

Chiral Nuclear Forces with Gradient Flow Regulator

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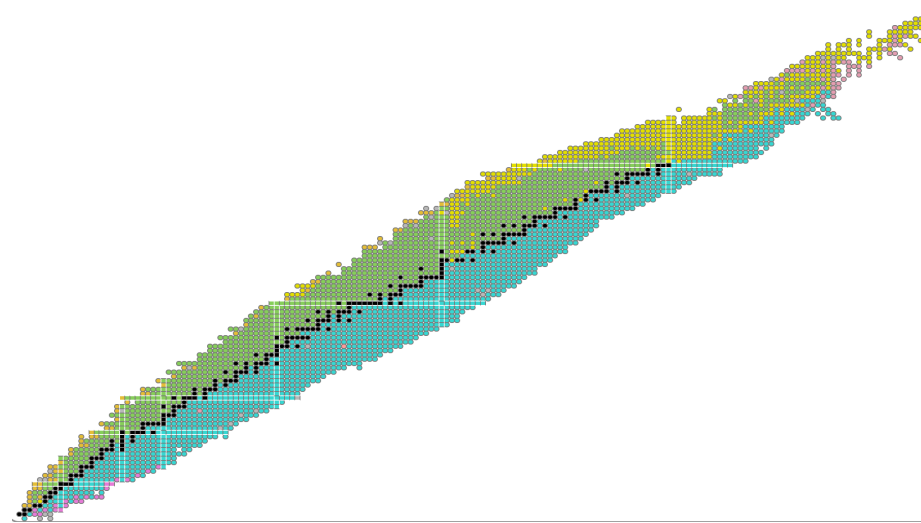
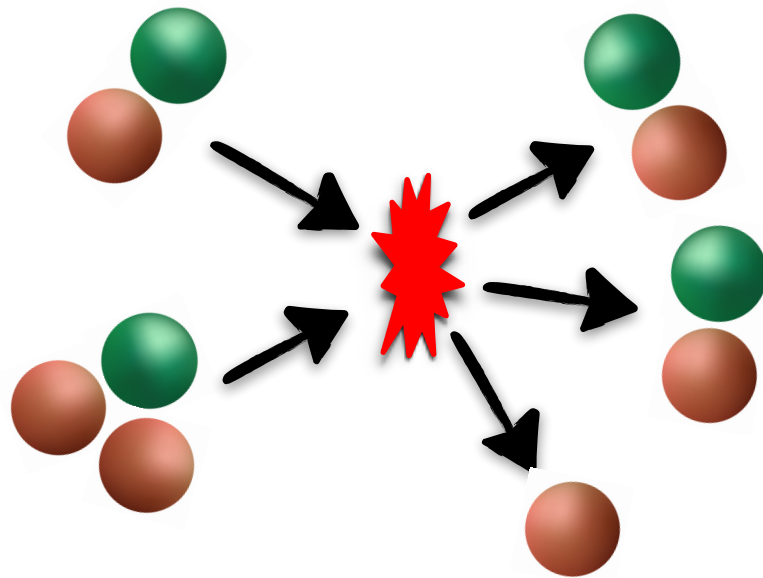
Standard Model EFT meets Chiral EFT
TRIUMF, Vancouver BC Canada
September 29, 2025



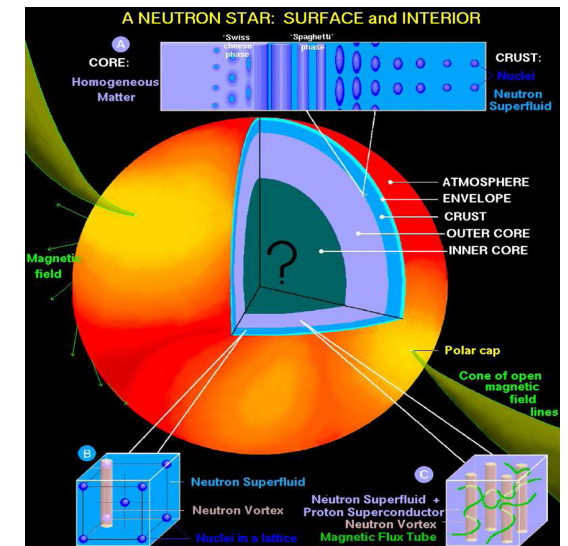
In collaboration with Evgeny Epelbaum

Outline

- Nuclear forces up to $N^4\text{LO}$
- Gradient-flow regularization of nuclear forces
- Path-integral method for derivation of nuclear forces
- Application: 3NF at $N^3\text{LO}$



Livechart, IAEA: <https://www.nds.iaea.org>

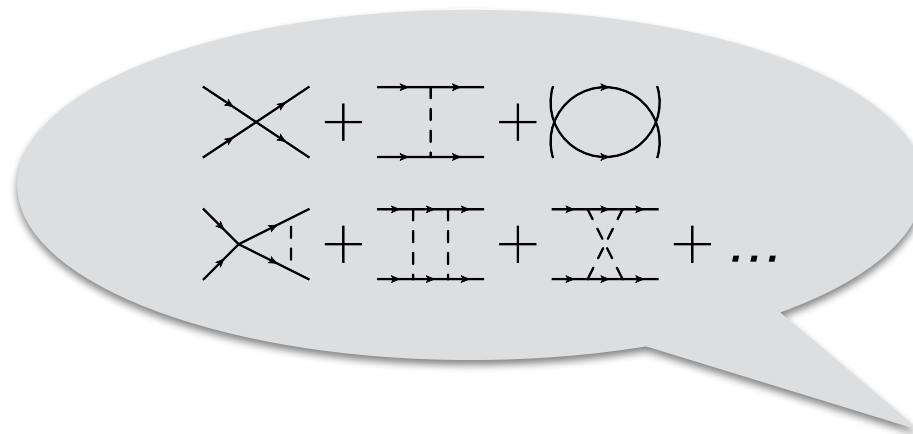


Lattimer: NAR54 (2010) 101

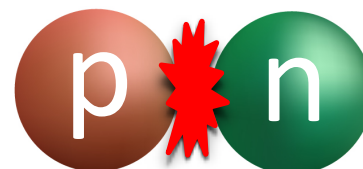
QM A-body problem

$$\left[\left(\sum_{i=1}^A \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3}) \right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \dots}_{\text{derived within ChPT}} \right] |\Psi\rangle = E|\Psi\rangle$$

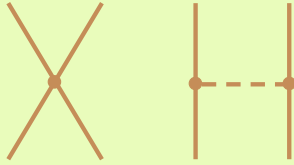
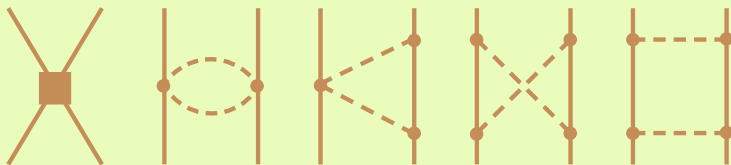
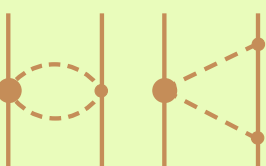
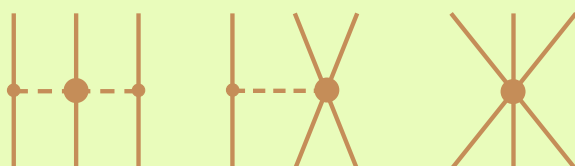
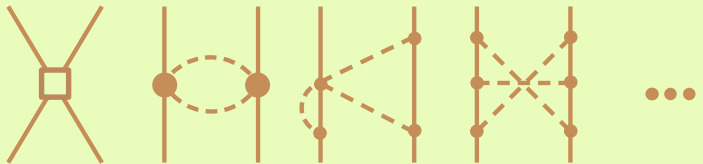
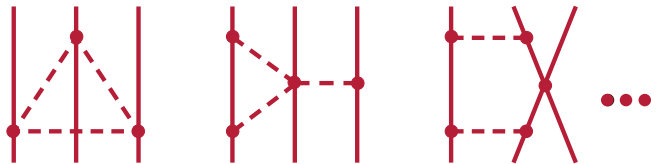
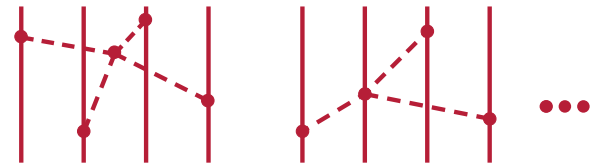
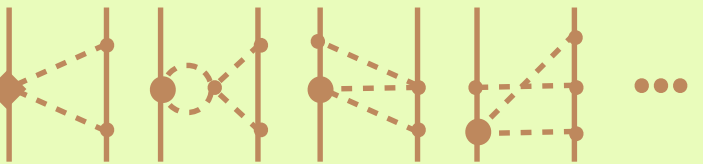
Weinberg '91



