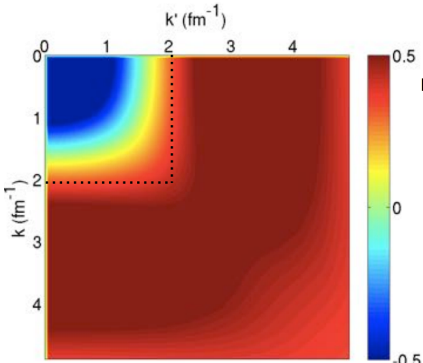
High Resolution:

$V(p,p' \gtrsim \overline{\Lambda}_\chi)$ components substantial

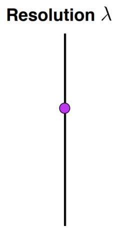
$\Longrightarrow$ Numerics needs fine grid to high momenta.

(a) TDAs with Similarity Renormalisation Group Transformation SRG: Szpigel/Perry 2000

Unitarity transformation $U\,V(p,p')\,U^\dagger$: Physics unchanged; reduce components $p \gtrsim \overline{\Lambda}_\chi$ χEFT breakdown scale.



Furnstahl



Medium Resolution:

$V(p,p' \gtrsim \Lambda_{\text{SRG}} > \overline{\Lambda}_\chi)$ dampened

$\Longrightarrow$ Grid sparser at high momenta.

2NI changed to “coarser grain” and additional 2NIs

$\Longrightarrow$ Iterated 2NIs induce 3NIs, 4NIs,. . .

3NI changed to “coarser grain” and additional 3NIs

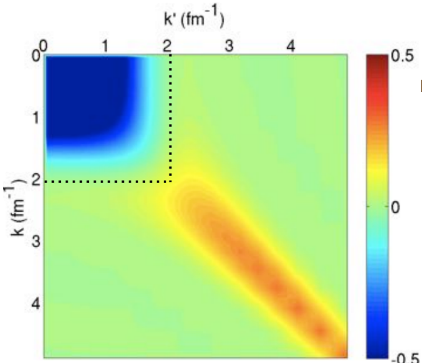
$\Longrightarrow$ Iterated 3NIs induce 4NIs, 5NIs,. . .

etc.

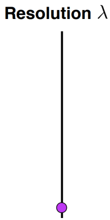
Physics unchanged only if *all* induced ANIs kept (i.e. U unitary).

(a) TDAs with Similarity Renormalisation Group Transformation SRG: Szpigel/Perry 2000

Unitarity transformation $U\,V(p,p')\,U^\dagger$: Physics unchanged; reduce components $p \gtrsim \overline{\Lambda}_\chi$ χEFT breakdown scale.



Furnstahl



Low Resolution:

$V(p,p' \gtrsim \Lambda_{\text{SRG}} \gtrsim \overline{\Lambda}_\chi)$ mostly zero.

$\Longrightarrow$ Grid concentrates on low momenta & diagonal.

2NI changed to "even coarser" (and additional) 2NIs

$\Longrightarrow$ Iterated 2NIs induce 3NIs, 4NIs,...

3NI changed to "even coarser" (and additional) 3NIs

$\Longrightarrow$ Iterated 3NIs induce 4NIs, 5NIs,...

etc.

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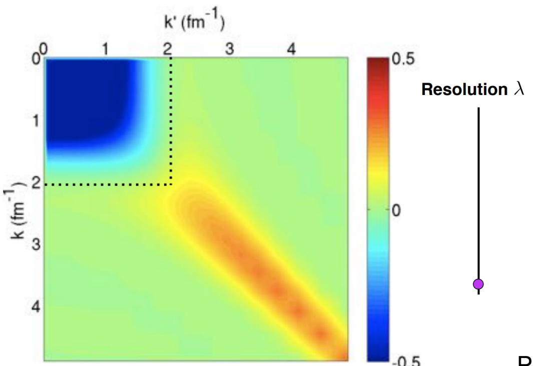

Furnstahl

Low Resolution:

$V(p, p' \gtrsim \Lambda_{\mathrm{SRG}} \gtrsim \overline{\Lambda}_\chi)$ mostly zero.

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2NI changed to "even coarser" (and additional) 2NIs

$\Longrightarrow$ Iterated 2NIs induce 3NIs, 4NIs,...

3NI changed to "even coarser" (and additional) 3NIs

$\Longrightarrow$ Iterated 3NIs induce 4NIs, 5NIs,...

etc.

Physics unchanged only if *all* induced ANIs kept (i.e. U unitary).

Induced 4NIs, 5NIs,... numerically expensive. $\Longrightarrow$ State Of The Art: Drop all 4NIs, 5NIs,...

Consistent with χEFT hierarchy of interactions 2NI>3NI>4NI>... at $p, p', q \sim m_\pi \ll \overline{\Lambda}_\chi$.

$\Longrightarrow$ Strictly speaking not unitary, but hopefully not substantial as long as $\Lambda_{\mathrm{SRG}} \gtrsim \overline{\Lambda}_\chi$.

Quality Criteria: Change of ^{6}Li binding energy, Form Factors and Compton MEs with Λ_{SRG}.

