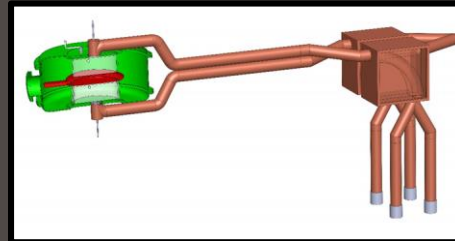
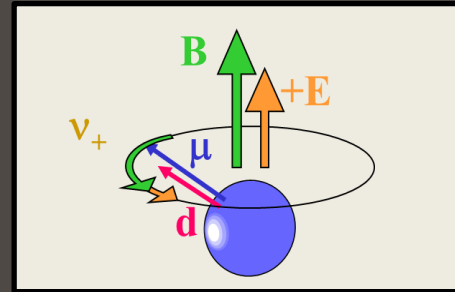


Monte Carlo EDM Simulations for the UCN Experiment at TRIUMF

February 18, 2017

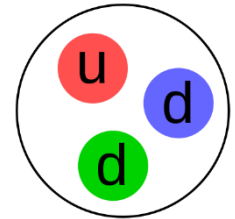
Sanmeet Chahal | University of Ottawa | TRIUMF



Project Overview

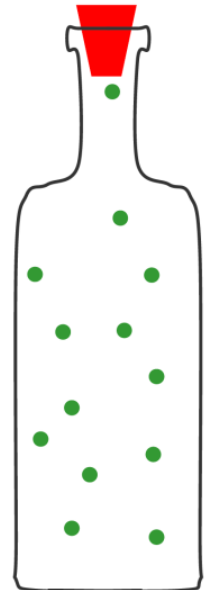


Goal: Measure the **electric dipole moment** (EDM) of the **neutron** (nEDM). Precision goal: 10^{-27} e•cm.



Motivation: Explain **matter/antimatter asymmetry**
Physics **beyond** the **Standard model**

Neutron type	Mean Energy (ev)	Velocity (m/s)	Temperature (K)
Fast	$> 500 \cdot 10^3$	$> 10^7$	$> 10\ 000$
Thermal	$25 \cdot 10^{-3}$	2200	300
Ultracold	$< 300 \cdot 10^{-9}$	< 10	< 0.002



1. EDM Measurement

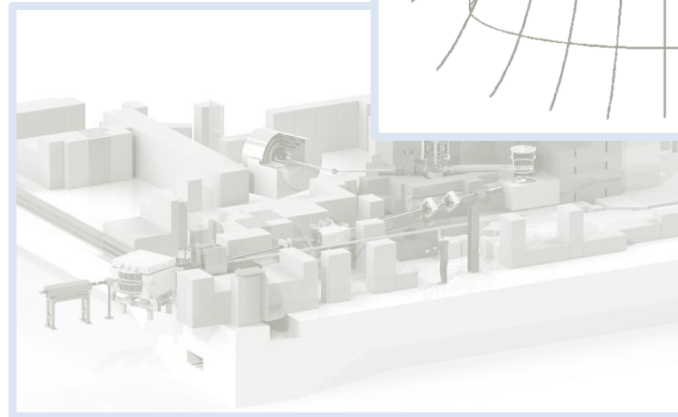
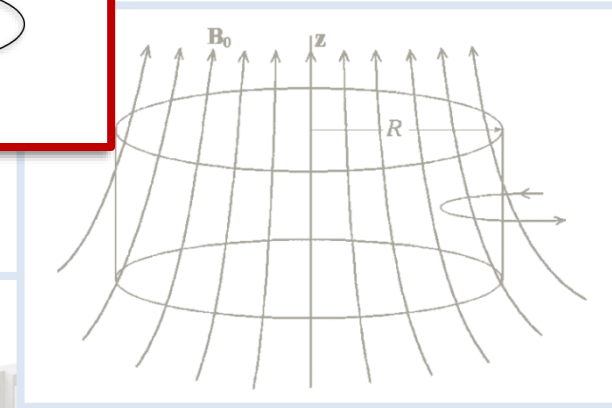
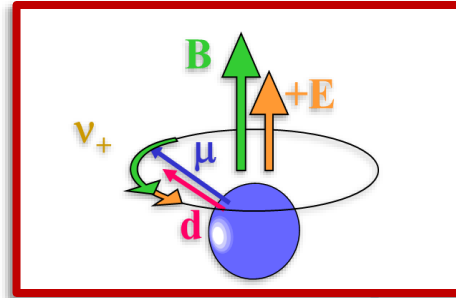
- Precession and EDM
- Ramsey Cycle
- Simulation Requirements

2. Simulation program

- PENTrack
- B_0 and E fields
- Geometric phase effect
- Benchmark tests

3. Cell Orientation Study

- Method
- Results

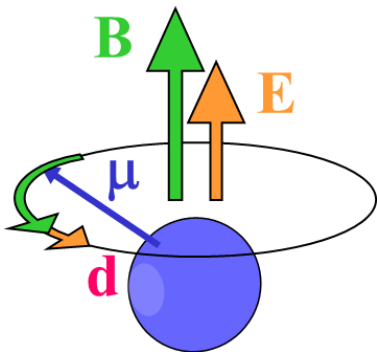


Precession and EDM

$$d_n = 0 \text{ e}\cdot\text{cm}$$

Apply **Electric (E)** fields

No change in ν !



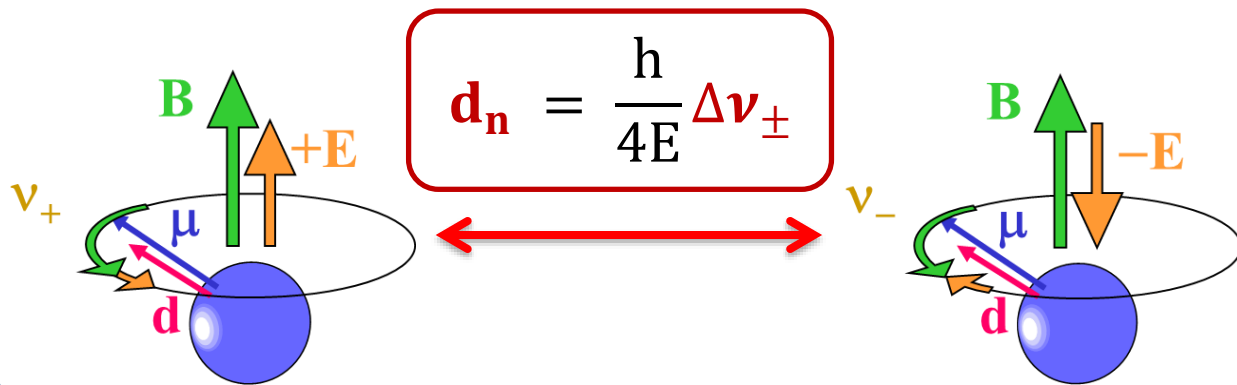
$$d_n \neq 0 \text{ e}\cdot\text{cm}$$

Parallel E field

Anti-parallel E field

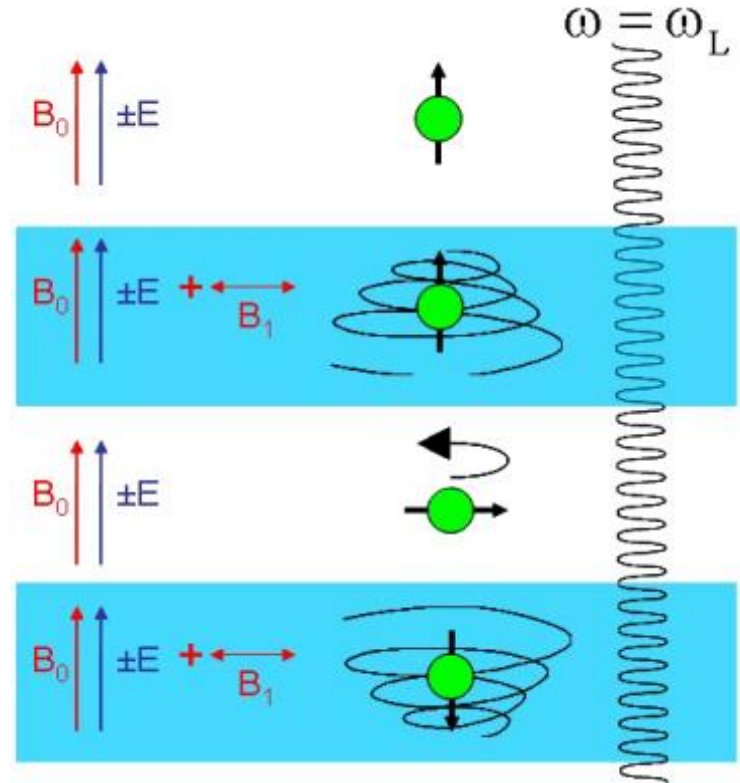
Precession (ν_+) is **faster**

Precession (ν_-) is **slower**



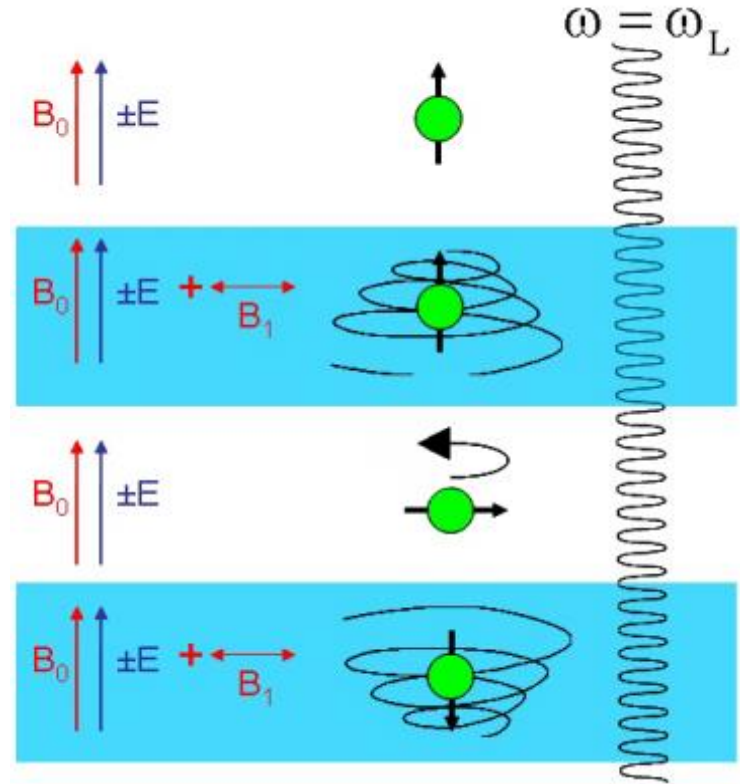
Ramsey Cycle

1. **Polarized** n & **constant** B_0 field.
2. B_1 pulse \rightarrow $\pi/2$ **spin flip**.
3. **Free precession** in transverse plane.
3. Second B_1 pulse \rightarrow $\pi/2$ **spin flip**.
4. **Count** neutrons' **spin state** \rightarrow flip **E field**.



Ramsey Cycle

1. **Polarized** n & **constant** B_0 field.
2. B_1 pulse $\rightarrow \pi/2$ spin flip.
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4. **Count** neutrons' **spin state** \rightarrow flip **E field**.