Chiral EFT is a systematic tool for derivation of nuclear forces below pion-production threshold



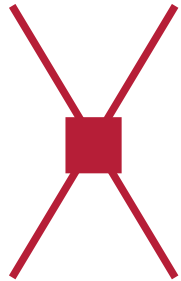
Chiral Expansion of the Nuclear Forces

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)	 Weinberg '90		
NLO (Q^2)	 Ordonez, van Kolck '92		
N ² LO (Q^3)	 Ordonez, van Kolck '92	 van Kolck '94; Epelbaum et al. '02	 Available matrix elements LENPIC '19
N ³ LO (Q^4)	 Kaiser '00 - '02	 [parameter-free] Bernard, Epelbaum, HK, Meißner, '08, '11	 [parameter-free] Epelbaum '06
N ⁴ LO (Q^5)	 Entem, Kaiser, Machleidt, Nosyk '15 Epelbaum, HK, Meißner '15	 Girlanda, Kievsky, Viviani '11 HK, Gasparyan, Epelbaum '12, '13 (short-range loop contrib. still missing)	 still have to be worked out

Adjustable Parameters in NN

Reinert, HK, Epelbaum PRL126 (2021) 092501

Couplings of short-range interactions are fixed from NN - data



LO [Q^0]: 2 operators (S-waves)
NLO [Q^2]: + 7 operators (S-, P-waves and ε_1)
N²LO [Q^3]: no new terms
N³LO [Q^4]: + 12 operators (S-, P-, D-waves and $\varepsilon_1, \varepsilon_2$)
N⁴LO [Q^5]: + 5 IB operators
N⁴LO⁺ [Q^6]: + 4 operators (F-waves)

of adjustable LECs = 25 IC + 5 IB + 3 π N constants = 33 parameters

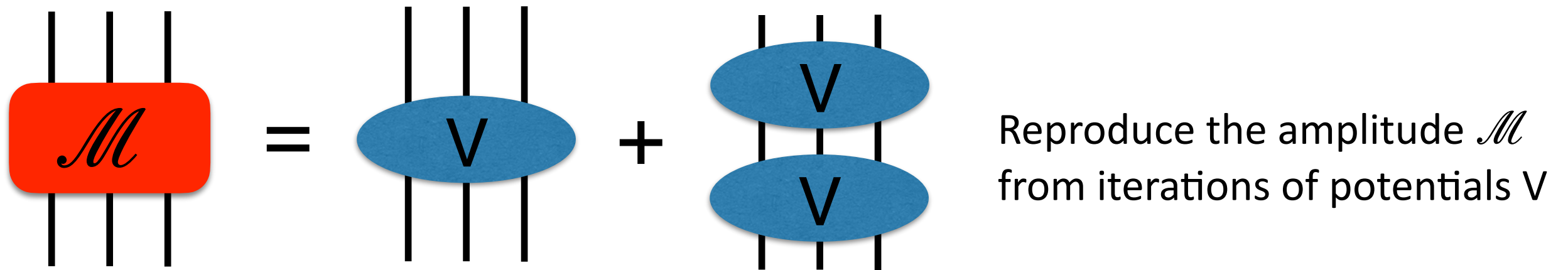
Summary on NN

- Employed a Bayesian approach to account for statistical and systematic uncertainties
- Extracted π N couplings from NN data within chiral EFT
- Achieved a statistically perfect description of NN data
 $\chi^2/\text{dat} = 1.005$ for ~ 5000 data in the energy range $E_{\text{lab}} = 0 - 280$ MeV

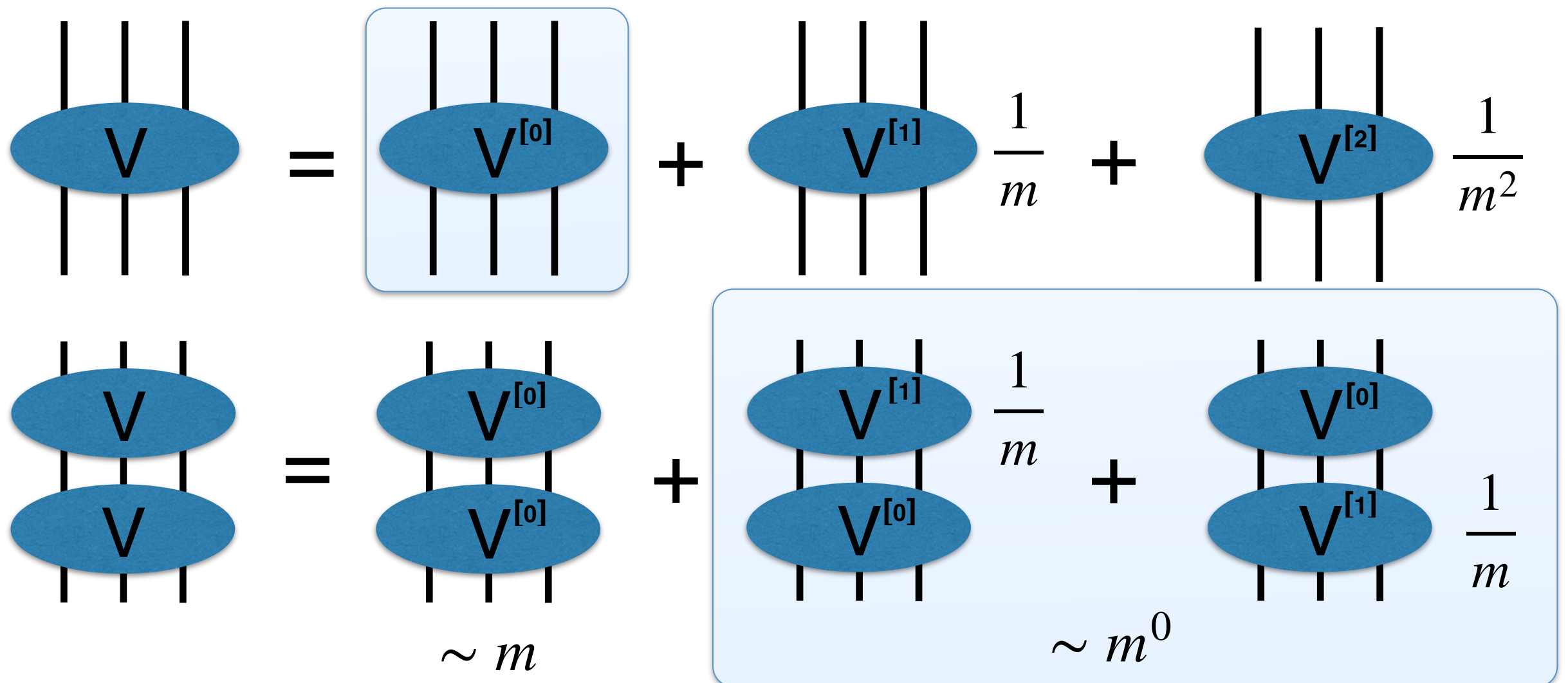
3NF at N3LO

Issues with Chiral Symmetry

3NF Amplitude from Iteration of Forces



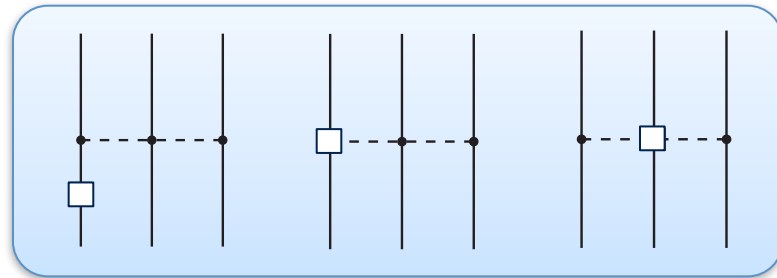
Free Green function scales as m/p^2



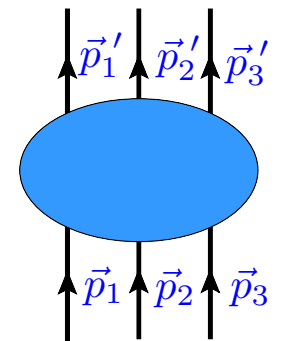
Call for Consistent Regularization

Violation of chiral symmetry at N3LO due to different regularizations: Dim. reg. vs cutoff reg.