Large Λ_{SRG} numerically expensive. $\Longleftrightarrow$ Small Λ_{SRG} bigger effects from missing induced ANI.

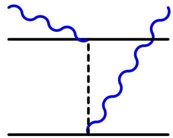
Empirically: $\Lambda_{\mathrm{SRG}} = \{1.88; 2.236; (3.0)\}\ \mathrm{fm}^{-1} = \{370; 447; (600)\}\ \mathrm{MeV}$ acceptable.

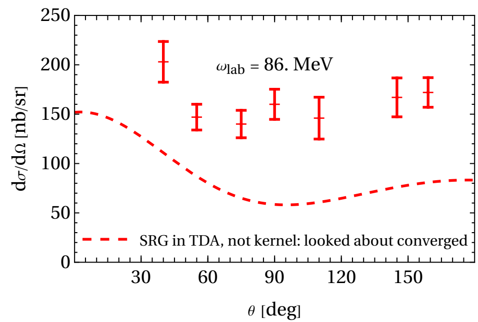
$\Longrightarrow$ **Cost-effective computation of SRG-transformed 1N and 2N TDAs** $\rho^{\mathrm{SRG}}_{1,2\mathrm{N}} = U\,\rho_{1,2\mathrm{N}}\,U^\dagger$.

(b) *Not* Transforming the Interaction Kernel: "SRG-And-Back"

Unitarity transformation $U\,V(p,p')\,U^\dagger$: Physics unchanged; reduce components $p \gtrsim \overline{\Lambda}_\chi$ χEFT breakdown scale.

Problem Mismatch: Transition Densities computed from SRG-transformed interactions,
un-transformed interactions inside kernels.

U  $U^\dagger$



Approaches:

(i) Just ignore. $\Longrightarrow$ missing factor $\times \approx 2$ in cross sections…

(ii) Usually done: SRG-transform kernel: $\mathcal{O}^{\rm SRG}_{n\rm N} = U\,\mathcal{O}_{n\rm N}\,U^\dagger$

$\Longrightarrow$ Must provide U with TDA, e.g. encode SRG in TDA file.

$\Longrightarrow$ CPU time for additional user step; **Murphy's-Law-errors**.

(iii) **New:** SRG back-transformation $U^\dagger\,\rho^{\rm SRG}_{1,2\rm N}\,U \approx \rho_{1,2\rm N}$. Nogga/Sun/(Long/hg) 2025

of TDAs from SRG-transformed space leads to TDAs in original momentum space.

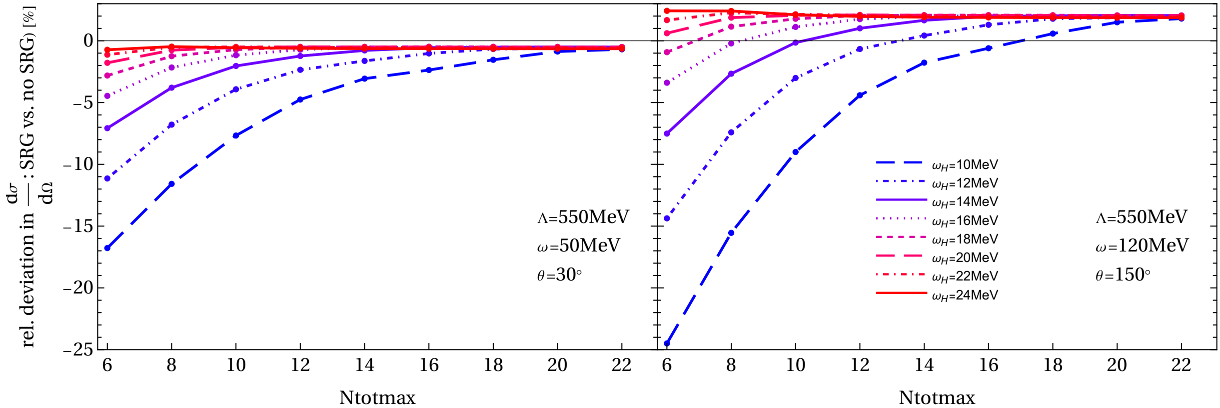
$\Longrightarrow$ No need to mess with kernel $\mathcal{O}_{n\rm N}$.

Still "truncated-SRG back". $\Longrightarrow$ Increased non-unitarity effects from "dropping" induced 4NI, 5NI,…

(c) Comparing Original Faddeev with "SRG-And-Back" NCSM in ^{4}He

Relative deviation of Compton cross sections in per-cent.

Small $q^2 = -(34\text{ MeV})^2$: mostly TDA diagonals. Large $q^2 = -(230\text{ MeV})^2$: mostly TDA off-diagonals.



%-level discrepancies: Original uses Faddeev methods. SRG-and-Back uses

(Ab-Initio) No Core Shell Method NCSM for $A \geq 4$ up to $\gtrsim 24$

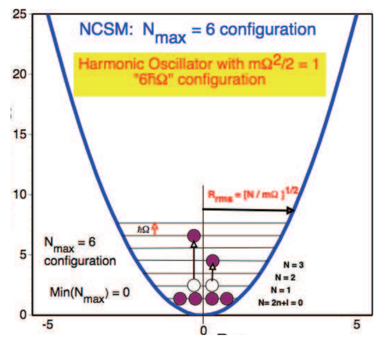
Barrett/Vary/. . . 1994-, Forssén, Ormand, Maris, Navratil, Roth, Stetcu, Nogga,. . .