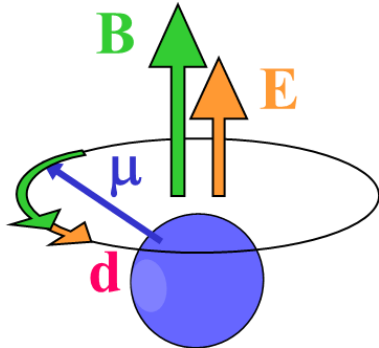


Fields

B_0 field – starting polarization

B_1 field – Do spin flip.

E field – Changes ν

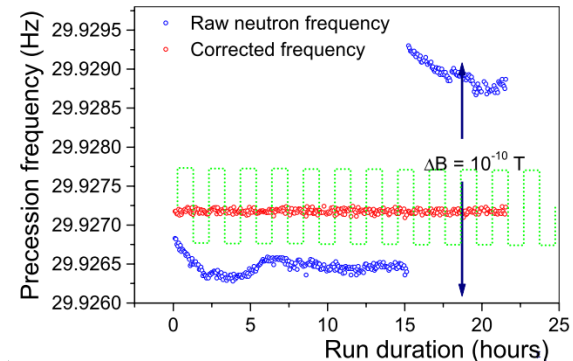


Other features

$v \times E$ effect : SR \rightarrow mot. B field

$$B_v = \frac{E \times v}{c^2}$$

Comagnetometer atoms – reduce $\Delta d_{f,n}^{\text{sys}}$



1. EDM Measurement

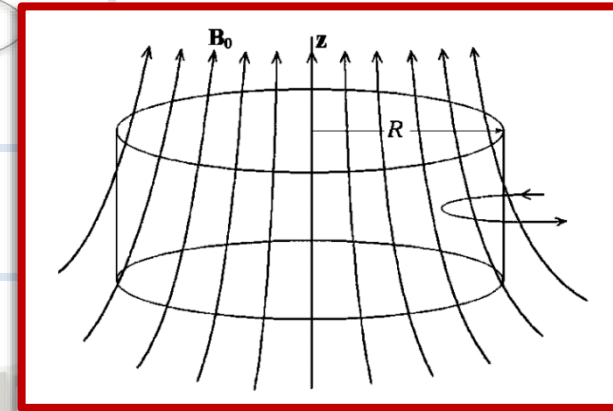
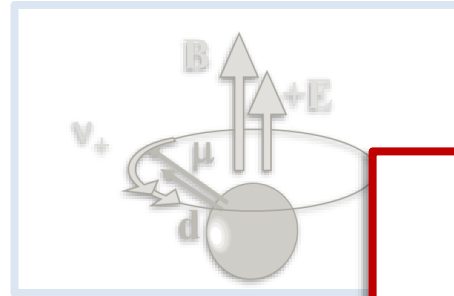
- Precession and EDM
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2. Simulation program

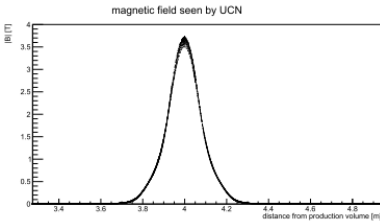
- PENTrack
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3. Cell Orientation Study

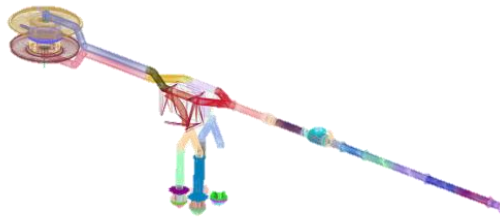
- Method
- Results



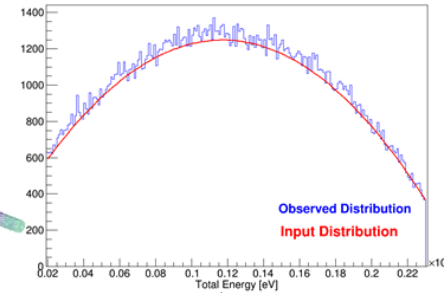
B & E Fields



Geometry



Energy spectrum



Equation of Motion

$$\ddot{\mathbf{x}} = \frac{1}{\gamma m} \left(1 - \frac{1}{c^2} \dot{\mathbf{x}} \otimes \dot{\mathbf{x}} \right) (P\mu\nabla|\mathbf{B}| - mge_z + q(\mathbf{E} + \dot{\mathbf{x}} \times \mathbf{B}))$$

PENTrack

Time Spin

$$\Delta v_{\pm}$$

d_n

Snapshot log

Hit log

End log

Spin log

Track log

B₀ Field

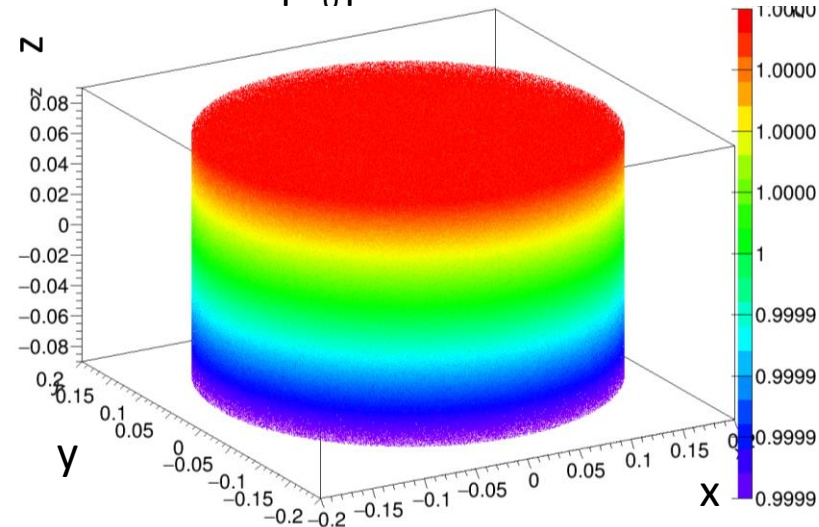
Axially **symmetric**, **static** B₀ field & **Maxwell** equations.

$$\nabla \cdot \mathbf{B} = 0$$
$$(\nabla \times \mathbf{B})_{\phi} = 0$$

$$B_z(r, z) \approx B_z(0, z) + \frac{\partial B_z(0, z)}{\partial z} r^2$$

$$B_r(r, z) \approx -\frac{r}{2} \frac{\partial B_z(0, z)}{\partial z}$$

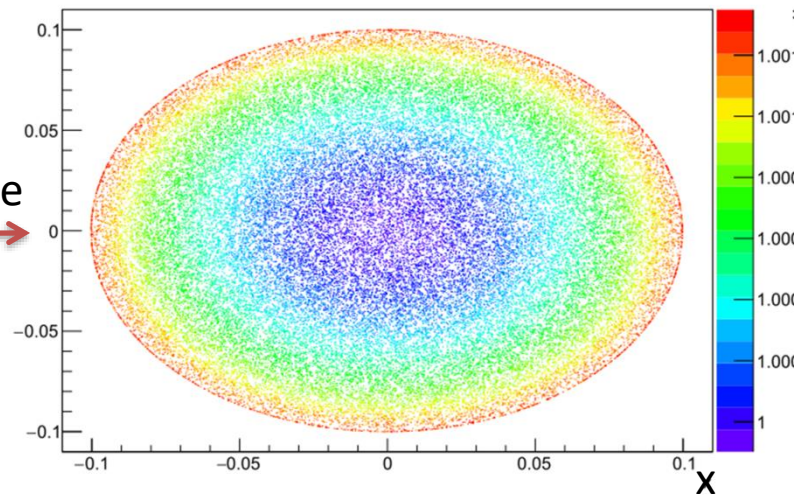
|B₀| in EDM cell



B_{abs}

r - dependence

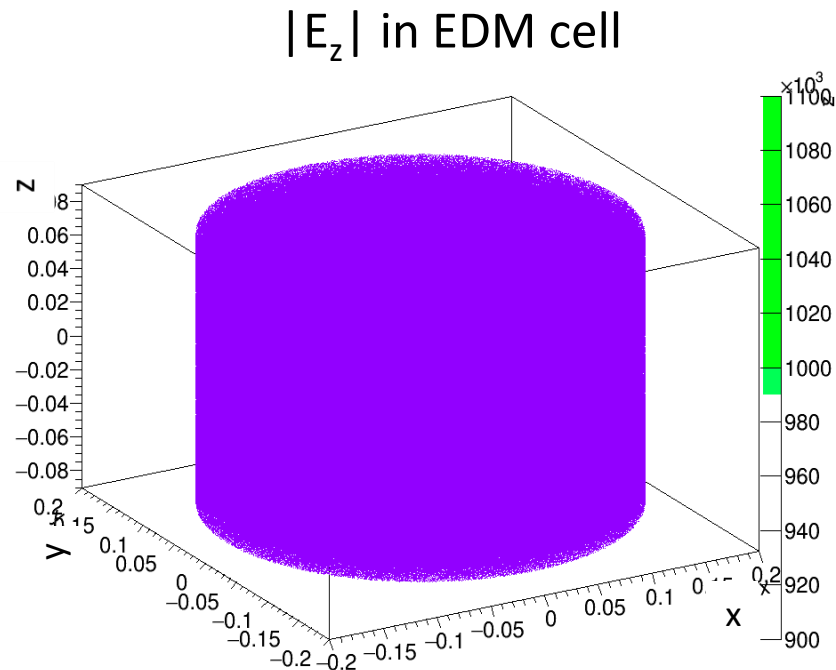
|B₀| in x-y plane



Ideal Model

- Static
- z-aligned
- Homogenous field (no gradient)
- vxE added

$$\mathbf{E} = \begin{bmatrix} 0 \\ 0 \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10^6 \text{ V/m} \end{bmatrix}$$



Geometric Phase Effect

$B_{xy} \rightarrow$ **shift** in ω_L



$$B_{xy} = \underbrace{B_{0xy}}_{\text{Gradient}} + \underbrace{B_v}_{v \times E} = -\frac{r}{2} \frac{\partial B_{0z}}{\partial z} + \frac{\mathbf{E} \times \mathbf{v}}{c^2}$$



$$\Delta\omega \propto \omega_{xy}^2$$

Geometric Phase Effect

$B_{xy} \rightarrow$ **shift** in ω_L

$$B_{xy} = \underbrace{B_{0xy}}_{\text{Gradient } v \times E} + \underbrace{B_v} = -\frac{r}{2} \frac{\partial B_{0z}}{\partial z} + \frac{\mathbf{E} \times \mathbf{v}}{c^2}$$

$$\Delta\omega \propto \omega_{xy}^2$$

$$\omega_{xy}^2 = (-\gamma B_{xy})^2$$

$$\omega_{xy}^2 = \gamma^2 \left(\frac{r}{2} \frac{\partial B_{0z}}{\partial z} \right)^2 + \left(\frac{\mathbf{E} \times \mathbf{v}}{c^2} \right)^2 + r \frac{\mathbf{E} \times \mathbf{v}}{c^2} \frac{\partial B_{0z}}{\partial z}$$

Geometric Phase Effect

$B_{xy} \rightarrow$ **shift** in ω_L

$$B_{xy} = \underbrace{B_{0xy}}_{\text{Gradient } vxE} + B_v = -\frac{r}{2} \frac{\partial B_{0z}}{\partial z} + \frac{\mathbf{E} \times \mathbf{v}}{c^2}$$

$$\Delta\omega \propto \omega_{xy}^2$$

$$\omega_{xy}^2 = (-\gamma B_{xy})^2$$

$$\omega_{xy}^2 = \gamma^2 \left(\frac{r}{2} \frac{\partial B_{0z}}{\partial z} \right)^2 + \left(\frac{\mathbf{E} \times \mathbf{v}}{c^2} \right)^2 + r \frac{\mathbf{E} \times \mathbf{v}}{c^2} \frac{\partial B_{0z}}{\partial z}$$