Epelbaum, HK, Reinert, *Front. in Phys.* 8 (2020) 98



← $1/m$ - corrections to TPE 3NF $\sim g_A^2$



$$V_{2\pi,1/m}^{g_A^2} = i \frac{g_A^2}{32mF_\pi^4} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3}{(q_1^2 + M_\pi^2)(q_3^2 + M_\pi^2)} \tau_1 \cdot (\tau_2 \times \tau_3) (2\vec{k}_1 \cdot \vec{q}_3 + 4\vec{k}_3 \cdot \vec{q}_3 + i [\vec{q}_1 \times \vec{q}_3] \cdot \vec{\sigma}_2)$$

$$\vec{q}_i = \vec{p}_i' - \vec{p}_i$$

$$\vec{k}_i = \frac{1}{2} (\vec{p}_i' + \vec{p}_i)$$

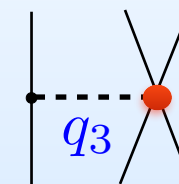
Naive local cut-off regularization of the current and potential

$$V_{2\pi,1/m}^{g_A^2,\Lambda} = V_{2\pi,1/m}^{g_A^2} \exp\left(-\frac{q_1^2 + M_\pi^2}{\Lambda^2}\right) \exp\left(-\frac{q_3^2 + M_\pi^2}{\Lambda^2}\right) \quad \& \quad V_{1\pi}^{Q^0,\Lambda} = -\frac{g_A^2}{4F_\pi^2} \tau_1 \cdot \tau_2 \frac{\vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2}{q_1^2 + M_\pi^2} \exp\left(-\frac{q_1^2 + M_\pi^2}{\Lambda^2}\right)$$

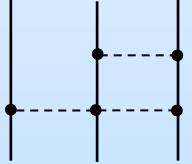
First iteration with OPE NN potential

$$V_{2\pi,1/m}^{g_A^2,\Lambda} \frac{1}{E - H_0 + i\epsilon} V_{1\pi}^{Q^0,\Lambda} + V_{1\pi}^{Q^0,\Lambda} \frac{1}{E - H_0 + i\epsilon} V_{2\pi,1/m}^{g_A^2,\Lambda} = \Lambda \frac{g_A^4}{128\sqrt{2}\pi^{3/2}F_\pi^6} (\tau_2 \cdot \tau_3 - \tau_1 \cdot \tau_3) \frac{\vec{q}_2 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3}{q_3^2 + M_\pi^2} + \dots$$

No such D-like term in chiral Lagrangian



The problematic divergence is canceled by the one $V_{2\pi-1\pi}$ if calculated via cutoff regularization

In dim. reg. $V_{2\pi-1\pi} =$  $+ \dots$ is finite

Symmetry Preserving Regulator

HK, Epelbaum, PRC 110 (2024) 4, 044004

Gradient-Flow Equation (GFE)

Balitsky, Yung, PL168B (1986) 113; Irwin, Manton, PLB 385 (1996) 187

Yang-Mills gradient flow in QCD: Lüscher, JHEP 04 (2013) 123

$$\partial_\tau B_\mu = D_\nu G_{\nu\mu} \text{ with } B_\mu|_{\tau=0} = A_\mu \text{ \& } G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

B_μ is a regularized gluon field

- Apply this idea to ChPT: HK, Epelbaum, PRC 110 (2024) 4, 044004

(Proposed in various talks by D. Kaplan for nuclear forces)

Introduce a smoothed pion field W with $W|_{\tau=0} = U$ satisfying GFE

$$\partial_\tau W = i w \text{EOM}(\tau) w \text{ with } w = \sqrt{W} \text{ and } \text{EOM}(\tau) = [D_\mu, w_\mu] + \frac{i}{2}\chi_- - \frac{i}{4}\text{Tr}(\chi_-)$$

$$w_\mu = i(w^\dagger(\partial_\mu - i r_\mu)w - w(\partial_\mu - i l_\mu)w^\dagger), \quad \chi_- = w^\dagger\chi w^\dagger - w\chi^\dagger w, \quad \chi = 2B(s + ip)$$

Note: The shape of regularization is dictated by the choice of the right-hand side of GFE

- Our choice is motivated by a Gaussian regularization of one-pion-exchange in NN

Gradient-Flow Equation

Analytic solution is possible of $1/F$ - expanded gradient flow equation:

$$W = 1 + i\tau \cdot \phi(1 - \alpha\phi^2) - \frac{\phi^2}{2} \left[1 + \left(\frac{1}{4} - 2\alpha \right) \phi^2 \right] + \mathcal{O}(\phi^5), \quad \phi_b = \sum_{n=0}^{\infty} \frac{1}{F^n} \phi_b^{(n)}$$

In the absence of external sources we have

$$[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)] \phi_b^{(1)}(x, \tau) = 0, \quad \phi_b^{(1)}(x, 0) = \pi_b(x)$$

$$[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)] \phi_b^{(3)}(x, \tau) = (1 - 2\alpha) \partial_\mu \phi^{(1)} \cdot \partial_\mu \phi^{(1)} \phi_b^{(1)} - 4\alpha \partial_\mu \phi^{(1)} \cdot \phi^{(1)} \partial_\mu \phi_b^{(1)} \\ + \frac{M^2}{2} (1 - 4\alpha) \phi^{(1)} \cdot \phi^{(1)} \phi_b^{(1)}, \quad \phi_b^{(3)}(x, 0) = 0$$

Iterative solution in momentum space: $\tilde{\phi}^{(n)}(q, \tau) = \int d^4x e^{iq \cdot x} \phi_b^{(n)}(x, \tau)$

$$\tilde{\phi}_b^{(1)}(q) = e^{-\tau(q^2 + M^2)} \tilde{\pi}_b(q)$$

$$\tilde{\phi}_b^{(3)}(q) = \int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4} \frac{d^4q_3}{(2\pi)^4} (2\pi)^4 \delta(q - q_1 - q_2 - q_3) \int_0^\tau ds e^{-(\tau-s)(q^2 + M^2)} e^{-s \sum_{j=1}^3 (q_j^2 + M^2)} \\ \times \left[4\alpha q_1 \cdot q_3 - (1 - 2\alpha) q_1 \cdot q_2 + \frac{M^2}{2} (1 - 4\alpha) \right] \tilde{\pi}(q_1) \cdot \tilde{\pi}(q_2) \tilde{\pi}_b(q_3)$$

Integration over momenta of pion fields with Gaussian prefactor introduces smearing

Iterative solution in Coordinate Space

$$\phi(x_\mu, \tau) = \text{[Diagram 1]} + \text{[Diagram 2]} + \dots$$

[integrated over $\vec{x}_1, t_1, \tau_1$]

Light-shaded area visualizes smearing in Euclidean space of size $\sim \sqrt{2\tau}$

Solid line stands for Green-function:

$$[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)] G(x - y, \tau - s) = \delta(x - y) \delta(\tau - s)$$

$$G(x, \tau) = \theta(\tau) \int \frac{d^4 q}{(2\pi)^4} e^{-\tau(q^2 + M^2)} e^{-i q \cdot x}$$

$$\phi_b^{(1)}(x, \tau) = \int d^4 y G(x - y, \tau) \pi_b(y)$$

$$\begin{aligned} \phi_b^{(3)}(x, \tau) = & \int_0^\tau ds \int d^4 y G(x - y, \tau - s) \left[(1 - 2\alpha) \partial_\mu \phi^{(1)}(y, s) \cdot \partial_\mu \phi^{(1)}(y, s) \phi_b^{(1)}(y, s) \right. \\ & \left. - 4\alpha \partial_\mu \phi^{(1)}(y, s) \cdot \phi^{(1)}(y, s) \partial_\mu \phi_b^{(1)}(y, s) + \frac{M^2}{2} \phi^{(1)}(y, s) \cdot \phi^{(1)}(y, s) \phi_b^{(1)}(y, s) \right] \end{aligned}$$

Properties under Chiral Transformation

Replace all pion fields in pion-nucleon Lagrangians $\mathcal{L}_{\pi N}^{(1)}, \dots, \mathcal{L}_{\pi N}^{(4)}$: $U \rightarrow W$

$$\mathcal{L}_{\pi N}^{(1)} = N^\dagger \left(D^0 + g u \cdot S \right) N \rightarrow N^\dagger \left(D_w^0 + g w \cdot S \right) N$$

Chiral transformation: by induction, one can show

$$U \rightarrow RUL^\dagger \rightarrow W \rightarrow RWL^\dagger$$

- Regularized pion fields transform under τ - independent transformations
- Nucleon fields transform in τ - dependent way