Harmonic Oscillator Basis with $\omega_H = \{14; 16; 18\}$ MeV

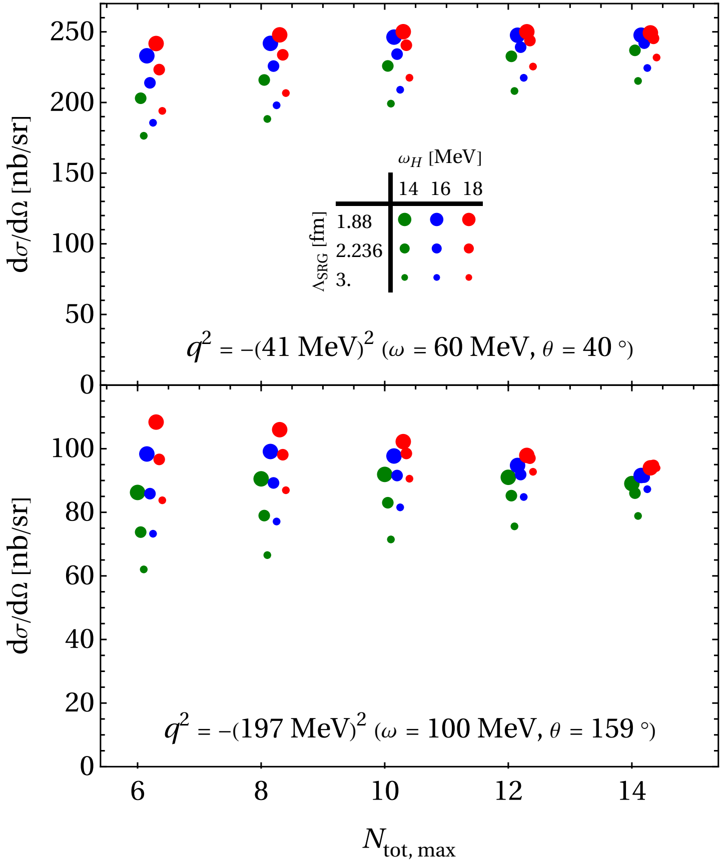
up to $N_{\text{tot,max}} = 6, \ldots, 14$, exact for $\omega_H, N_{\text{tot,max}} \to \infty$.

Quality Criteria: Change of ^{6}Li binding energy, Form Factors,

Compton MEs with $\Lambda_{\text{SRG}}, \omega_H, N_{\text{tot,max}}$.



Vary 2013



$A \geq 6$: Traditional way punitively expensive.

$\Longrightarrow$ “SRG-And-Back” and NCSM both must find “sweet spot” in $\Lambda_{\text{SRG}}, \omega_H, N_{\text{tot,max}}$:

Small enough for efficient computation,
Large enough for NCSM extrapolation,
small induced 4NIs, 5NIs,. . .

Quality Criteria:

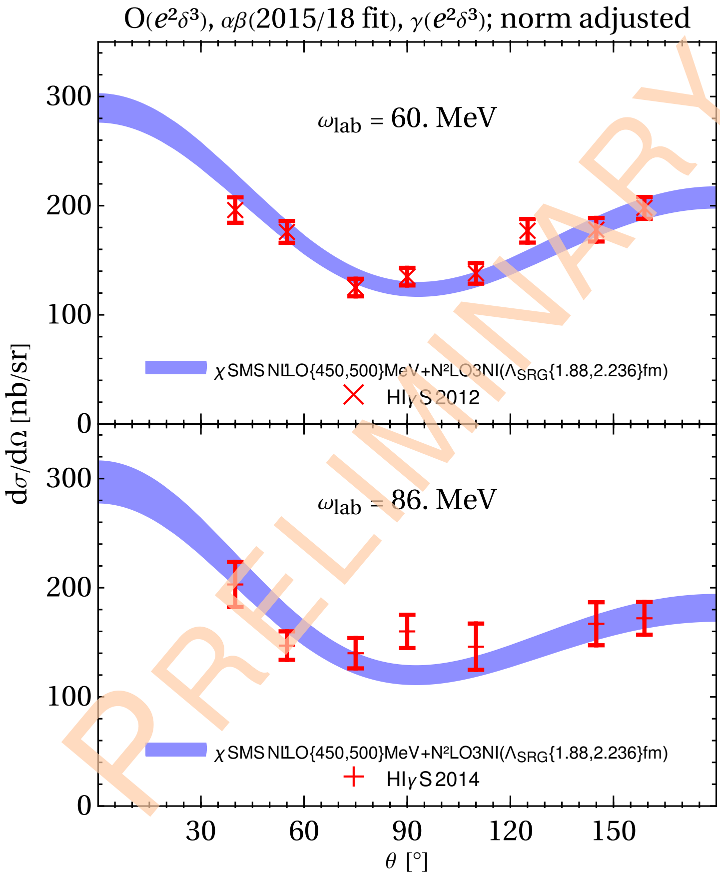
$B(^6\text{Li})$, Form Factors, Compton ME variation with $\Lambda_{\text{SRG}}, \omega_H, N_{\text{tot,max}}$.