Linear in E \rightarrow mimics real EDM

Benchmark Tests

False EDM:

$$d_{f,n} = -\frac{\hbar}{4} \left(\frac{\langle \frac{\partial B_z(0,z)}{\partial z} \rangle_V}{B_{0z}^2} \right) \frac{v_{xy}^2}{c^2} \left[1 - \frac{\omega_r^{*2}}{\omega_0^2} \right]^{-1} \rightarrow d_{f,n} = -\frac{\hbar}{4} \left(\frac{\langle \frac{\partial B_z(0,z)}{\partial z} \rangle_V}{B_{0z}^2} \right) \frac{v_{xy}^2}{c^2} \left[1 - \frac{\omega_r^{*2}}{\omega_0^2} \right]^{-1}$$

Constant
Variable

Pendelbury et al. :

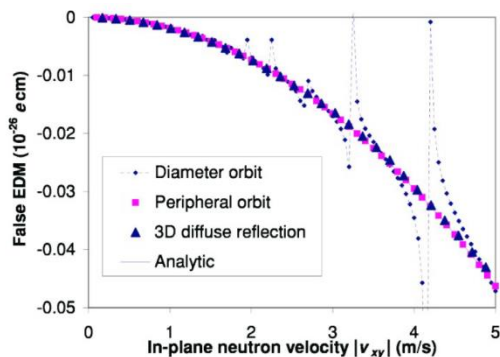
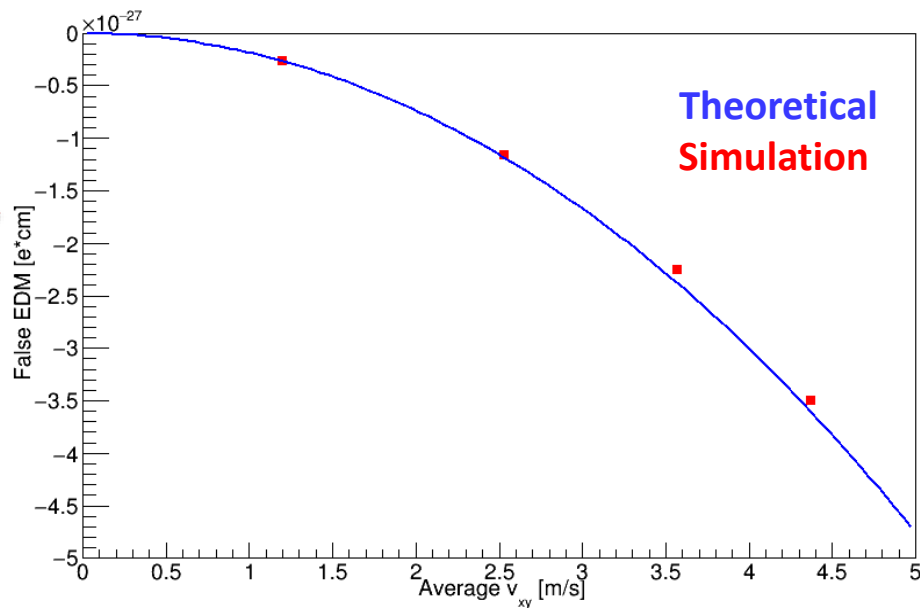


FIG. 11. (Color online) False EDM's obtained by computer simulation in the $|\omega_r| < |\omega_0|$ case. The results shown are for 2D specular reflection following peripheral and diameter orbits and for 3D diffuse reflection. The analytic result of Eq. (29) is shown as a smooth curve. Other parameters were $\partial B_{0z}/\partial z = 1$ nT/m and $B_0 = 1$ μ T.

Chahal:

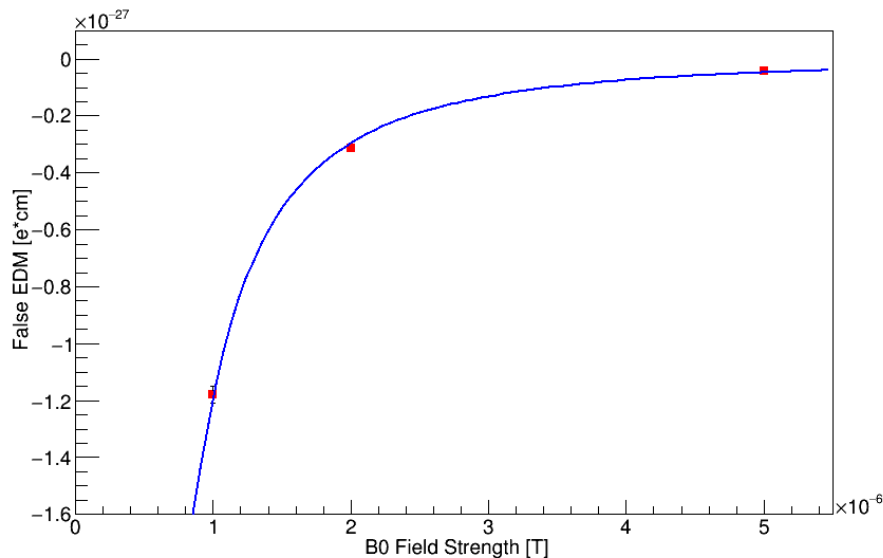


Benchmark Tests



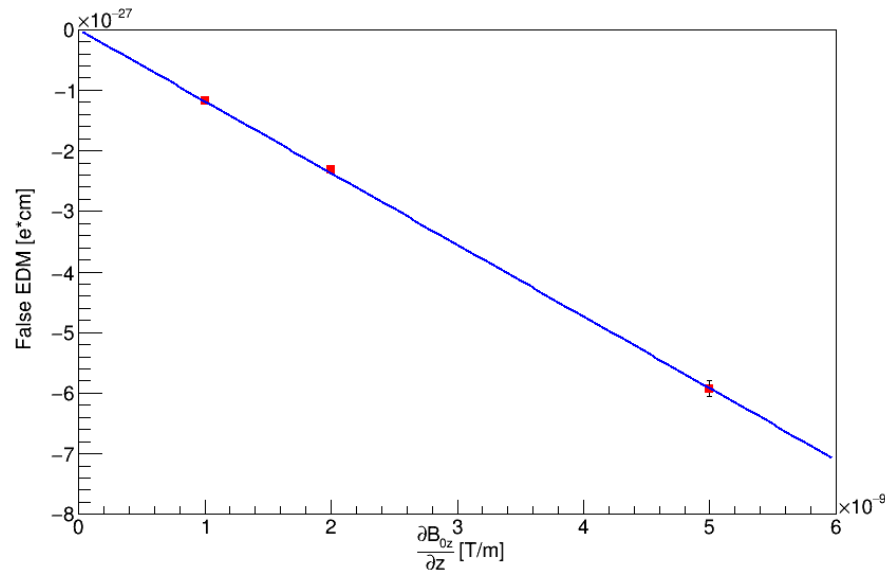
$$d_{f,n} = -\frac{\hbar}{4} \left(\frac{\left\langle \frac{\partial B_z(0,z)}{\partial z} \right\rangle_V}{B_{0z}^2} \right) \frac{v_{xy}^2}{c^2} \left[1 - \frac{\omega_r^{*2}}{\omega_0^2} \right]^{-1}$$

Constant
Variable



$$d_{f,n} = -\frac{\hbar}{4} \left(\frac{\left\langle \frac{\partial B_z(0,z)}{\partial z} \right\rangle_V}{B_{0z}^2} \right) \frac{v_{xy}^2}{c^2} \left[1 - \frac{\omega_r^{*2}}{\omega_0^2} \right]^{-1}$$

Constant
Variable



1. EDM Measurement

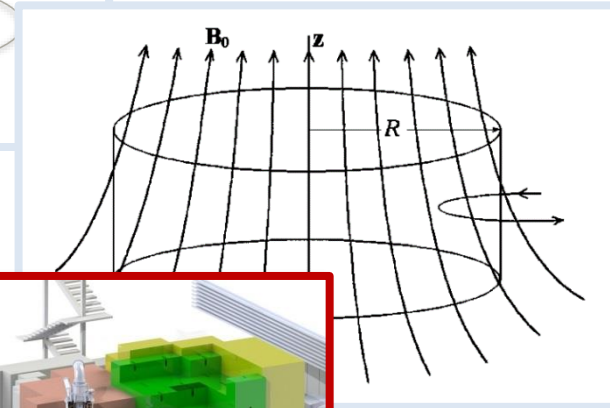
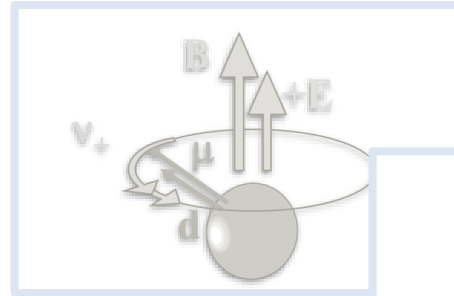
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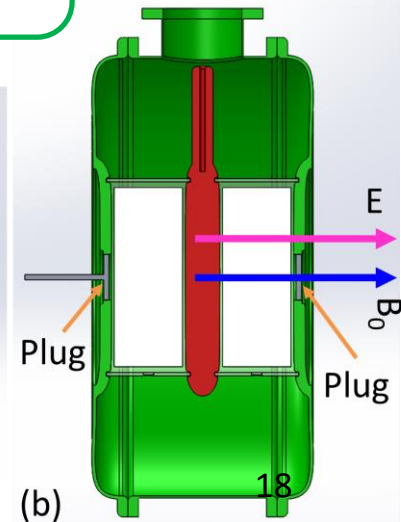
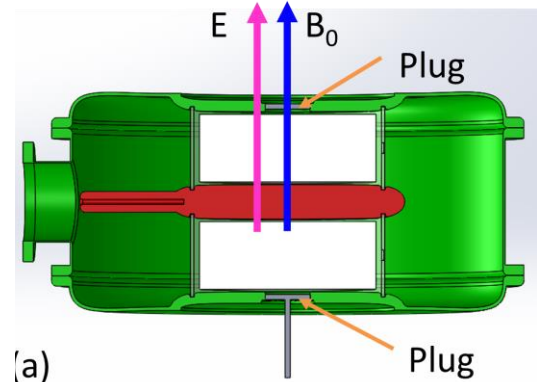
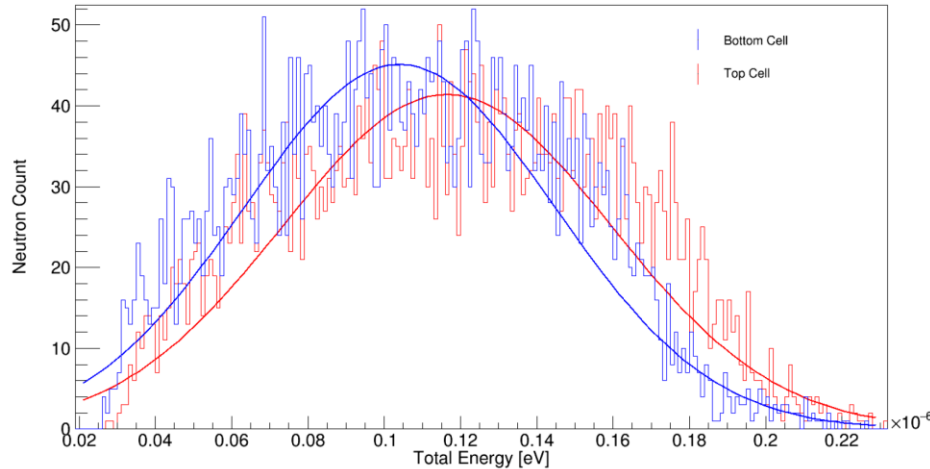
- Description
- Results