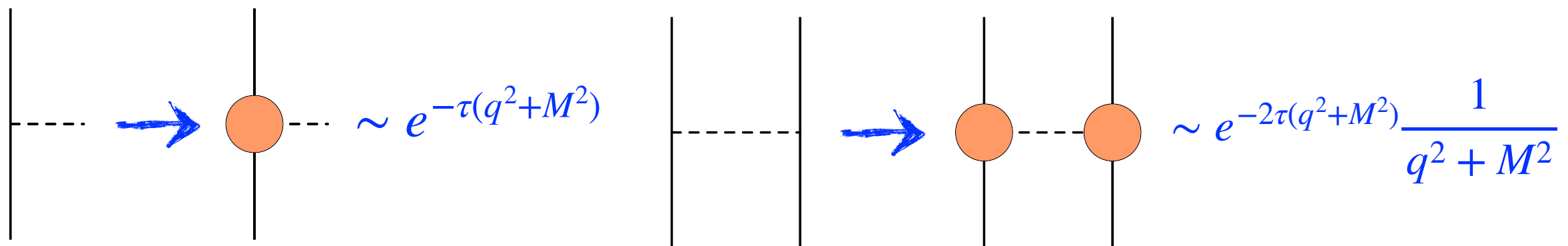
$$N \rightarrow KN, \quad K = \sqrt{LU^\dagger R^\dagger R} \sqrt{U} \rightarrow N \rightarrow K_\tau N, \quad K_\tau = \sqrt{LW^\dagger R^\dagger R} \sqrt{W}$$

Regularization for Nuclear Forces

To regularize long-range part of the nuclear forces and currents

- Leave pionic Lagrangians $\mathcal{L}_\pi^{(2)}$ & $\mathcal{L}_\pi^{(4)}$ unregularized (essential)
- Replace all pion fields in pion-nucleon Lagrangians $\mathcal{L}_{\pi N}^{(1)}, \dots, \mathcal{L}_{\pi N}^{(4)}$: $U \rightarrow W$

$$\mathcal{L}_{\pi N}^{(1)} = N^\dagger \left(D^0 + g u \cdot S \right) N \rightarrow N^\dagger \left(D_w^0 + g w \cdot S \right) N$$



For $\tau = \frac{1}{2\Lambda^2}$ this regulator reproduces SMS regularization of OPE

Quantization of regularized Interactions

Difficulties in formulation of regularized chiral EFT

- Gradient-flow prescription $U \rightarrow W$ in pion-nucleon Lagrangian leads to higher number of time-derivative dependence in the interaction

$$W = 1 + i\tau \cdot \phi(1 - \alpha\phi^2) - \frac{\phi^2}{2} \left[1 + \left(\frac{1}{4} - 2\alpha \right) \phi^2 \right] + \mathcal{O}(\phi^5), \quad \phi_b = \sum_{n=0}^{\infty} \frac{1}{F^n} \phi_b^{(n)}$$

$$\phi_b^{(1)}(x) = \exp(\tau[\partial_\mu \partial_\mu - M^2]) \pi_b(x) = \exp(\tau[\partial_0^2 + \vec{\nabla}^2 - M^2]) \pi_b(x)$$

→ Canonical quantization of the pion-field becomes difficult
(Ostrogradski - approach, Constrains, ...)

- In ChPT in zero- and one-nucleon sector vertices with higher number of derivatives can appear and would lead to similar problem

Solution: use path-integral quantization Gasser, Leutwyler, *Annals Phys.* 158 (1984) 142;
Bernard, Kaiser, Kambor, Meißner, *Nucl. Phys. B* 388 (1992) 315

Can we choose a path-integral formulation in two- and more-nucleon sector?

Path-Integral Framework for Derivation of Nuclear Forces

HK, Epelbaum, PRC110 (2024) 4, 044003

Illustration fo Yukawa-like Model

We start with generating functional:

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN][D\pi] \exp\left(i \int d^4x (\mathcal{L} + \eta^\dagger(x)N(x) + N^\dagger(x)\eta(x))\right)$$

Yukawa-like toy-model:

$$\mathcal{L} = N^\dagger \left(i \frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} + \frac{g}{2F} \vec{\sigma} \cdot \vec{\nabla} \boldsymbol{\pi} \cdot \boldsymbol{\tau} \right) N + \frac{1}{2} (\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - M^2 \boldsymbol{\pi}^2)$$

- Perform a Gaussian path-integral over the pion fields

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN] \exp\left(i S_N + i \int d^4x (\eta^\dagger(x)N(x) + N^\dagger(x)\eta(x))\right)$$

$$S_N = \int d^4x N^\dagger(x) \left(i \frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} \right) N(x) - V_{NN} \quad \leftarrow \text{Non-instant one-pion-exchange interaction}$$

$$V_{NN} = -\frac{g^2}{8F^2} \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \boldsymbol{\tau}] N(x) \Delta_F(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \boldsymbol{\tau}] N(y)$$

with non-instant pion propagator: $\Delta_F(x) = \int \frac{d^4q}{(2\pi)^4} \frac{e^{-iq \cdot x}}{q^2 - M^2 + i\epsilon}$

Instant Interactions from Path-Integral

To transform V_{NN} into an instant form we rewrite a pion propagator

$$\frac{1}{q_0^2 - \omega_q^2} = -\frac{1}{\omega_q^2} + \frac{1}{q_0^2 - \omega_q^2} + \frac{1}{\omega_q^2} = -\frac{1}{\omega_q^2} + q_0^2 \frac{1}{\omega_q^2} \frac{1}{q_0^2 - \omega_q^2}, \quad \omega_q = \sqrt{\vec{q}^2 + M^2}$$

In coordinate space this corresponds to $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2} \Delta_{FS}(x)$ with

$$\Delta_S(x) = - \int \frac{d^4 q}{(2\pi)^4} \frac{e^{-i q \cdot x}}{\omega_q^2} = - \delta(x_0) \int \frac{d^3 q}{(2\pi)^3} \frac{e^{i \vec{q} \cdot \vec{x}}}{\omega_q^2}, \quad \Delta_{FS}(x) = \int \frac{d^4 q}{(2\pi)^4} \frac{e^{-i q \cdot x}}{\omega_q^2 (q_0^2 - \omega_q^2)}$$

• The decomposition $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2} \Delta_{FS}(x)$ can be generalized

$$G(x) = \int \frac{d^4 q}{(2\pi)^4} e^{-i q \cdot x} \tilde{G}(q_0^2, q^2) \text{ and } \tilde{G}(q_0^2, q^2) \text{ is differentiable at } q_0 = 0$$

$$\text{Defining } G_S(x) = \int \frac{d^4 q}{(2\pi)^4} e^{-i q \cdot x} \tilde{G}(0, q^2) \text{ and } G_{FS}(x) = \int \frac{d^4 q}{(2\pi)^4} e^{-i q \cdot x} \frac{\tilde{G}(q_0^2, q^2) - \tilde{G}(0, q^2)}{q_0^2}$$

$$\rightarrow G(x) = G_S(x) - \frac{\partial^2}{\partial x_0^2} G_{FS}(x)$$

Instant Interactions from Path-Integral

Perform an instant decomposition of the pion propagator $\Delta_F(x) = \Delta_S(x) - \frac{\partial^2}{\partial x_0^2} \Delta_{FS}(x)$

$$V_{NN} = -\frac{g^2}{8F^2} \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \vec{\tau}] N(x) \Delta_F(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \vec{\tau}] N(y)$$

$$\rightarrow V_{NN} = V_{OPE} + V_{FS}$$

$$V_{OPE} = -\frac{g^2}{8F^2} \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \vec{\tau}] N(x) \Delta_S(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \vec{\tau}] N(y) \quad \text{is instant}$$

$$V_{FS} = \frac{g^2}{8F^2} \int d^4x d^4y \vec{\nabla}_x \cdot [N^\dagger(x) \vec{\sigma} \vec{\tau}] N(x) \frac{\partial^2}{\partial x_0^2} \Delta_{FS}(x-y) \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \vec{\tau}] N(y) \quad \text{is non-instant}$$

V_{FS} is time-derivative dependent and thus can be eliminated by a non-polynomial field redefinition

$$N(x) \rightarrow N'(x) = N(x) + i \frac{g^2}{8F^2} \int d^4y [\vec{\sigma} \vec{\tau} N(x)] \cdot [\vec{\nabla}_x \frac{\partial}{\partial x_0} \Delta_{FS}(x-y)] \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \vec{\tau} N(y)]$$

$$N^\dagger(x) \rightarrow N'^\dagger(x) = N^\dagger(x) - i \frac{g^2}{8F^2} \int d^4y \vec{\nabla}_y \cdot [N^\dagger(y) \vec{\sigma} \vec{\tau} N(y)] [\vec{\nabla}_y \frac{\partial}{\partial y_0} \Delta_{FS}(y-x)] \cdot [N^\dagger(x) \vec{\sigma} \vec{\tau}]$$

Instant Interactions from Path-Integral

Non-local field transformations remove time-derivative dependent two-nucleon interactions but generate time-derivative dependent three-nucleon interactions.