Well-converged Compton cross sections for
$\Lambda_{\text{SRG}} = \{1.880, 2.234\}\ \text{fm}^{-1}$,
$\omega_H = 18\ \ \text{MeV}, N_{\text{tot,max}} = 14$:

$\pm 1.5\%$ at low q, $\pm 3\%$ at high q.

(e) Test: TDA with "SRG-And-Back" NCSM for Compton Scattering

Long/hg/Nogga/Sun
in preparation



Using PDG 2018 polarisabilities:

Angular dependence well reproduced,

but some interesting data points.

HIGS 2023/2024

Theory Uncertainties $60\ \text{MeV}: \pm 4\%$, $86\ \text{MeV}: \pm 7\%$ *Preliminary*

Method uncertainty (small):

variation with $\Lambda_{\text{SRG}}, \omega_H, N_{\text{tot,max}}$

χEFT uncertainty dominates:

from order-by-order & variation with potential $60\ \text{MeV}: \pm 3\%$, $86\ \text{MeV}: \pm 6\%$ *Preliminary*

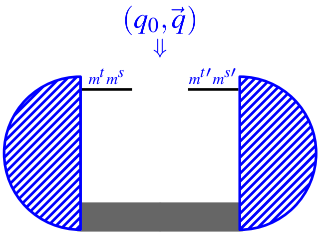
χSMS NN(N^{4}LO)+3N(N^{2}LO) at cutoffs

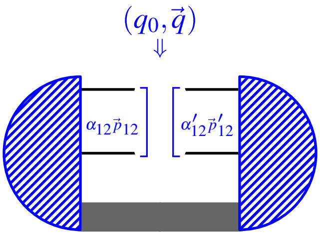
$\Lambda = \{450; 500\}$ MeV (^{6}Li binding!)

$\Lambda_{\text{SRG}} = \{1.88; 2.236\}\ \text{fm}^{-1}$ ($B(^6\text{Li})$!)

(f) Recap: Transition-Density Amplitude (TDA) Method

applied to Dark Matter in d, ^{3}He, ^{4}He:
de Vries/Köber/Nogga/Shain [2310.11343]





$A-1$ **spectator nucleons**
1 **Active Nucleon:**
one-body density

$A-2$ **spectator nucleons**
2 **Active Nucleons:**
two-body density

only depends on quantum numbers of actives and mom. transfer

Idea: Split calculation into

kernel: interaction with n active nucleons	|	**structure: $A-n$ spectators**
recycle same reaction for different nuclei	|	**recycle same nucleus for different reactions**
Compton on ^{3}He, ^{3}H, ^{4}He, ^{6}Li, ...	|	**^{4}He Compton, π prod., FFs, dark matter,...**

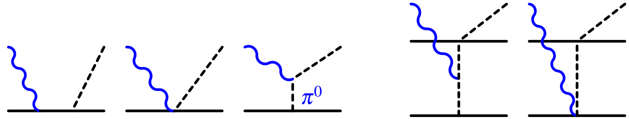
χEFT hierarchy of few-body interactions: onebody, twobody $\gg$ threebody $\gg$ fourbody...

(g) ^{3}He Pion Photo-/Electro-Production at Threshold with TDAs

So far: same Compton kernels, different nuclei – Now: recycle TDAs with different kernels.

LO $\gamma^{(*)}\ {}^3$He $\to \pi^0\ {}^3$He at threshold

kernels: Beane/...1997



One-Body, same kernel multipole responses	transverse, isoscalar$\pm$isovector		longitudinal, isoscalar$\pm$isovector	
	$F_T^{1N,S+V}$	$F_T^{1N,S-V}$	$F_L^{1N,S+V}$	$F_L^{1N,S-V}$
Lenkewitz/...2011 AV18+UIX	$+0.017(13)(3)$	$1.480(26)(3)$	$-0.079(14)(8)$	$1.479(26)(8)$
Braun/...2019 Idaho	$+0.041(2)$	$1.544(77)$	——	——
Long/... χSMS [400-550] TDA	$-0.044(11)$	$1.473(20)$	$-0.032(10)$	$1.473(20)$

Two-Body, same kernel multipole responses	transverse	longitudinal
	F_T^{2N}	F_L^{2N}
Lenkewitz/...2011 AV18+UIX	$-29.3\ \mathrm{fm}^{-1}$	$-22.9\ \mathrm{fm}^{-1}$
Braun/...2019 Idaho	$-27.1(33)\ \mathrm{fm}^{-1}$	——
Long/... χSMS [400-550] TDA	$-30.6(5)\ \mathrm{fm}^{-1}$	$-24.2(4)\ \mathrm{fm}^{-1}$

First with theory uncertainties (vary χSMS cutoff); smaller numerical errors – smaller overall error: numerics!

No complex Jacobi-coordinate algebra; numerically fast kernel, simple to programme.

Implemented algorithm stable against integrable singularities above thresholds for generic on-shell pion propagator.