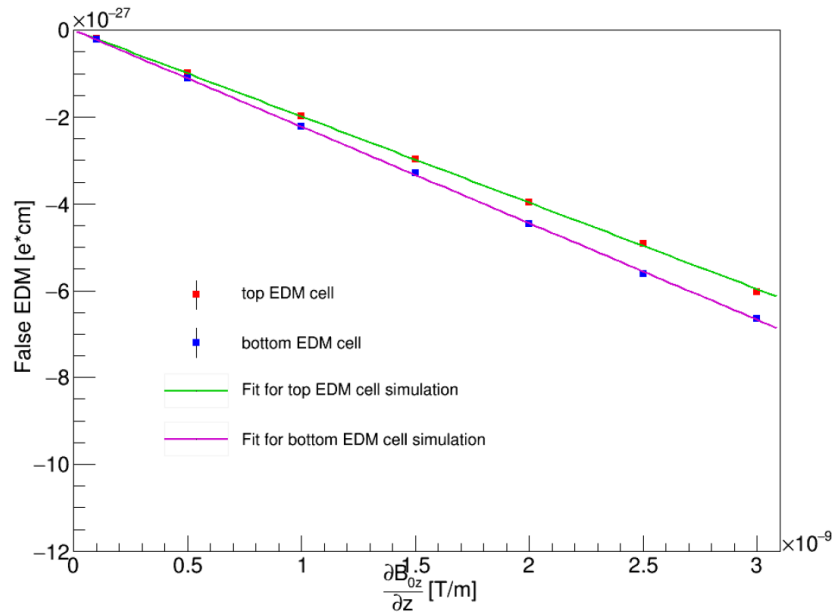
Optimum cell orientation

Goal: Determine effect of cell orientation on the $d_{f,n}$.

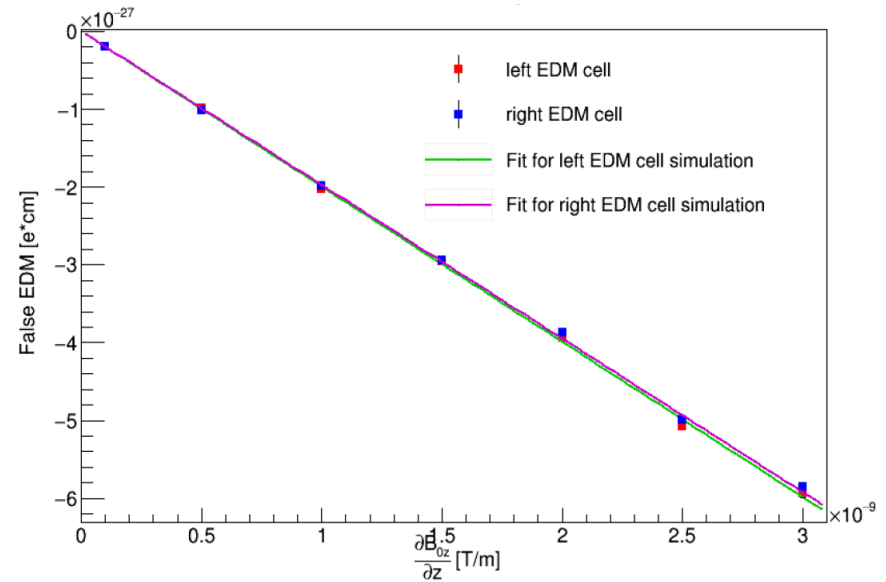
Procedure: 1. Energy spectrum from filling efficiency simulation.
2. Determine $d_{f,n}$ in both orientations.



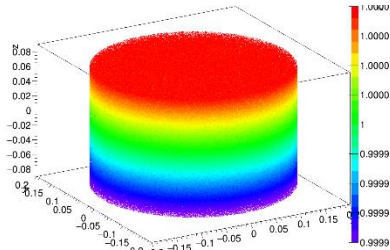
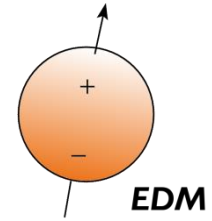
Vertical orientation



Horizontal orientation

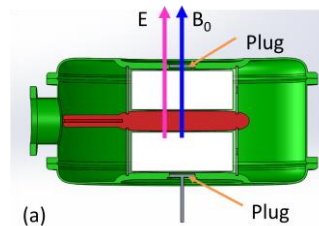


Basic nEDM simulation **requirements in place**



Implementation **benchmarked with published results**

Utility demonstrated with cell orientation study



Thank you! Questions?



TRIUMF: Alberta | British Columbia |
 Calgary | Carleton | Guelph | Manitoba
 | McMaster | Montréal | Northern
 British Columbia | Queen's | Regina |
 Saint Mary's | Simon Fraser | Toronto |
 Victoria | Winnipeg | York



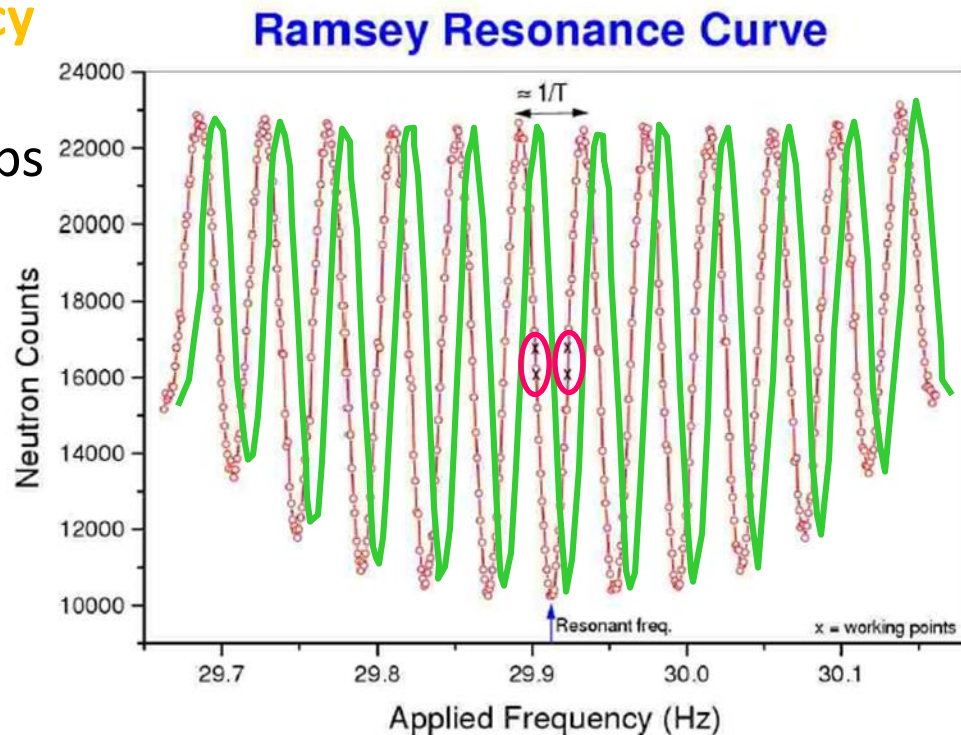
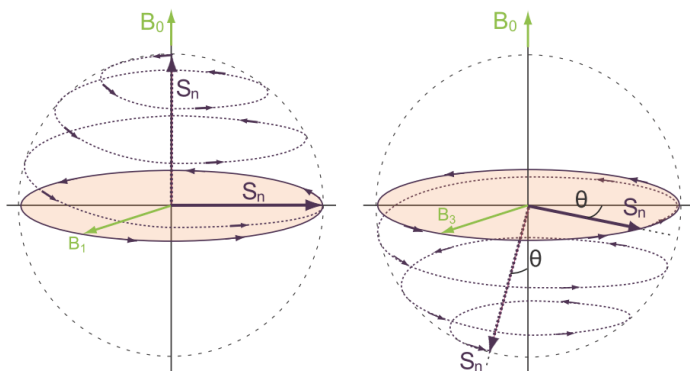
- [1] Golub R, Richardson D, Lamoreaux S K. 1991. *Ultra Cold Neutrons*, First edition. Adam Hilger: IOP Publishing Ltd.
- [2] Losekamm, Martin. *Monte Carlo Simulations and High Voltage Tests for the Future UCN Source and nEDM Experiment at TRIUMF* [Bachelor's thesis]. Technische Universität München, 2013.
- [3] Schreyer, W. 2011. Monte Carlo-simulations for the neutron lifetime experiment PENELOPE [dissertation]. Technische Universität München.
- [4] Knecht, Andreas. 2009. Towards a New Measurement of the Neutron Electric Dipole Moment [dissertation]. University of Zurich.
- [5] Zheng, Wangzhi. 2012. Experiments of Search for Neutron Electric Dipole Moment and Spin-Dependent Short-Range Force [dissertation]. Duke University.
- [6] Adachi, T, Altieri, E et al. 2015. International UCN Source and nEDM Experiment at TRIUMF Conceptual Design Report 2015 [internal report]. TRIUMF: Vancouver.
- [7] Picker, R. 2008. PENELOPE and AbEx On the Way Towards a New Precise Neutron Lifetime Measurement [dissertation]. Technische Universität München.

- [8] Abulnaga, M. *Winter 2015 CO-OP Term Report*. Vancouver: TRIUMF, 2015.
- [9] Barré, E. *Developing the UCN experimental facility at TRIUMF*. Vancouver: TRIUMF, 2014.
- [10] Kaiser, H. “Course notes: Neutron Optics and Neutron Interferometry”. Bloomington: IUCF, 2006.
- [11] Lambert, J.H. *Photometria sive de mensura et gradibus luminis colorum et umbra* (Eberhard Klett, Augsburg, 1760).
- [12] Heule, Stefan. *Production, Characterization and Reflectivity Measurements of Diamond-like Carbon and other Ultracold Neutron Guide Materials* [dissertation]. Universität Zürich, 2008.
- [13] Atchison, F., Daum, M., Henneck R., et al. *Diffuse reflection of ultracold neutrons from low roughness surfaces*, Eur. Phys. J. A 44 (2010) 23.
- [14] Pierre, E (personal communication, May 26 - June 22, 2015).
- [15] Lloyd, E. *Guide Topologies Parameters*. Vancouver: TRIUMF, 2014.
- [16] Chin, M. *UCN guide topologies and kink* (Internal report). Vancouver: TRIUMF, 2015.