These contributions can be eliminated by similar field transformations

$$Z[\eta^\dagger, \eta] = \int [DN'^\dagger][DN'] \det \left(\frac{\delta(N'^\dagger, N')}{\delta(N^\dagger, N)} \right) \exp \left(i S_{N(N^\dagger, N')} + i \int d^4x (\eta^\dagger(x) N(N'^\dagger, N')(x) + N(N'^\dagger, N')^\dagger(x) \eta(x)) \right)$$

$$\simeq \int [DN'^\dagger][DN'] \det \left(\frac{\delta(N'^\dagger, N')}{\delta(N^\dagger, N)} \right) \exp \left(i S_{N(N^\dagger, N')} + i \int d^4x (\eta^\dagger(x) N'(x) + N'^\dagger(x) \eta(x)) \right)$$


Equivalence theorem: nucleon pole-structure is unaffected by the field-transf.

$$S_{N(N^\dagger, N')} = \int d^4x N'^\dagger(x) \left(i \frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} \right) N'(x) - V_{OPE} + \mathcal{O}(g^4)$$

$$V_{OPE} = -\frac{g^2}{8F^2} \int d^4x d^4y \vec{\nabla}_x \cdot [N'^\dagger(x) \vec{\sigma} \vec{\tau}] N'(x) \Delta_S(x-y) \vec{\nabla}_y \cdot [N'^\dagger(y) \vec{\sigma} \vec{\tau}] N'(y)$$



Instant one-pion-exchange interaction

One-Loop Corrections to Interaction

One loop corrections to NN & NNN interaction come from functional determinant

$$\det \left(\frac{\delta(N'^{\dagger}, N')}{\delta(N^{\dagger}, N)} \right) = \exp \left(\text{Tr} \log \frac{\delta(N'^{\dagger}, N')}{\delta(N^{\dagger}, N)} \right)$$

Due to non-local structure of field transformations $\det \left(\frac{\delta(N'^{\dagger}, N')}{\delta(N^{\dagger}, N)} \right) \neq 1$

$$S_{N(N^{\dagger}, N')} = \int d^4x N'^{\dagger}(x) \left(i \frac{\partial}{\partial x_0} + \frac{\vec{\nabla}^2}{2m} + \frac{3g^2 M^3}{32\pi F^2} \right) N'(x) - V_{OPE} + \mathcal{O}(g^4)$$



Nucleon mass-shift **Langacker, Pagels, PRD 10 (1974) 2904**
is reproduced from functional determinant

Note: The Z-factor of the nucleon is equal to one. This is due to the replacement

$$\eta^{\dagger} N + N^{\dagger} \eta \rightarrow \eta^{\dagger} N' + N'^{\dagger} \eta \quad \text{in the generating functional } Z[\eta^{\dagger}, \eta]$$

The original Z-factor of the nucleon is reproduced if we remove this replacement

$$Z = 1 - \frac{9M^2 g^2}{2F^2} \left(\bar{\lambda} + \frac{1}{16\pi^2} \left(\log \frac{M}{\mu} + \frac{1}{3} - \frac{\pi M}{2\mu} \right) \right)$$

Path-integral Approach

We start with generating functional:

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN][D\pi] \exp\left(i \int d^4x (\mathcal{L}_\pi + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \mathcal{L}_{NNN} + \eta^\dagger(x)N(x) + N^\dagger(x)\eta(x))\right)$$

- Integrate over pion fields via loop-expansion of the action
 - ➔ expansion of the action around the classical pion solution
- Perform instant decomposition of the remaining interactions between nucleons
- Perform nucleon-field redefinitions to eliminate non-instant part of the interaction
- Calculate functional determinant to get loop corrections to few-nucleon forces

Checks in dimensional regularization

Unitary transformation (Okubo) & path-integral approaches lead to the same chiral EFT nuclear forces up to N⁴LO

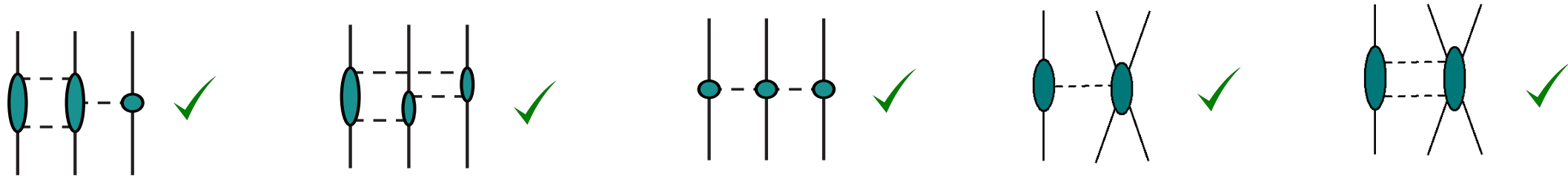
Status Report on 3NF

3NF up to N⁴LO

	Long - range			Short - range			
NLO							
N ² LO	van Kolck '94, Epelbaum et al. '02						
N ³ LO				...			...
	Ishikawa, Robilotta, PRC76 (07); Bernard, Epelbaum, HK, Meißner, PRC77 (08); PRC84 (11)			Bernard, Epelbaum, HK, Meißner, PRC84 (11)			
N ⁴ LO				...			
	HK, Gasparyan, Epelbaum PRC85 (12); PRC87 (13)			Work in progress			
	Girlanda, Kievsky, Viviani, PRC84 (11)						
	2π - 1π	ring	2π				

Status Report on 3N at N³LO

- We calculated all long- and short-range contributions to 3NF & 4NF at N³LO



3NF's are given in terms of integrals over Schwinger parameters

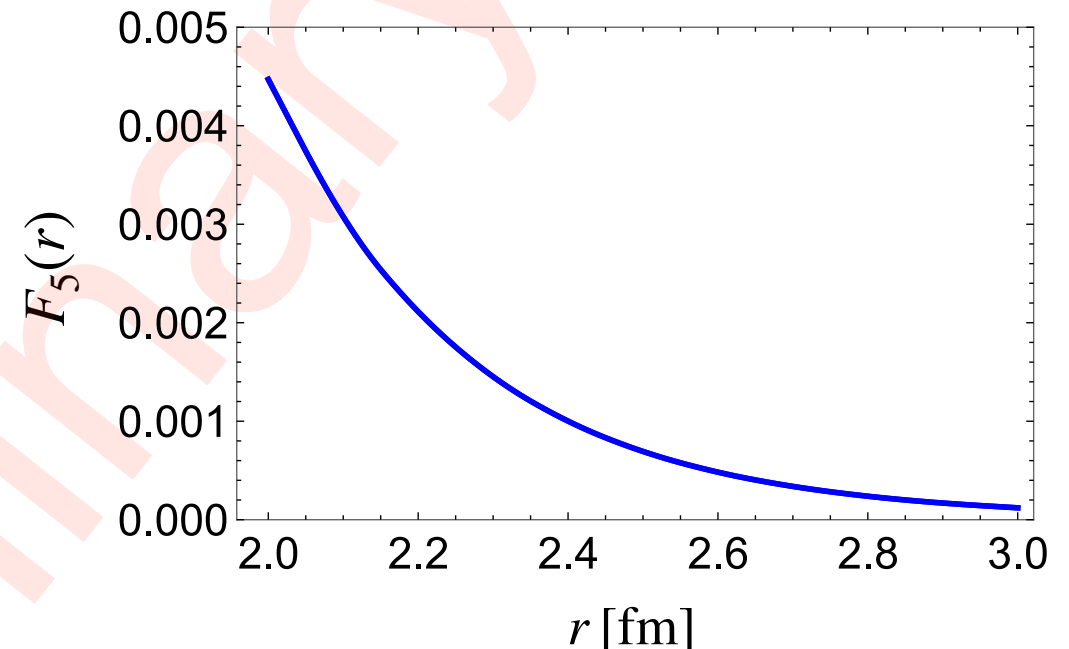
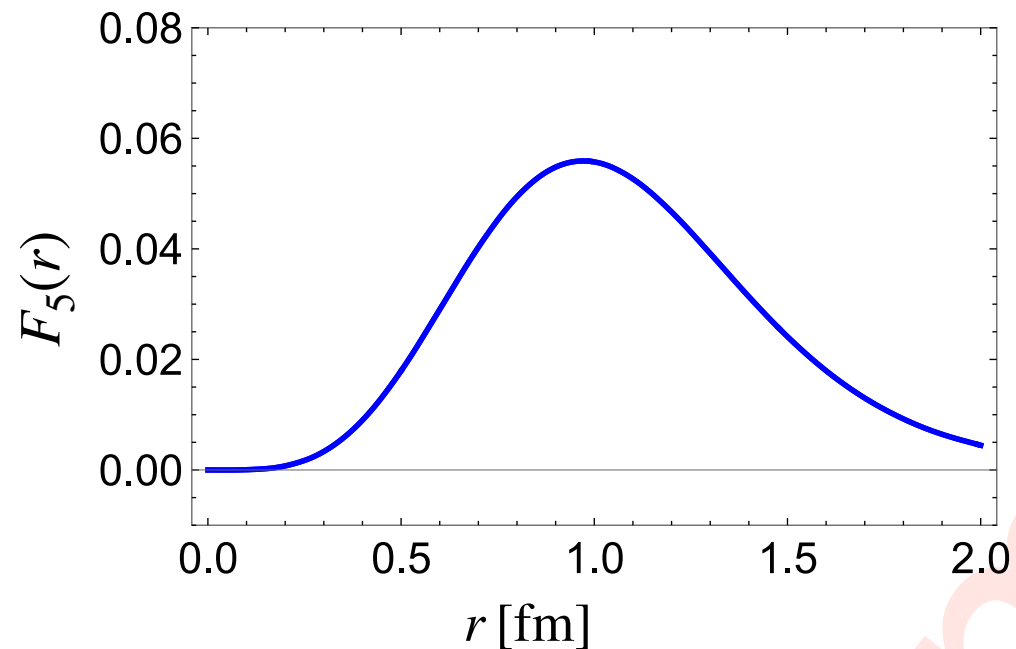
$$V_{3N}^{2\pi-1\pi} = \tau_1 \cdot \tau_2 \times \tau_3 \vec{q}_1 \cdot \vec{\sigma}_1 \times \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3 \frac{e^{-\frac{q_3^2 + M_\pi^2}{\Lambda^2}}}{q_3^2 + M_\pi^2} \left(-\frac{g_A^4}{F_\pi^6} \frac{q_1}{2048\pi} \int_0^\infty d\lambda \operatorname{erfi}\left(\frac{q_1 \lambda}{2\Lambda\sqrt{2+\lambda}}\right) \frac{\exp\left(-\frac{q_1^2 + 4M_\pi^2}{4\Lambda^2}(2+\lambda)\right)}{2+\lambda} + \dots \right) + \dots$$