Working now on: above-threshold; $A \geq 4$; $A \geq 3$ pion scattering at & above threshold. Long PhD 2026

(a) Where We Are

hg/McGovern/Nogga/Phillips EPJA 60 (2024) 132 [2401.16995]
Long/hg/Nogga/Sun/dots in progress

Transition Density Amplitude Method splits probe of nucleus A with momentum transfer $\vec{q}$
into *n-Body kernel* (interaction of probes with n active nucleons)
and *n-Body TDA* (n active nucleons in background of $A-n$ spectators).
Recycling: same $(A,\vec{q})$ TDAs for different kernels – same kernels for different $(A,\vec{q})$ TDAs.
TDAs generated via highly efficient, well-developed few-body techniques (NCSM, SRG,...).
TDAs make few-nucleon computations fast, freely available, reliable, easy to use, modify, adapt.

Onebody & Twobody Density Repository via `python` script at **pypi.org/project/nucdens/** (ongoing) from:
χSMS[400-550]2NN^{4}LO +3NN^{3}LO Reinert/...2018 Maris 2021 (for $A \geq 4$ with $\Lambda_{\text{SRG}} = \{1.88; 2.236; 3.0\}\ \text{fm}^{-1}$, a few higher);
Idaho N^{3}LO +3NFa/b Entem/Machleidt 2003; for checks (numerics of hard-core) AV18+UIX Wiringa/...1995,...

SRG-And-Back: SRG becomes technical detail to get TDA, no need to modify "standard" kernels.

Quality Criteria: Converged binding, Form Factors, Compton MEs varying potential (+SRG: $\Lambda_{\text{SRG}}, \omega_H, N_{\text{tot,max}}$).

Readily Available TDAs for ^{3}He, ^{3}H, ^{4}He full coverage: $q^2 = -([0;240]\ \text{MeV})^2$ up to pion thresholds.
Kernels: N^{3}LO Compton below m_π; LO $\gamma^{(*)}\pi^0$ and $\pi\pi$ at threshold.

It Works Compton on ^{3}He, ^{4}He, ^{6}Li: converges; agrees with data. Threshold $^3\text{He}\,(\gamma^{(*)},\pi^0)^3\text{He}$.

Work In Progress TDAs for ^{6}Li, deuteron, all $A \leq 6$ up to $q^2 = -(\sim 400\ \text{MeV})^2$ well beyond pion thresholds.
Kernels: above pion threshold: N^{3}LO Compton; NLO $\gamma^{(*)}\pi^0$ and $\pi\pi$.

Plans $A > 6$: CNO and beyond with NCSM ($A \lesssim 24$?) $A > 14$ (also) with other methods, e.g. Shell Model.

(b) Where This Can Go

We Are Looking for Applications! (neutrino scattering, gravitational FFs, Dark Matter, (B)SM,. . .)

Collaborate on (B)SM in low-energy few-nucleon systems (`Fortran` subroutines nice but not necessary).

Contact hgrie@gwu.edu .

Intent Lower access barrier to *ab-initio* Nuclear Physics with a robust, user-friendly, well-documented Public-Domain `Fortran` Code Suite convoluting *user-defined kernels* with TDA downloaded from repo.

$\Longrightarrow$ Community effort to produce TDAs for any nucleus in standardised format?

Supporting drive for standardised exchange of kernels, codes, matrix elements,. . .

cf. Heinz/Duguet/hg/Hergen/Tichai: *Perspectives on Open-Source Toolchain for Ab Initio Nuclear Physics*, in [2202.01105]

Aspirations

So far onebody and twobody TDAs; χiral Hierarchy $\Longrightarrow$ 3body important for $q \sim m_\pi$ (huge storage requirement)?

For $\omega, |\vec{q}| \lesssim \dfrac{1}{R_A} \to 0$: also intermediate-state coherent A-nucleon rescattering (Compton,. . .). $\Longrightarrow$ Not TDAs.

So far no transmutation/charge-exchange ${}^{Z}\!A \to {}^{Z\pm 1}\!A$: $\beta^\pm/\mu^\pm$ decay/capture, $n\gamma \to p\pi^-$, $(pp)\gamma \to (np)\pi^+$.

So far TDAs from iterated potential. $\Longrightarrow$ Cannot address renormalisability, large cutoff variations,. . .

$\Longrightarrow$ Produce TDAs with iterated LO, higher orders in perturbation?!

The efficient person gets the job done right. The effective person gets the right job done.

There is always a well-known solution

to every human problem ——

neat, plausible,

H. L. Mencken

There is always a well-known solution

to every human problem ——

neat, plausible, and wrong.