Backup Slides

Ramsey Resonance Curve

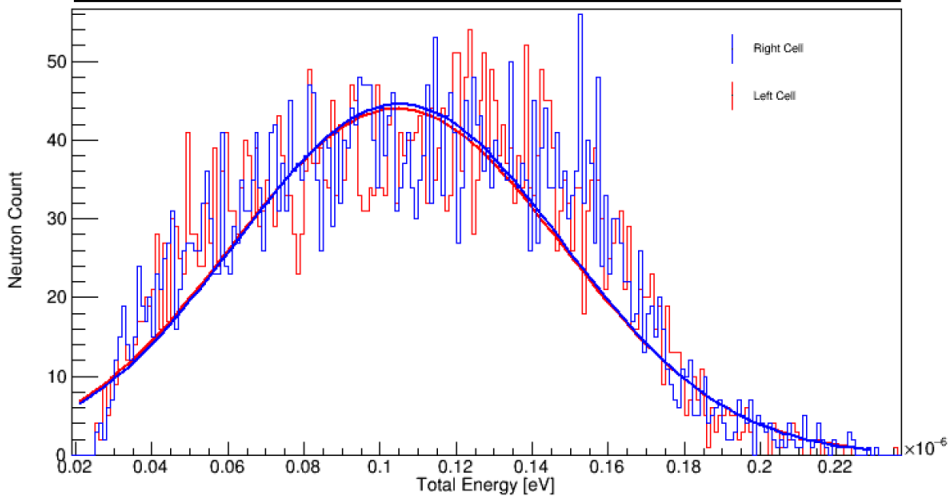
1. B_1 pulse at **non-resonance frequency**
2. **Probability of spin flip** $\neq 1$.
3. At **resonance** = **max** # of neutron flips
4. **Fit** to **Ramsey curve** to find ν_+ or ν_-
5. Use $d_n = \frac{h}{4E} \Delta\nu_{\pm}$



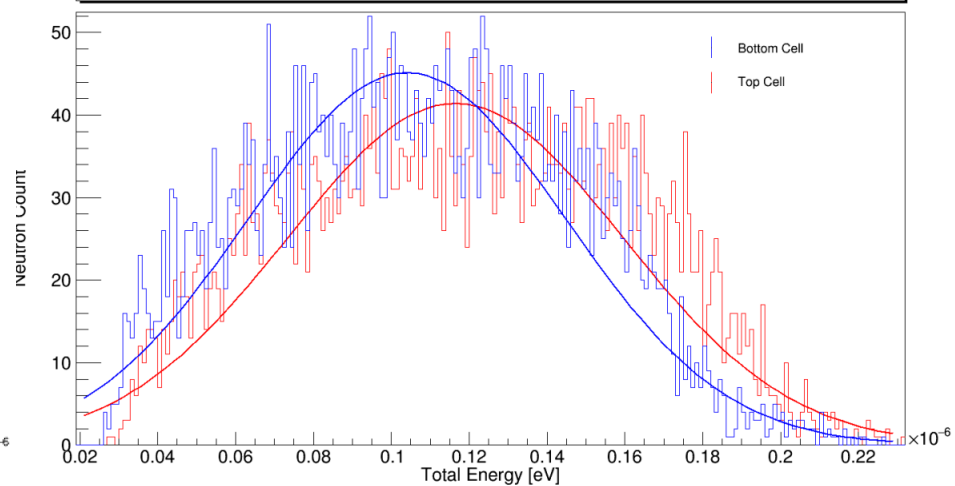
Total Energy Distribution



Horizontal Cells UCN Energy Distribution

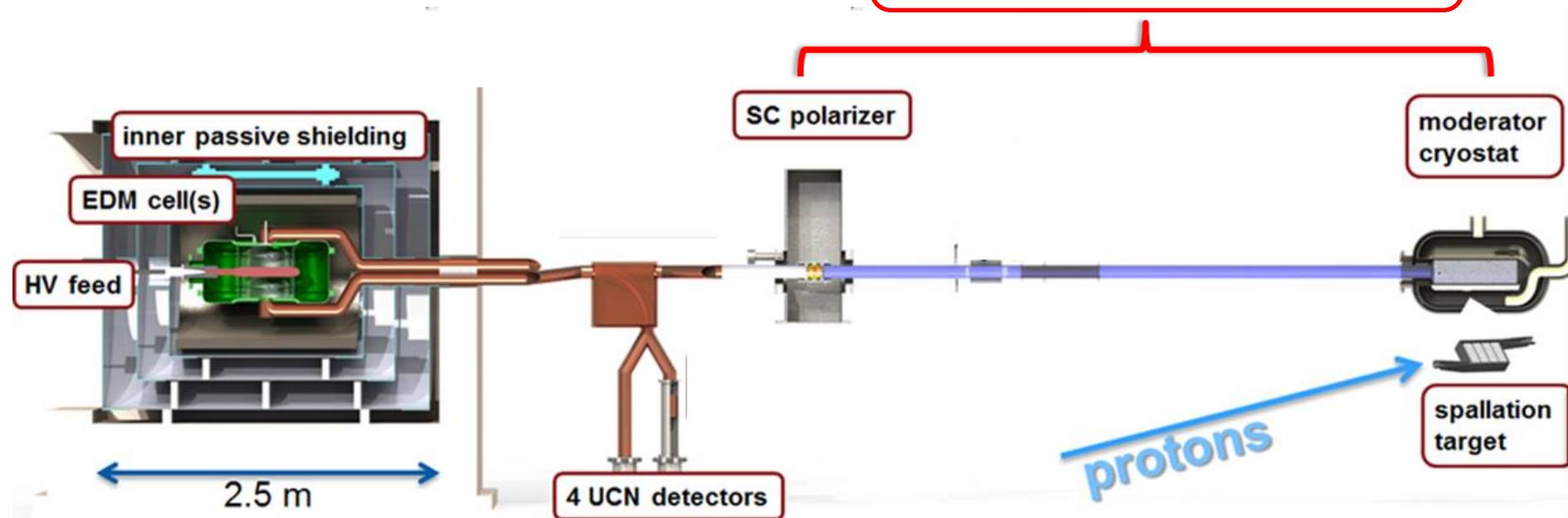


Vertical Cells UCN Energy Distribution

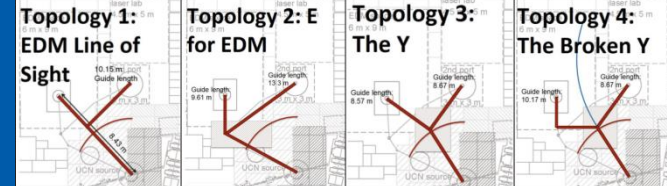


UCN Experiment

UCN produced in **UCN source**



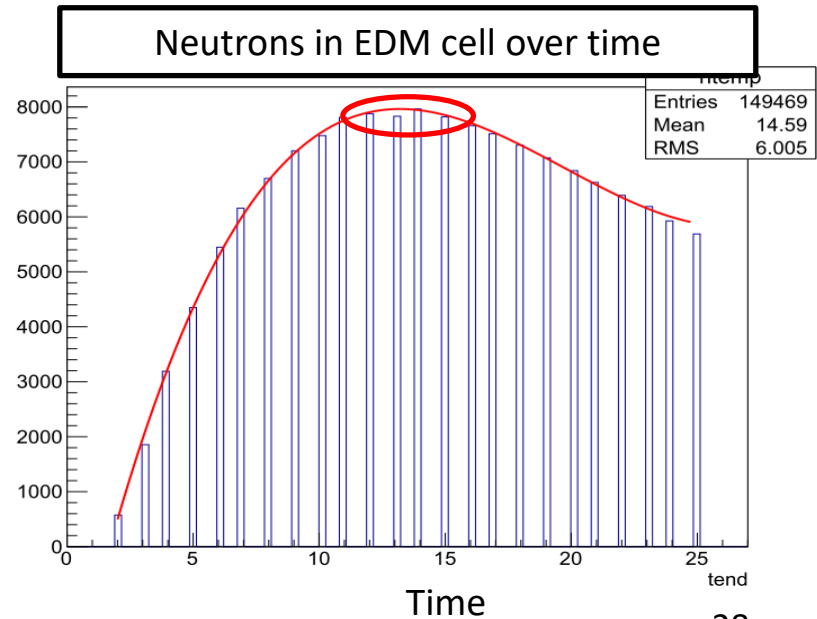
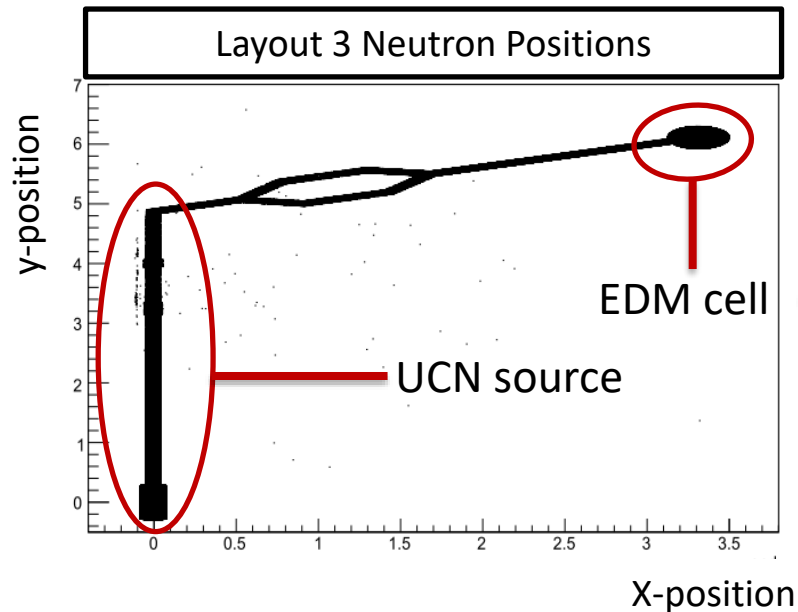
Procedure



PENTrack → **Snapshot** of
neutron **positions**

Histogram **neutron**
count in **EDM cell**

Find **maximum**,
Compare all layouts



Simulation Purpose

“Determine **optimum** guide layout”