Dimension of integrals over Schwinger parameters depends on topology

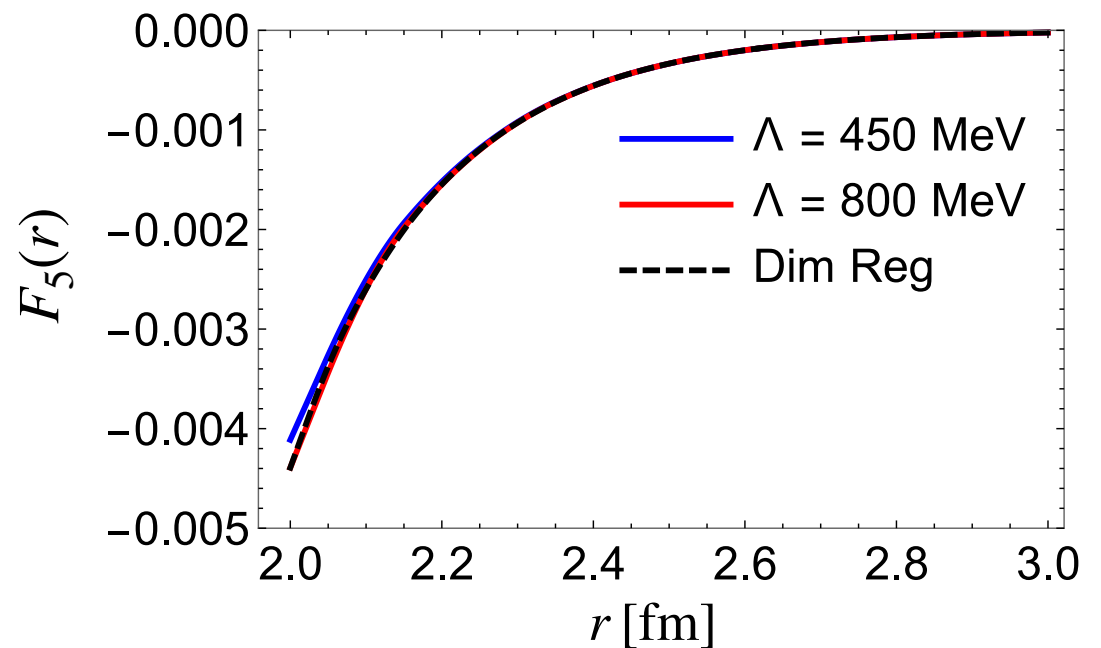
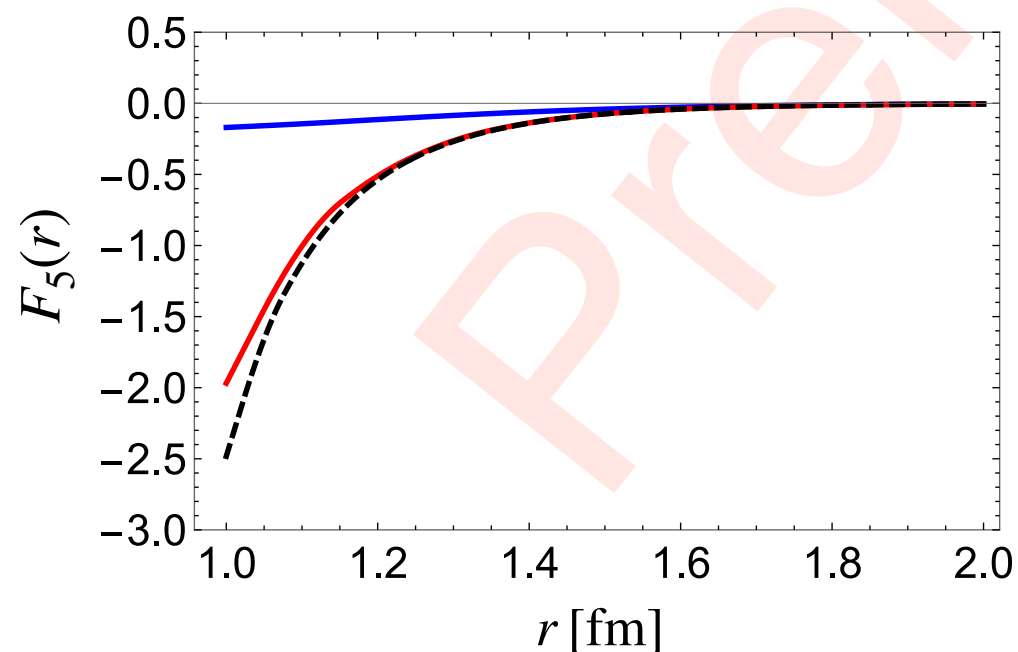
Space			
Momentum	2	1	3
Coordinate	4	1	0

Selected Profile Functions

$$V_{3N}^{\text{ring}} = F_1(r_{12}, r_{23}, r_{13}) + \dots + \tau_2 \cdot \tau_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2 F_5(r_{12}, r_{23}, r_{13}) + \dots \quad F_5(r) = F_5(r, r, r) \text{ [MeV]}$$

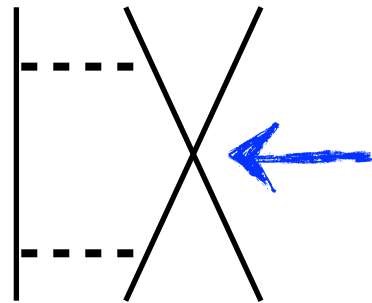


At $\Lambda \rightarrow \infty$ regularized 3NF reproduce dim. reg. results from [Bernard et al. PRC77 \(08\)](#)



Short Range 3NF at N³LO

Complication in calculation of short-range 3NF due to non-local regulator of LO NN



Non-local regulator of short-range NN at LO introduces additional momentum in loop functions

Structure functions of short-range 3NF can become complex

Time-reversal transformation (T): $\vec{\sigma}_j \rightarrow -\vec{\sigma}_j$, $\tau_j^y \rightarrow -\tau_j^y$, $\vec{q}_j \rightarrow \vec{q}_j$, $\vec{k}_j \rightarrow -\vec{k}_j$

Hermitian conjugation (h.c.): $\vec{\sigma}_j \rightarrow \vec{\sigma}_j$, $\tau_j \rightarrow \tau_j$, $\vec{q}_j \rightarrow -\vec{q}_j$, $\vec{k}_j \rightarrow \vec{k}_j$

$$\exp\left(-\frac{(2(\vec{k}_2 - \vec{k}_3) + \vec{q}_2)^2}{8\Lambda^2}\right) + \exp\left(-\frac{(2(\vec{k}_2 - \vec{k}_3) - \vec{q}_2)^2}{8\Lambda^2}\right) \quad \text{Invariant under T and h.c.}$$

$$i \left[\exp\left(-\frac{(2(\vec{k}_2 - \vec{k}_3) + \vec{q}_2)^2}{8\Lambda^2}\right) - \exp\left(-\frac{(2(\vec{k}_2 - \vec{k}_3) - \vec{q}_2)^2}{8\Lambda^2}\right) \right] \quad \text{Invariant under T and h.c.}$$

- Combination of these functions are allowed to appear in structure functions

➔ Structure functions might be complex: not related to unitarity cut (phase)

Short Range 3NF at N³LO

Complex structure functions of short-range part of 3NF require complex PWD

Solution 1: Is there a nucleon-field transformation which would make 3NF's real?