H. L. Mencken

(a) “Mene, Tekel, Upharsin”: Weinberg’s Pragmatic Proposal

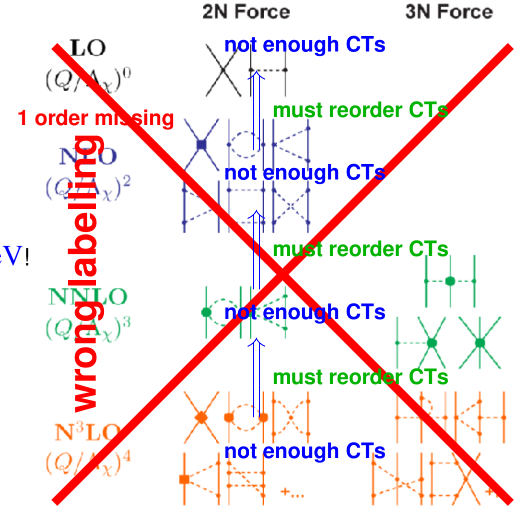
Pragmatic, widely used (“Everybody Does It”).
But *conceptually inconsistent*:

– Not renormalised in low partial waves with attractive tensor.
Nogga/Timmermans/van Kolck 2005

– ${}^1\mathrm{S}_0$: m_π^2-dependence of CT and divergence do not match.

$$V_C \to \frac{m_\pi^2}{\pi^2\, r} \text{ but } C_0 \sim m_\pi^0 \text{ bites you for } m_\pi \neq 140\ \mathrm{MeV}!$$

Kaplan/Savage/Wise 1996, Beane/Bedaque/Savage/van Kolck 2002



underlying plot by Epelbaum, butchered by hg

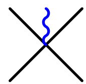
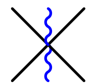
Not Just LO Reg/Ren Problem: ricochets through orders.

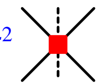
$\Longrightarrow$ WPP underestimates number of CTs per order.

$\Longrightarrow$ WPP at alleged order Q^n not as accurate as thought:
Accurate only to lower order $Q^{n-1,2,3,\ldots}$.

Fitting may obscure the problem. . .

Not Just NN Problem: 2N currents promoted. Phillips/Valderrama 2015

$\Longrightarrow$ Gauged & gauge-invariant currents earlier, e.g. at LO $\vec{p}\cdot\vec{p}\,'C_P \to$  

$\Longrightarrow$ Chiral-gauged NN currents earlier: $D_2 m_\pi^2$ in LO ${}^1\mathrm{S}_0 \xrightarrow{\chi\mathrm{sym}} D_2\pi^2$  LO in πNucleus.

We may be unable to say whose PC is right, but we have evidence whose is wrong. WPP is; it’s *In-Effective*.

Still, use it pragmatically to develop numerics & first glimpses at final theory – with *caveat on systematics*!

(b) NN χEFT Power Counting Comparison

prepared for Orsay Workshop by Grießhammer 7.3.2013
based on and approved by the authors in private communications

Derived with explicit & implicit assumptions; contentious issue.

All but WPP: RGE as construction principle, but different approximations at short-range lead to variant interpretations.

Proposed order Q^n at which counter-term enters *differs*. $\Longrightarrow$ Predict *different* accuracy, # of parameters.

order	**Weinberg** (Pragm. Prop.) PL**B251** (1990) 288 etc.	**Birse** PR**C74** (2006) 014003 etc.	**Pavon Valderrama et al.** PR**C74** (2006) 054001 etc.	**Long/Yang** PR**C86**(2012) 024001 etc.
Q^{-1}	LO of $^1S_0$, $^3S_1$, OPE	plus $^3D_1$, $^3SD_1$	plus $^3P_{0,2}$, $^3D_2$	plus $^3P_{0,2}$
$Q^{-\frac{1}{2}}$	none	LO of $^3P_{0,1,2}$, $^3PF_2$, $^3F_2$, $^3D_2$	LO of $^3SD_1$, $^3D_1$, $^3PF_2$, $^3F_2$	none
Q^0	none	NLO of $^1S_0$		
$Q^{\frac{1}{2}}$	none	NLO of $^3S_1$, $^3D_1$, $^3SD_1$	none	none
Q^1	LO of $^3SD_1$, $^1P_1$, $^3P_{0,1,2}$; NLO of $^1S_0$, $^3S_1$	none	none	LO of $^3SD_1$, $^1P_1$, $^3P_1$, $^3PF_2$; NLO of $^3S_1$, $^3P_0$, $^3P_2$; N^{2}LO of $^1S_0$
# at Q^{-1}	2	4	5	4
# at Q^0	$+0$	$+7$	$+5$	$+1$
# at Q^1	$+7$	$+3$	$+0$	$+8$
total at Q^1	9	14	10	13

With same χ^2/d.o.f., the *self-consistent* proposal with least parameters *wins*: minimum information bias.

Still, use any *pragmatically* to develop numerics & first glimpses at final theory – with caveat on systematics!

(c) Even Weinberg Can Be Wrong

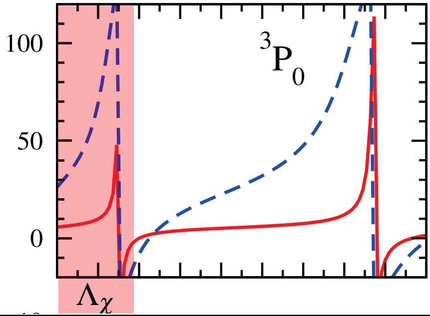
Consistency check **of Weinberg's proposal: Observables cut-off dependent at LO?**
Need 4 *new*, momentum-dependent LECs for low attractive triplets: $^3P_{0,2}$, $^3D_{2,3}$.