Want: High **neutron densities**

$$\sigma_{\text{EDM}} = \frac{\hbar}{2\alpha T E \sqrt{N}}$$

α – visibility

T – observation time

E – Electric field

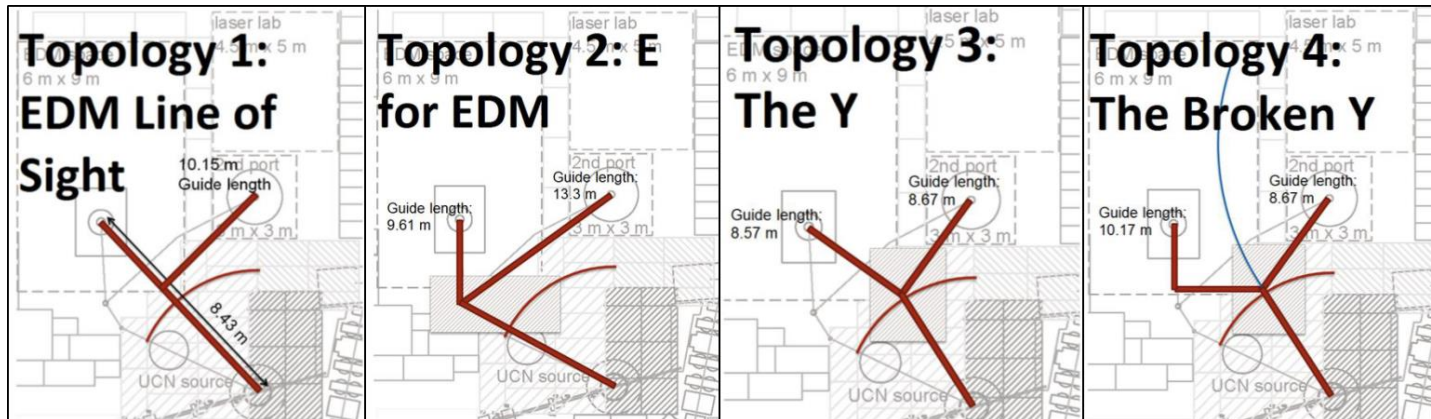
N – number of neutrons

Don't want: High **radiation**

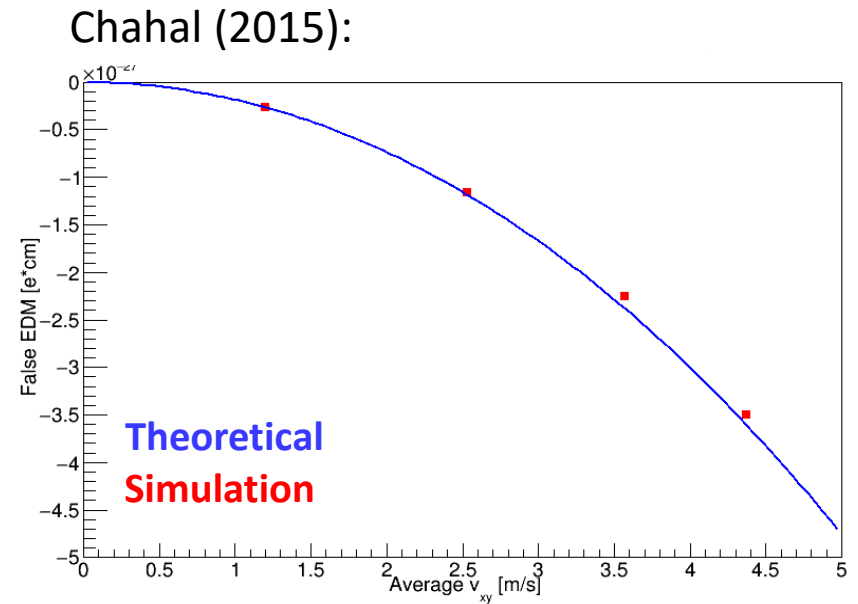
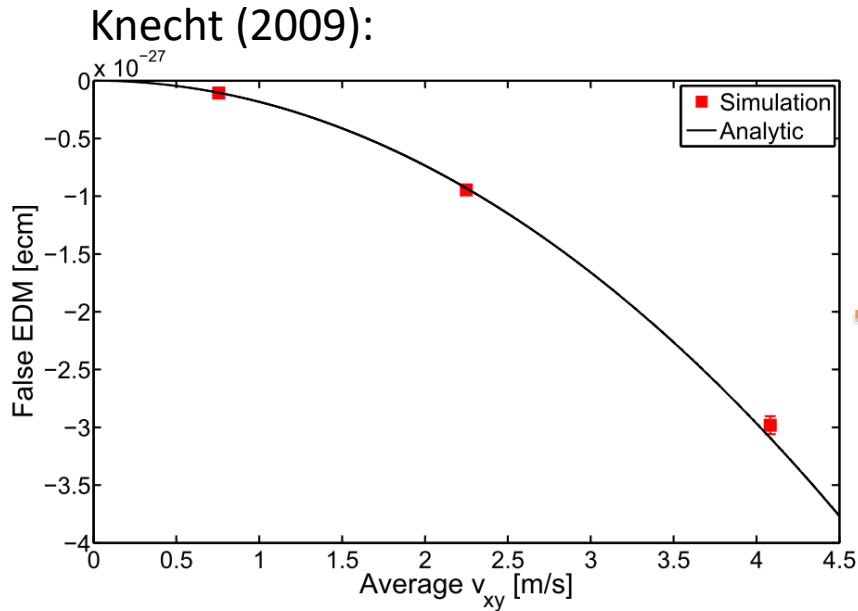
in experimental area.



No access to experiment.



False EDM vs. average planar velocity

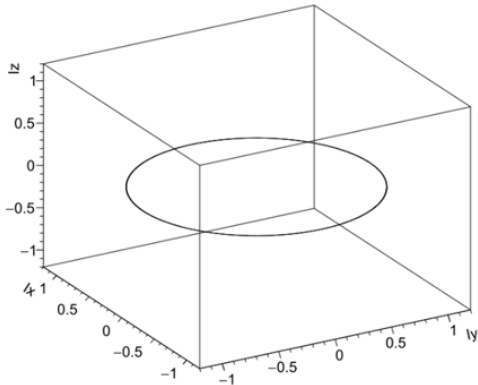


Neutron Precession Test

Axially **symmetric**, **static** B_0 field & **Maxwell** equations.

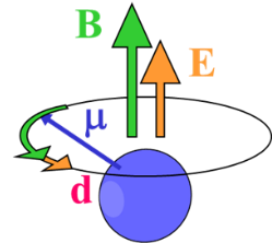
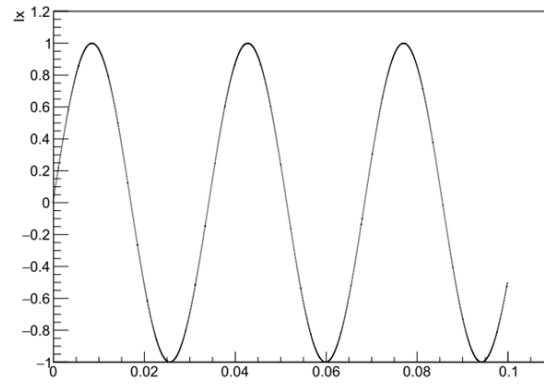
$$B_z(r, z) \approx B_z(0, z) + \frac{\partial B_z(0, z)}{\partial z}$$

lz:lx:ly

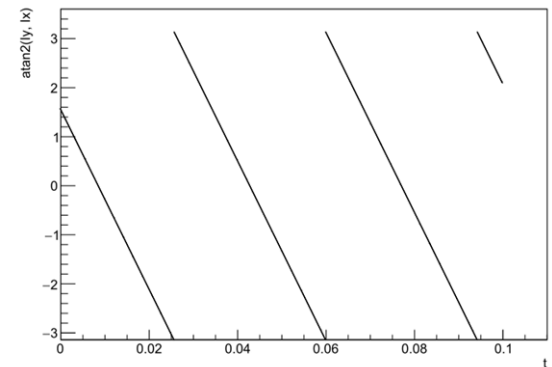


$$B_r(r, z) \approx -\frac{r}{2} \frac{\partial B_z(0, z)}{\partial z}$$

lx:t



atan2(ly, lx):t



$$\Delta\omega_{L, \text{sim}} = 72.2 \pm 3.5 \text{ pHz} \rightarrow \text{for } d_n = 10^{-27} \text{ e} \cdot \text{cm} \text{ need } \Delta\omega_L < 9 \text{ nHz}$$

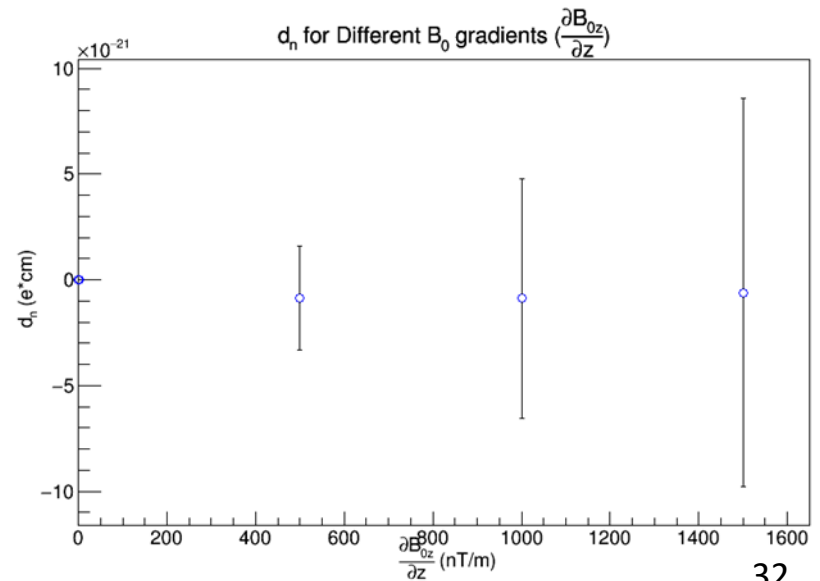
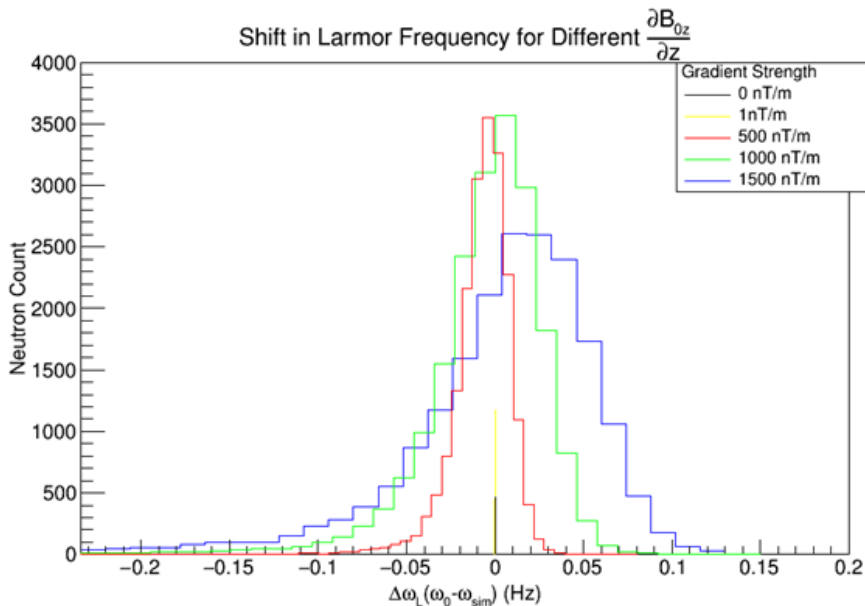
Gradient Comparison

Test $\Delta\nu$ in different gradients

low energy n \rightarrow lower in cell \rightarrow lower B_0 \rightarrow $\Delta\nu < 0$

Energy 

EDM cell



B₀ Field Formulas



B_x

$$B_x(x, y, z) = -\frac{x}{2} \frac{\partial B_z(0,0)}{\partial z}$$

$$\frac{\partial B_x}{\partial x} = -\frac{1}{2} \frac{\partial B_z(0,0)}{\partial z}$$

$$\frac{\partial B_x}{\partial y} = 0, \quad \frac{\partial B_x}{\partial z} = 0$$

B_y

$$B_y(x, y, z) = -\frac{y}{2} \frac{\partial B_z(0,0)}{\partial z}$$

$$\frac{\partial B_y}{\partial x} = 0, \quad \frac{\partial B_y}{\partial z} = 0$$

$$\frac{\partial B_y}{\partial y} = -\frac{1}{2} \frac{\partial B_z(0,0)}{\partial z}$$