Idea: Constrain field transformations needed to make interactions instant

Every ϵ_{ijk} in field transformations should be accompanied with an „ i “

→ Indeed, we achieved with these transformations an instant 3NF and get real structure functions for short-range 3NF

Solution 2: Change the regulator of short-range NN interaction at LO to local one

→ Short-range 3NF's at N³LO becomes local and automatically real

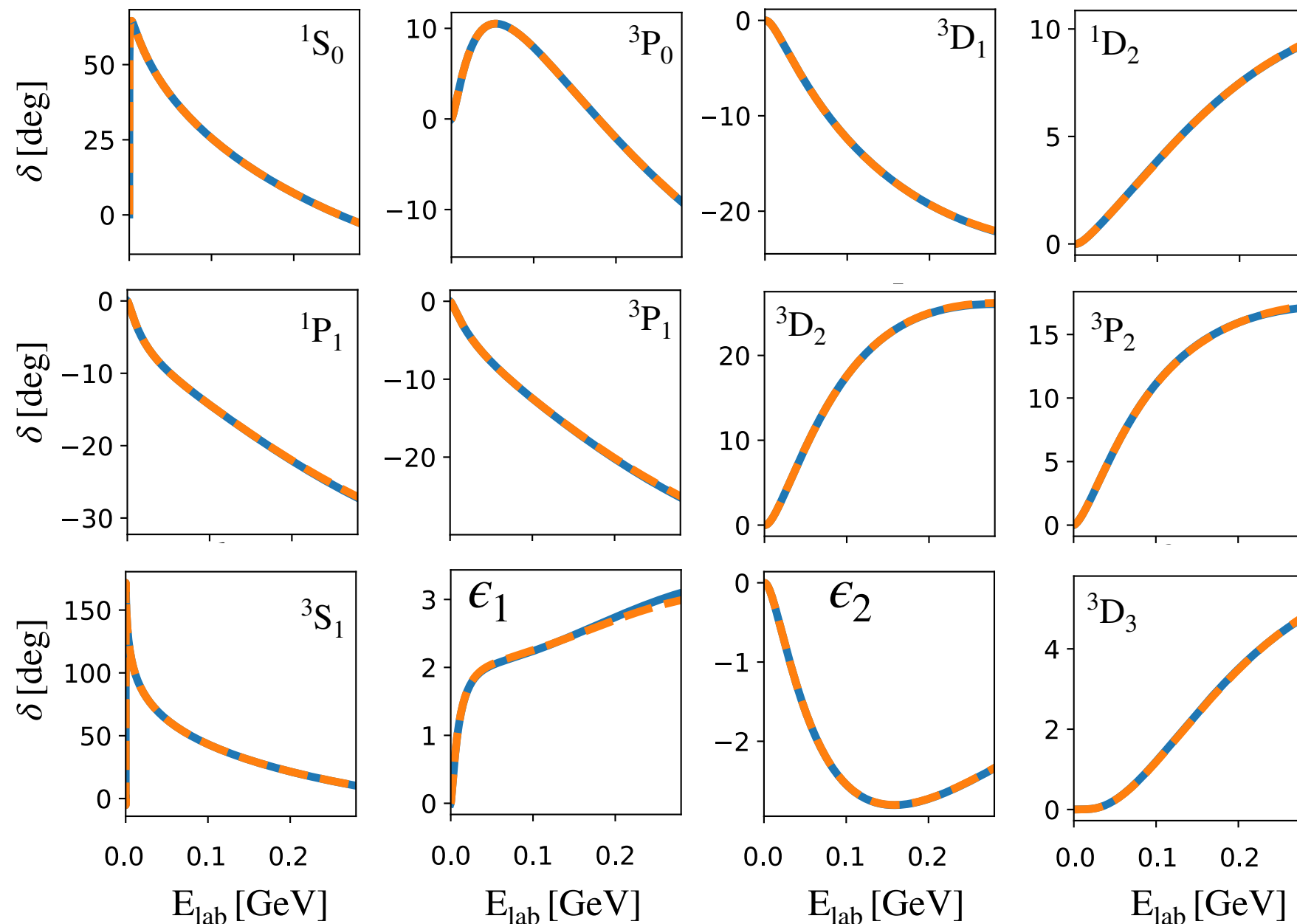
→ Expressions for local short-range 3NF's at N³LO are simpler

→ PWD of local 3NF's is less expensive

But: we need to generate a new NN force

NN phase shifts and mixing angles

Heihoff et al. : forthcoming



$\Lambda = 450 \text{ MeV}$

● Quality of nuclear force does not change when we change the regulator of the LO short-range NN interaction

● Local regularization of the LO short-range NN leads to simpler 3NF at N³LO

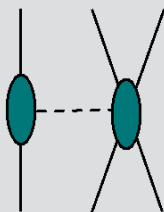
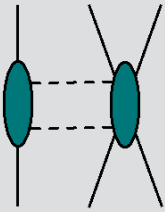
— Local short-range regulator at LO: $\exp(-q^2/\Lambda^2)$, $\chi^2 = 1.0069$

- - Non-local short-range regulator at LO: $\exp(-(p'^2 + p^2)/\Lambda^2)$, $\chi^2 = 1.0062$

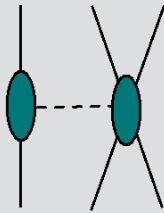
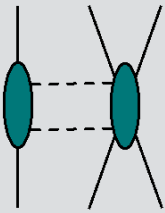
Short Range 3NF at N³LO

We followed both paths and provide two versions of 3NF

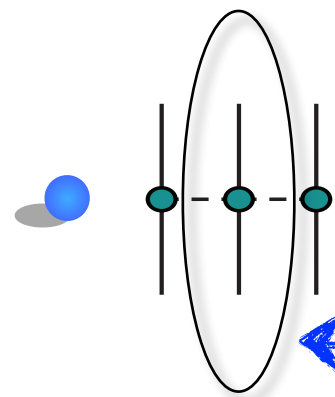
Version 1: Non-local short-range 3NF which can be used with SMS potential