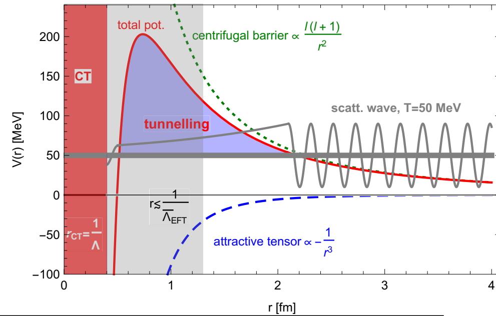
Low attractive P/D-wave triplets: Weinberg predicts zero LECs at LO (momentum-independence).

phase-shift δ(cut-off Λ):

$E_{\text{lab}} = 10$ MeV (solid)
50 MeV (dashed)
100 MeV (dotted)
190 MeV (dash-dotted)

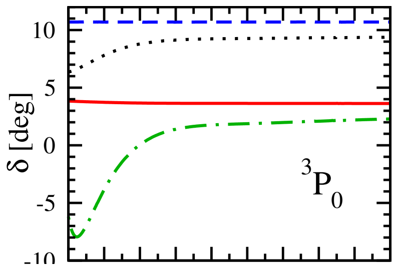
$$V(r) \propto -\frac{\#}{r^3} + \frac{l(l+1)}{r^2}$$



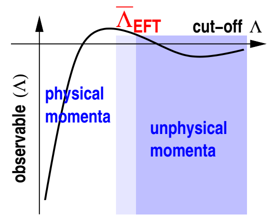


Without CT cutoff-dependent even for $\Lambda \approx \bar{\Lambda}_\chi$. $\Longrightarrow$ Short-distance missing. $\Longrightarrow$ *Not renormalised.*

Λ-Dependence With Counter Term



"Cutoff Democracy":
Any cutoff $\Lambda \gtrsim \bar{\Lambda}_{\text{EFT}}$ is equally valid as it probes momenta beyond range where description makes sense.
Cutoff Λ unphysical
Breakdown Scale $\bar{\Lambda}_{\text{EFT}}$ physical



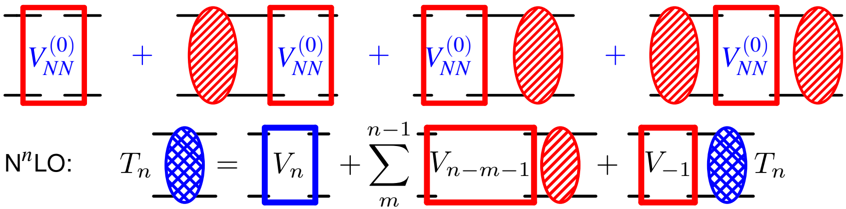
(d) Do Contributions to *Observables* Decrease With Increasing Order?

$\Longrightarrow$ Find radius of convergence $k \lesssim \overline{\Lambda}_{\text{EFT}}$, systematically estimate truncation error (Bayes) –
and *only then* compare to data: beware of confirmation bias.

Corrections in $Q \ll 1$ by "strict perturbation" about LO (Distorted-Wave Born; efficient way Vanasse 1305.0283**):**
$\Longrightarrow$ Power-counting of amplitudes (observables); simple, no resummation artefacts.

NLO:

N^{n}LO:



Use/Develop More Strict-Perturbation Methods! cf. hg notes Trento 2018-21;

$\longrightarrow$ Oliver Thim; $\Delta\mathcal{O}_0 = \dfrac{\mathcal{O}[V_{-1} + \epsilon\, V_0] - \mathcal{O}[V_{-1}]}{\epsilon}$ with $\epsilon \to 0$ Shi/.../Long/... PRC 106 (2022) 015505 [2205.02000]; ...