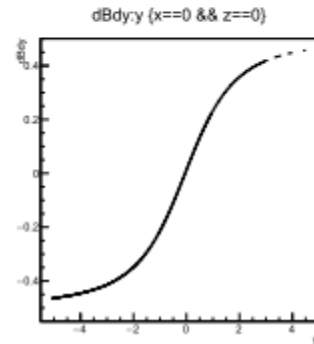
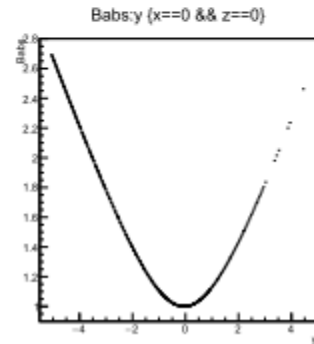
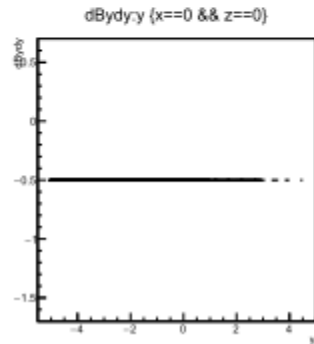
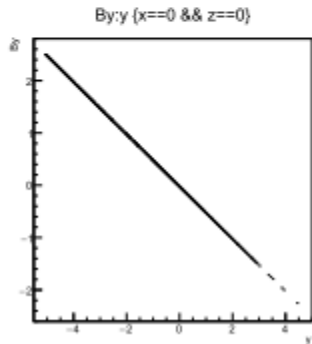
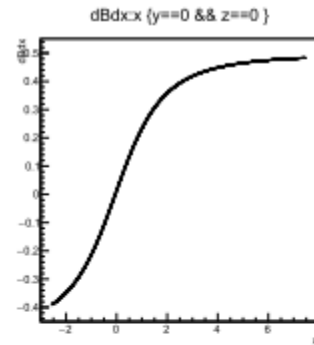
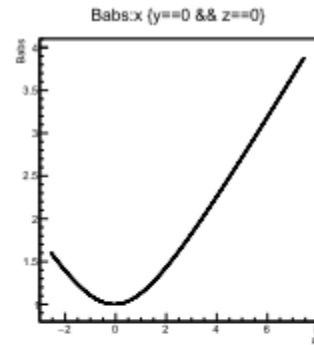
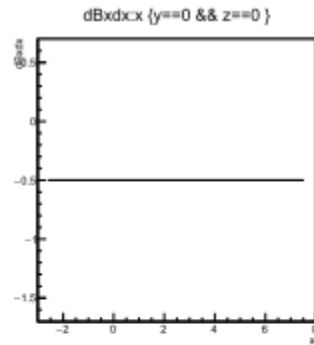
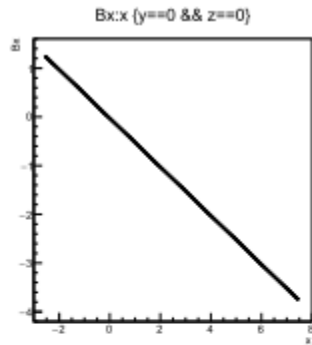
B_z

$$B_z = B_z(0,0) + \frac{\partial B_z(0,0)}{\partial z} z$$

$$\frac{\partial B_z}{\partial x} = \frac{\partial B_z}{\partial y} = 0$$

$$\frac{\partial B_z}{\partial z} = \frac{\partial B_z(0,0)}{\partial z}$$

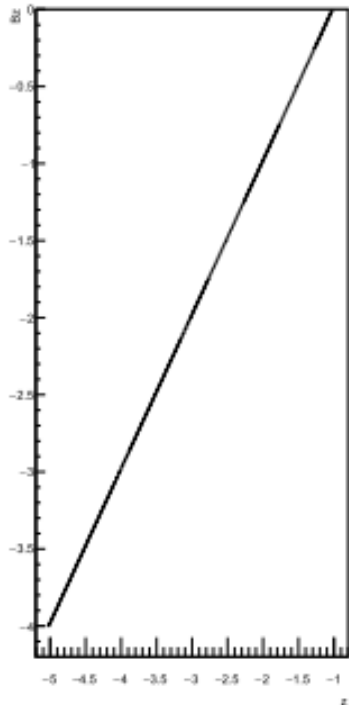
Single Variable B_0 field Tests



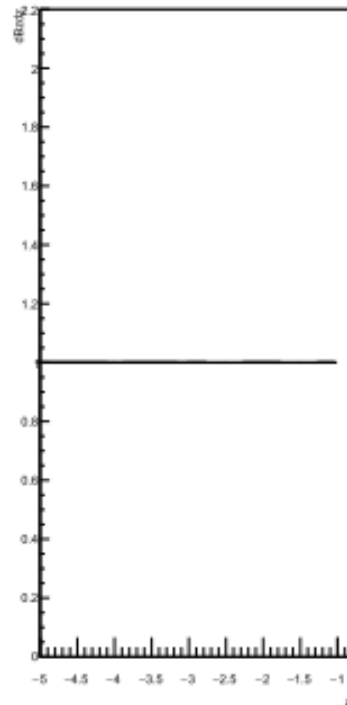
Single Variable B_0 field



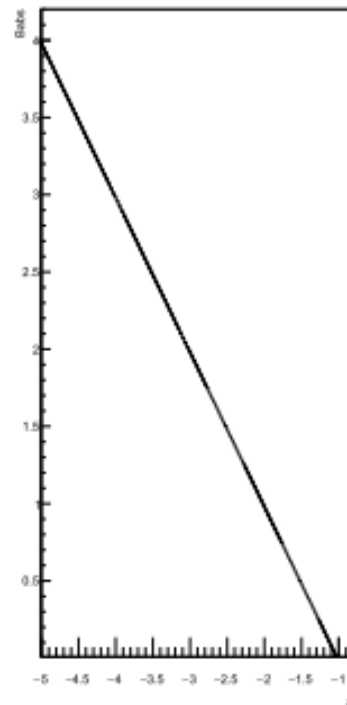
$B_z:z$ ($y==0$ && $x==0$)



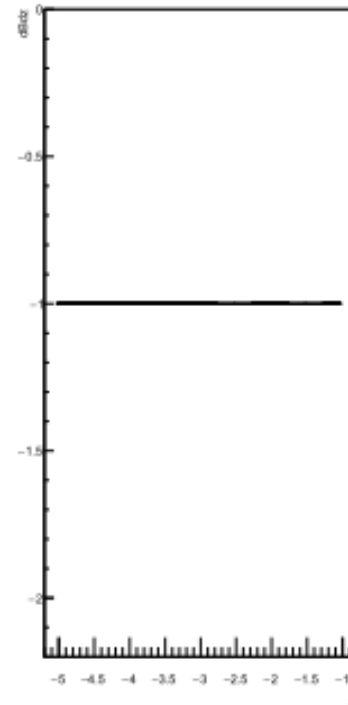
$\frac{dB_z}{dz}:z$ ($y==0$ && $x==0$)

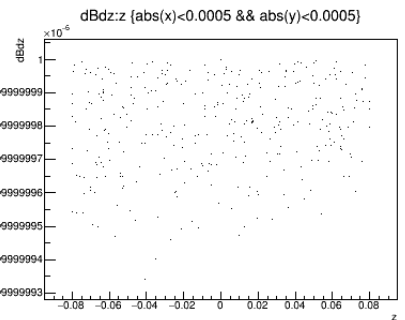
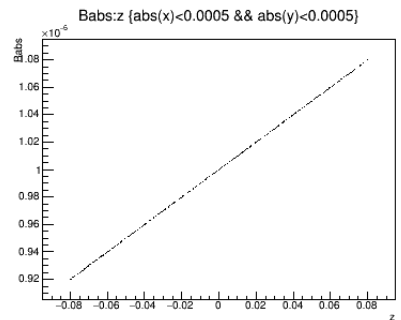
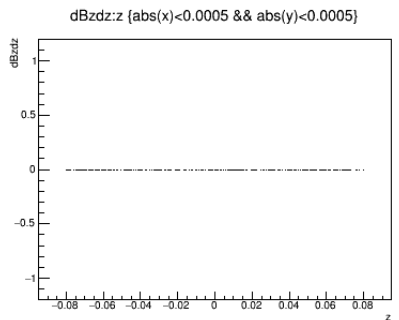
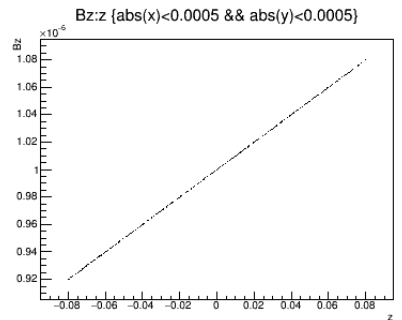
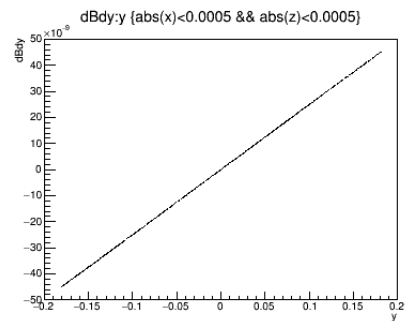
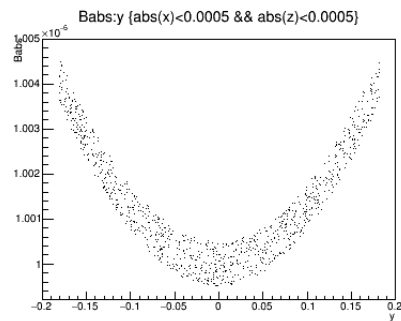
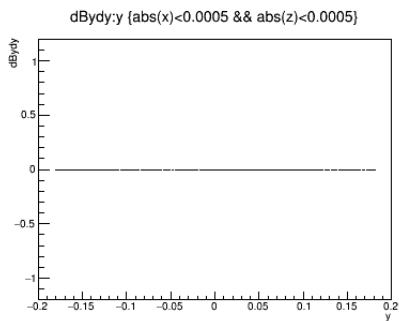
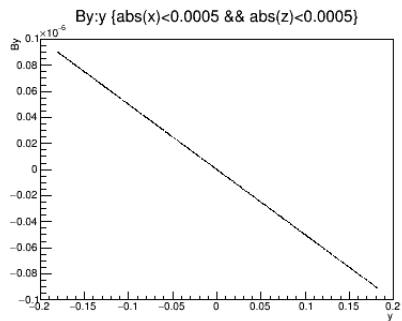
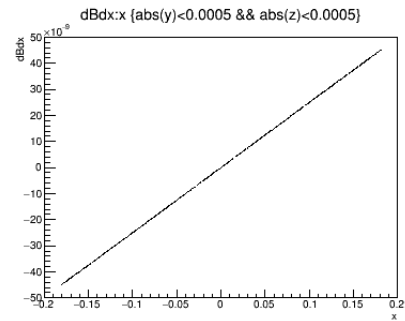
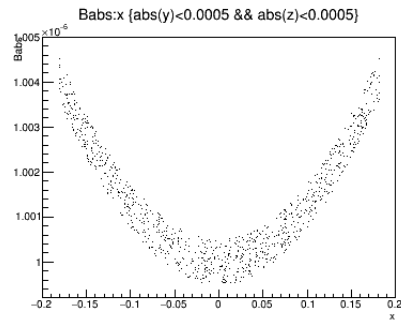
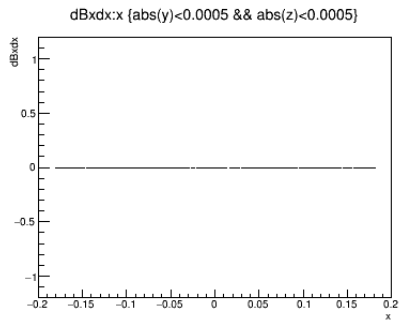
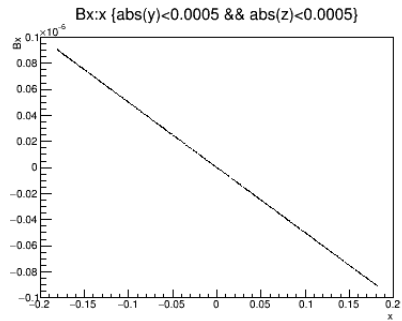