Space			2.4 MB
Momentum	1	1	

Version 2: Local short-range 3NF to be used with the new NN potential

Space			0.4 MB
Momentum	1	1	
Coordinate	0	0	

Pion-Nucleon Scattering up to Q^3

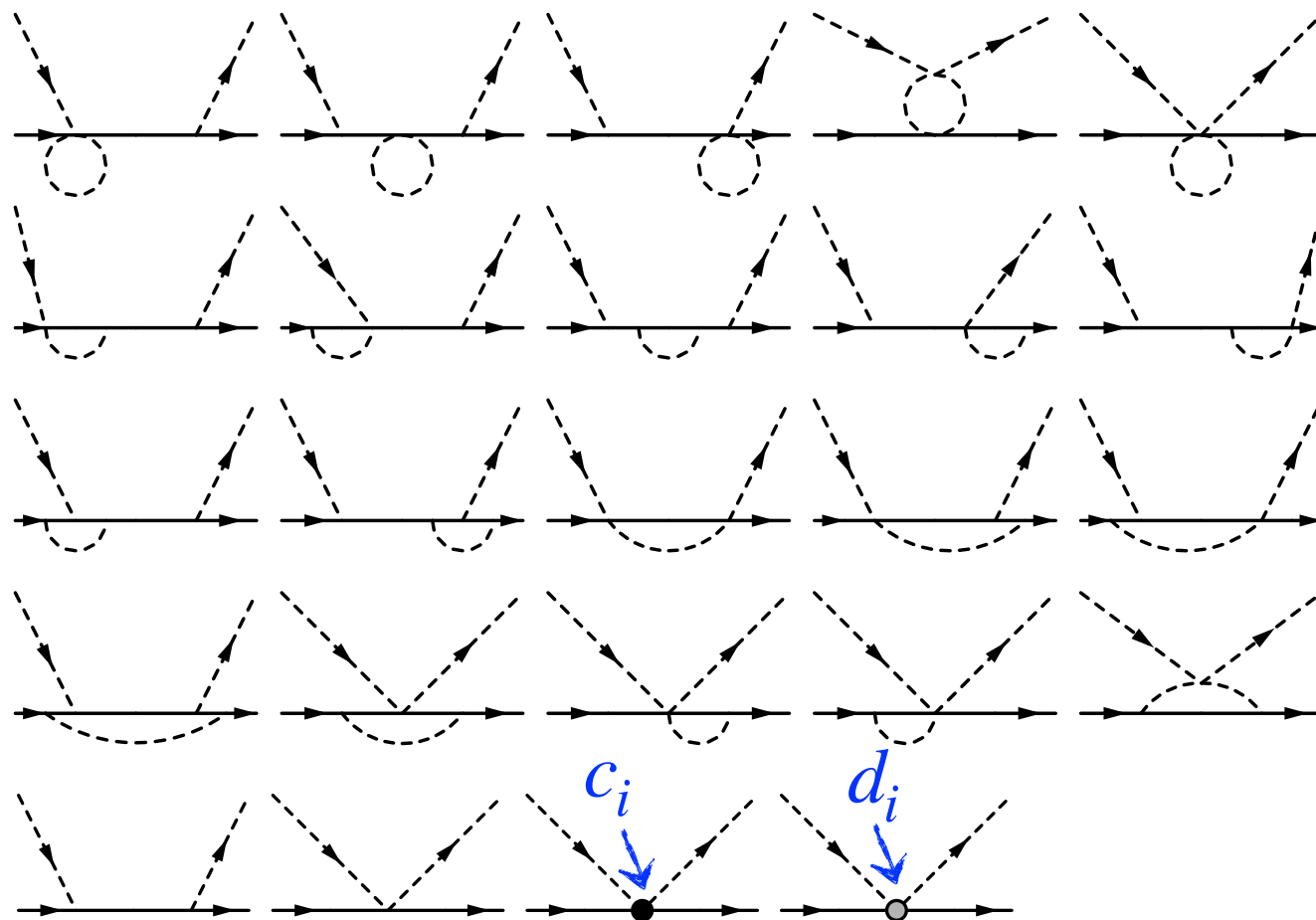


TPE topology includes pion-nucleon amplitude as a subprocess

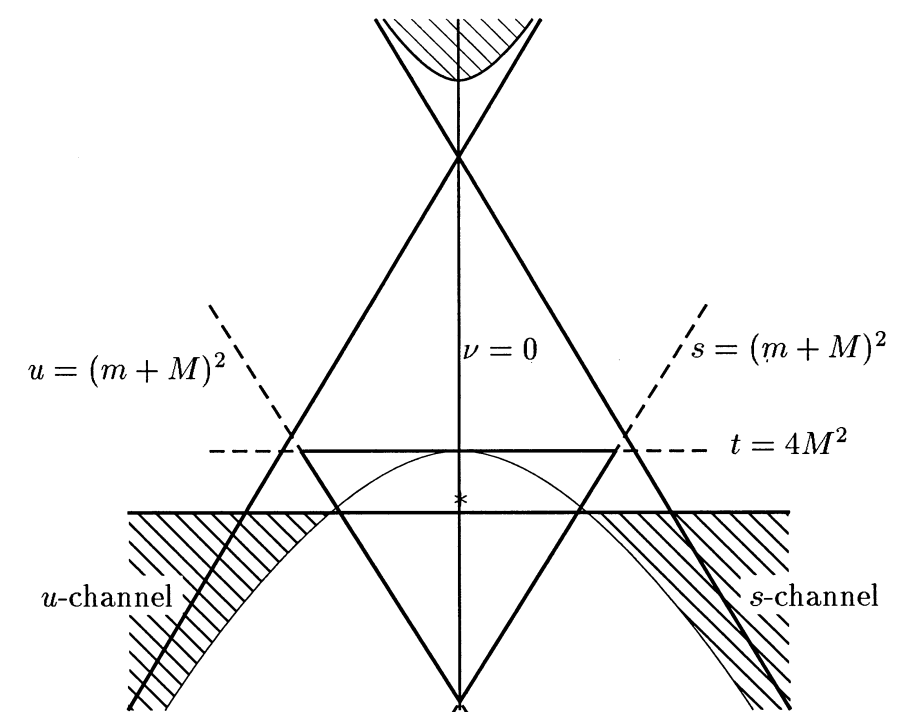
← Pion-nucleon amplitude with gradient-flow regulator depends on c_i 's

Calculation of pion-nucleon scattering with gradient-flow regulator required

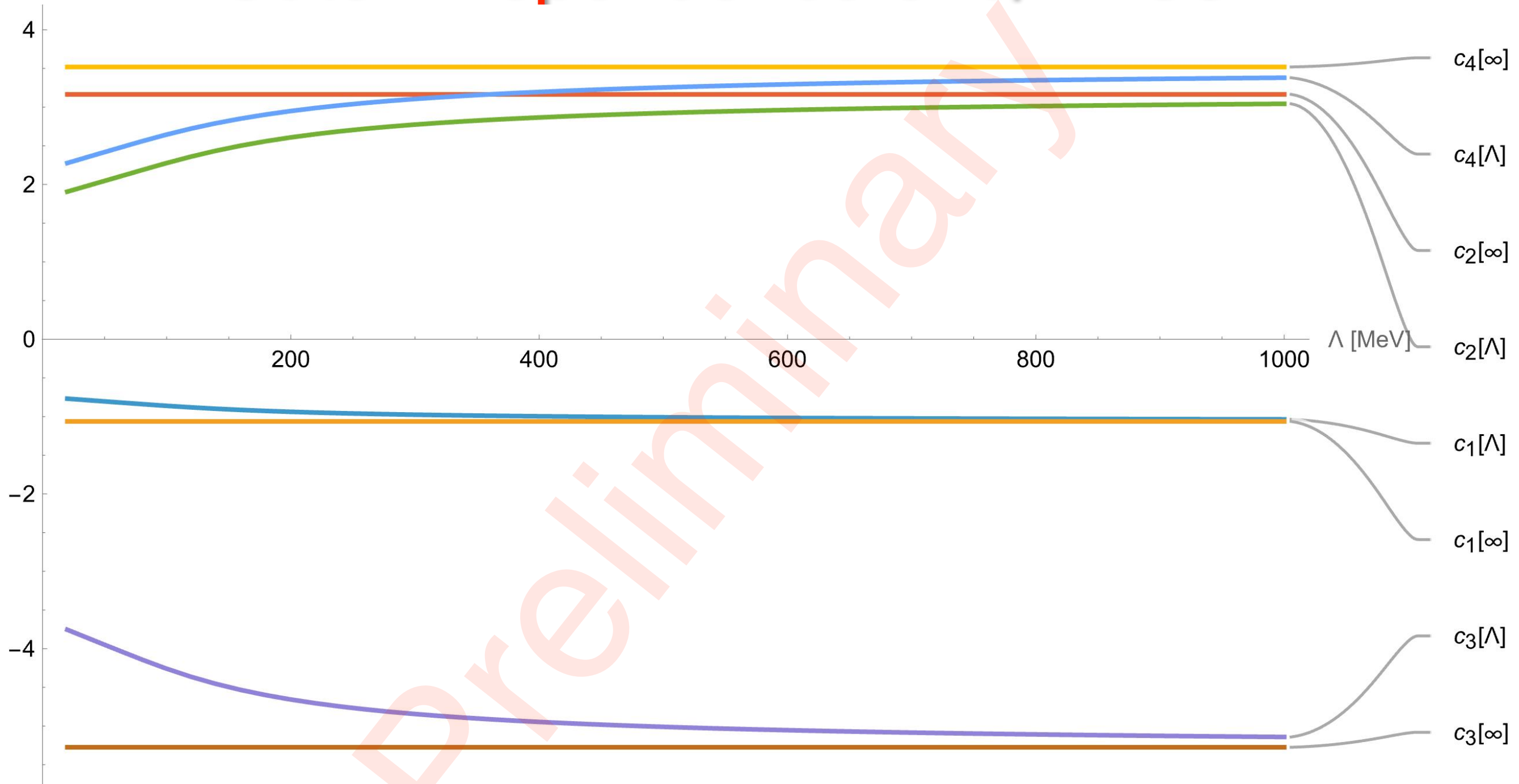
→ Patrick Walkowiak's master thesis



Fit LECs to pion-nucleon sub-threshold coefficients which are determined from Roy-Steiner equation



Cutoff Dependence of c_i LECs



- Saturation towards dim-reg results ($\Lambda \rightarrow \infty$) is fast
- For $\Lambda \sim 500$ MeV the absolute value of c_i is smaller compared to c_i in dim-reg.

Summary

Field-theoretical Methods for derivation of nuclear forces at high order

- Gradient-flow regularization is a promising tool for symmetry preserving derivation of nuclear forces
- Path-integral approach is worked out for derivation of nuclear forces
Can deal with higher number of time-derivatives of pion-fields.

Application: 3NF at N³LO

- Calculation of gradient-flow regularized 3NF at N³LO is finished
- Two versions for short-range 3NF at N³LO
 - With non-local regulator in LO NN (SMS potential)
 - With local regulator in LO NN (new NN required)

Outlook

- Partial wave decomposition (PWD): K. Hebeler, A. Nogga & K. Topolnicki

PWD is computationally more expensive, due to higher dimension of integrals over Schwinger parameters

- NN force up to $N^2\text{LO}$ within gradient-flow regularization: NN at chiral limit