Contrast to Popular "Partially-Resummed Perturbation" Weinberg 1990

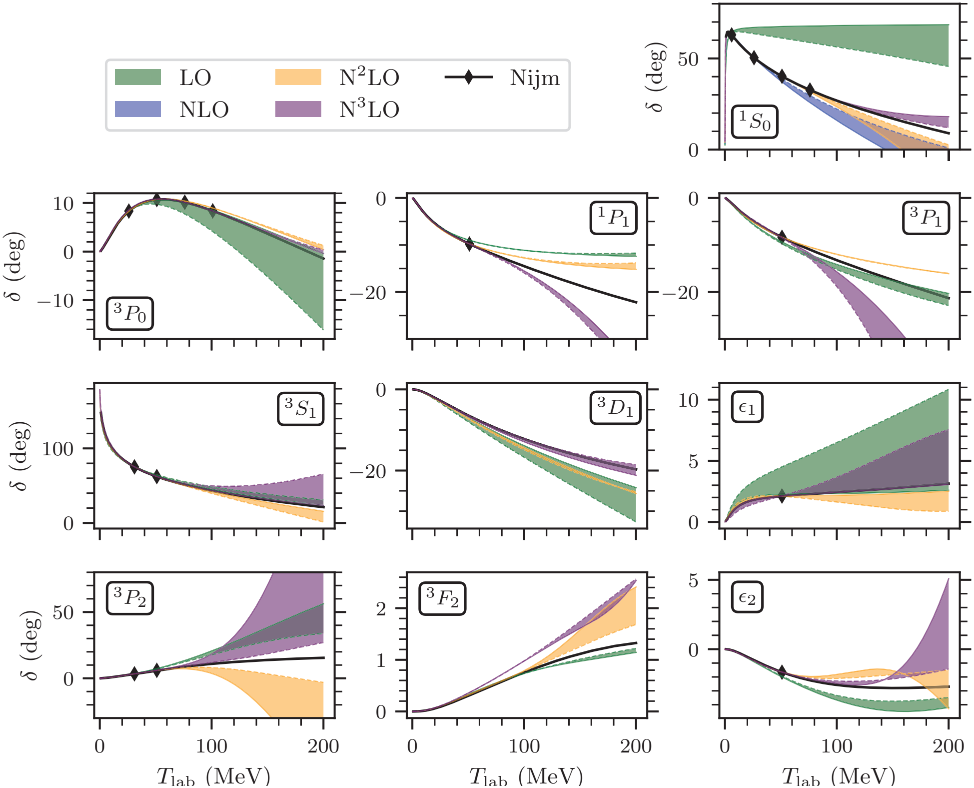
Power-count V_{NN} & iterate $\Longrightarrow T = \dfrac{V_{\text{LO}} + V_{\text{NLO}} + \dots}{1 - (V_{\text{LO}} + V_{\text{NLO}} + \dots)\, G_{NN}}$.

$\Longrightarrow$ Obscures PC in observables, unphysical poles around $\overline{\Lambda}_{\text{EFT}}$: artefacts, wrong causal structure.

$\Longrightarrow$ Limited to small cutoff variation range $\Lambda \approx \overline{\Lambda}_{\text{EFT}} \pm 20\%$, implementation & numerics more difficult.

Works *under assumption that* expansion indeed small, e.g. $1 + x + x^2 + x^3 - \dfrac{1}{1-x} = x^4 + \mathcal{O}(x^5)$ *if* $|x| < 1$.
But resummed version loses control over convergence test & over which interactions needed for
Λ-independence (UV changed!). May still provide some guidance/insight – but beware of missed CTs!

Thim/Ekström/Forssén PRC 109 (2024) 064001
[2402.15325] → Oliver Thim's talk



Order-by-order convergence ✓—- Uncertainty corridors from $\Lambda \in [500; 2500]$ MeV

(e) Not An Ivory Tower Exercise: Beyond-SM for $0\nu\beta\beta$, EDM and Dark Matter

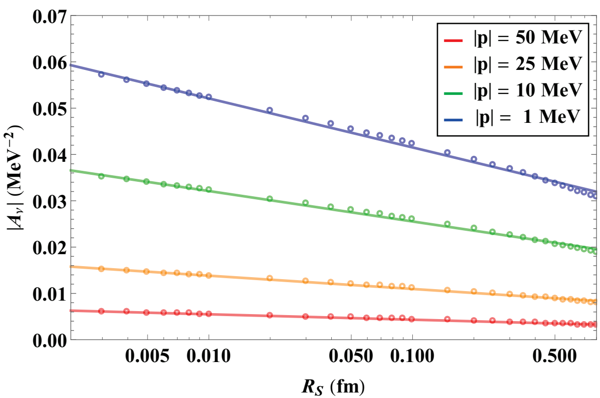
$\Longrightarrow$ “Unexpected” $2\mathrm{N}$ currents to absorb cutoff-dependence/restore RG-invariance & symmetries.

LO In Nuclear Matrix Elements for Dark Matter Detection: Hoferichter/Klos/Schwenk PLB 746 (2015) 410 [1503.04811]
de Vries/Köber/Nogga/Shain [2310.11343]

NLO In Nuclear Matrix Elements for Strong-CP Violation: de Vries/Gnech/Shain PRC 103 (2021) 012501 [2007.04927]

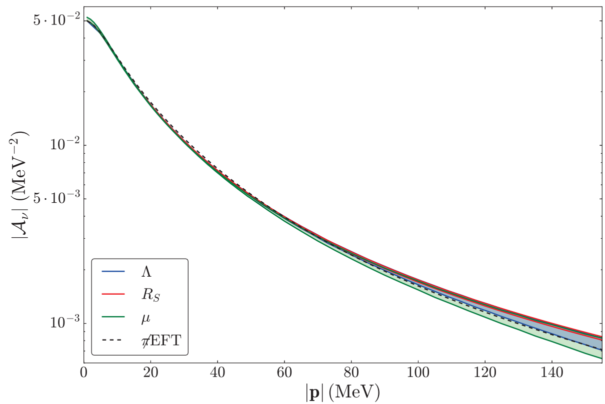
LO In Nuclear Matrix Elements for Neutrinoless Double-Beta Decay Detection:

Cirigliano/Dekens/de Vries/Graesser/Mereghetti/Pastore/van Kolck PRL120 (2018) 202001 [1802.10097]



RG/cutoff-dependence at fixed k without CT

(Weinberg’s Pragmatic Proposal)



RG/cutoff-corridor with k with CT

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