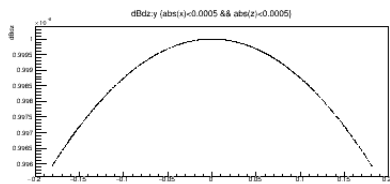
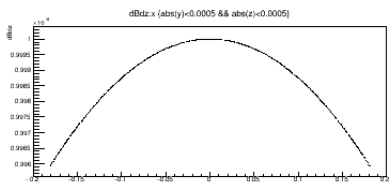
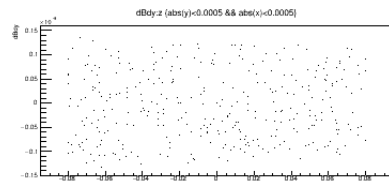
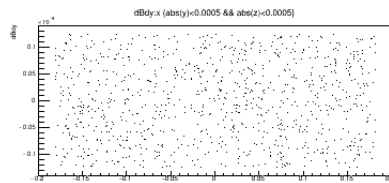
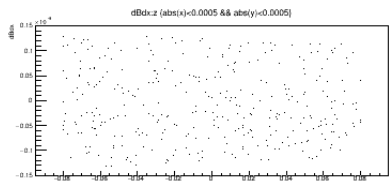
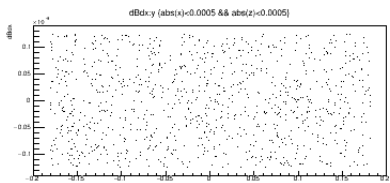
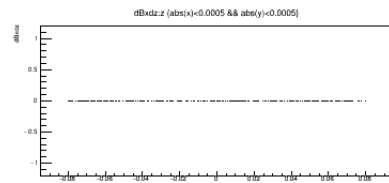
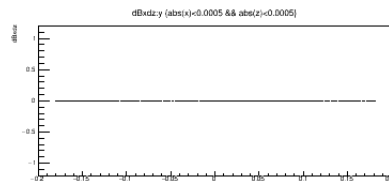
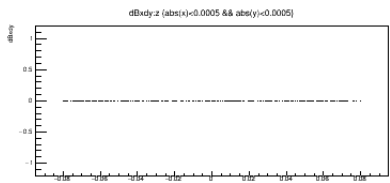
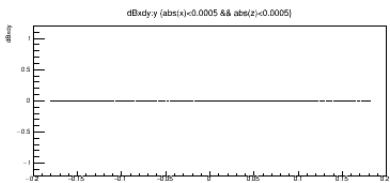
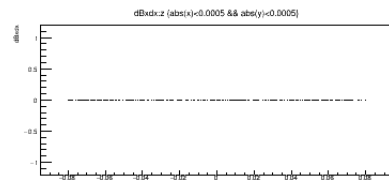
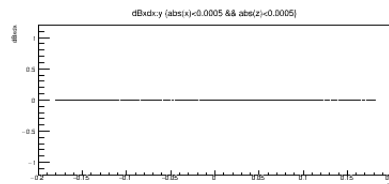
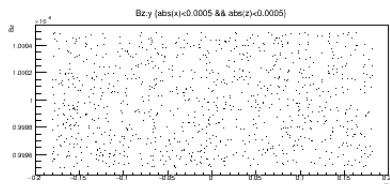
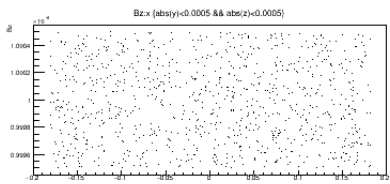
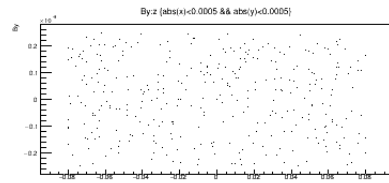
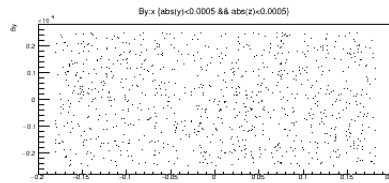
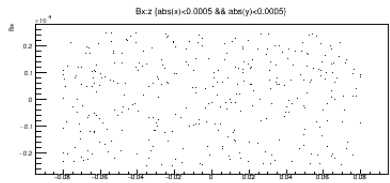
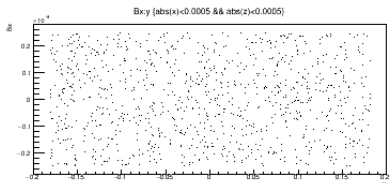
$B_{abs}:z$ ($y==0$ && $x==0$)



$\frac{dB_{abs}}{dz}:z$ ($y==0$ && $x==0$)





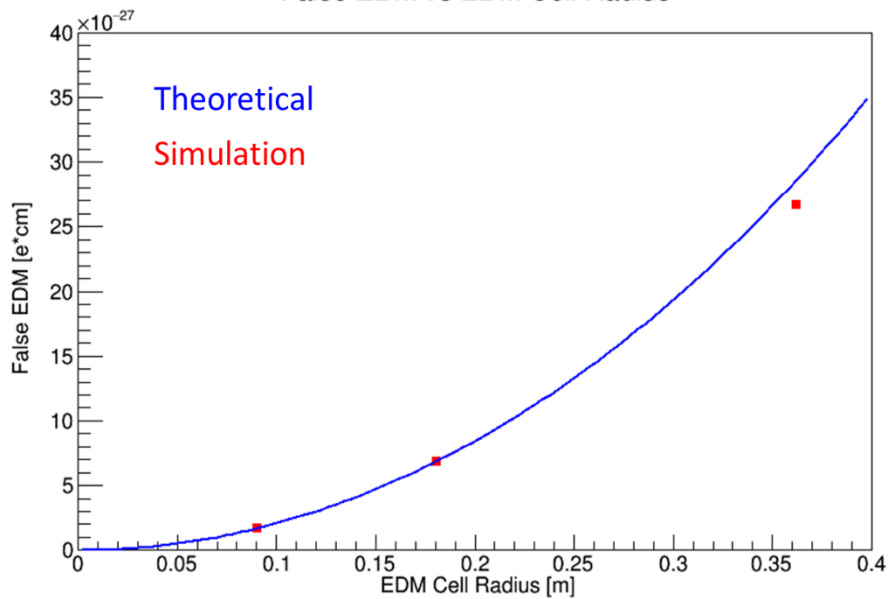


Benchmark Test



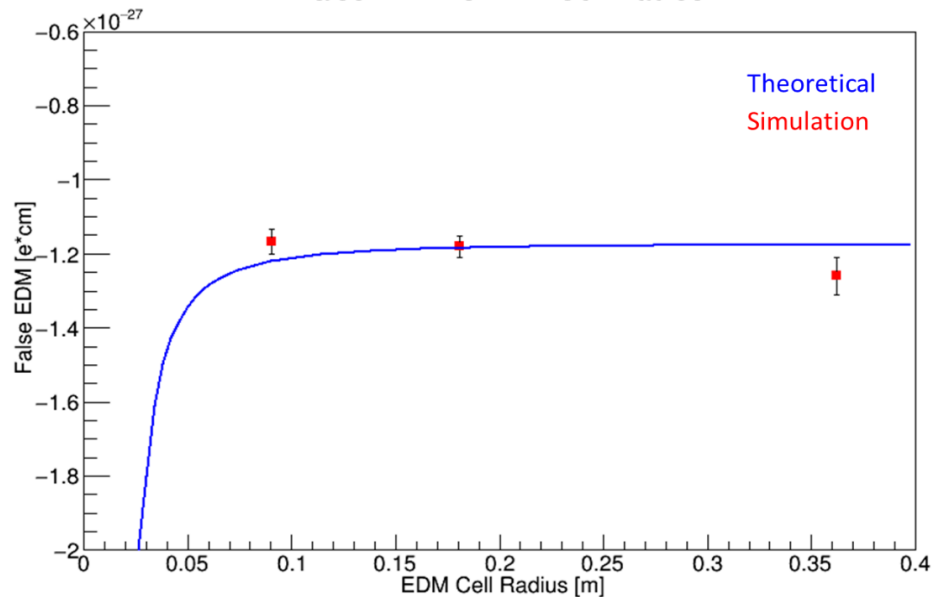
Mercury

False EDM vs EDM Cell Radius



Neutron

False EDM vs EDM Cell Radius

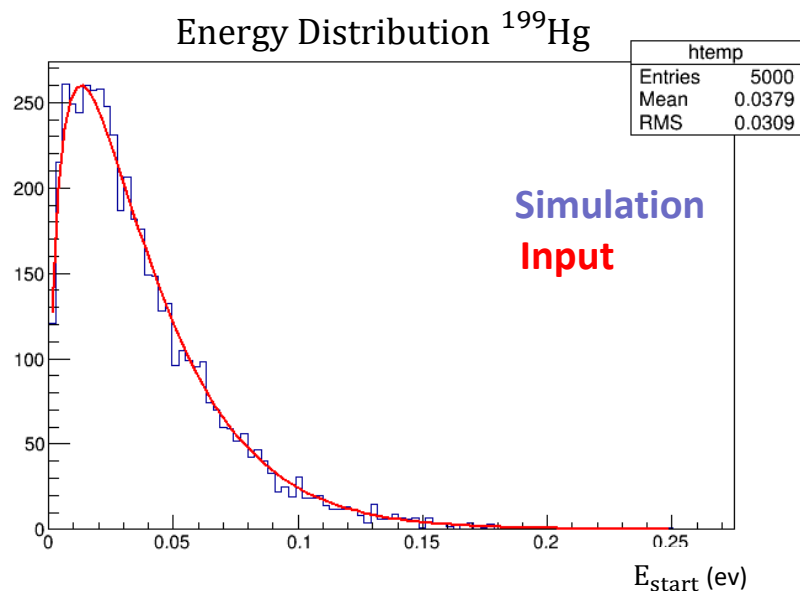


Comagnetometers (^{199}Hg)

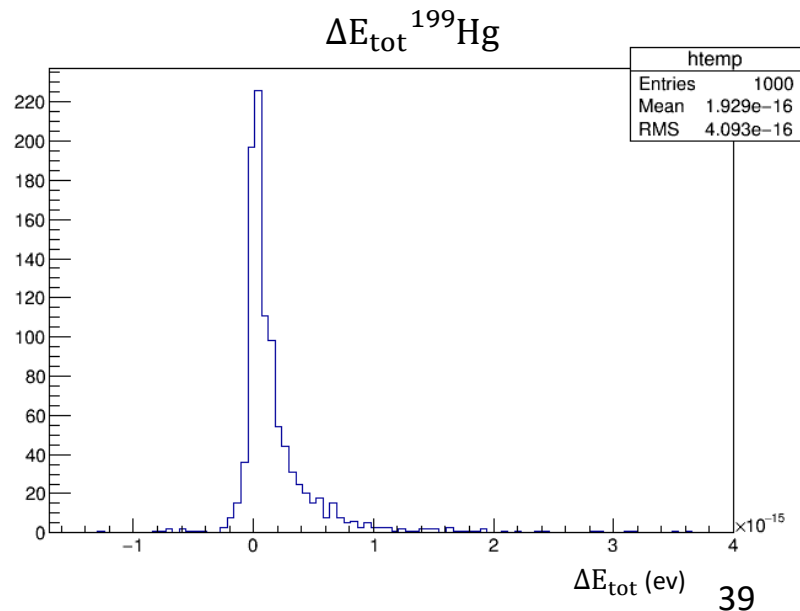


PENTrack: only p^+ , e^- , and n \longrightarrow Add xenon and mercury atoms

Maxwell-Boltzmann Energy Spectrum



Energy conservation: $\frac{\Delta E}{E} \cong 10^{-12}$ eV

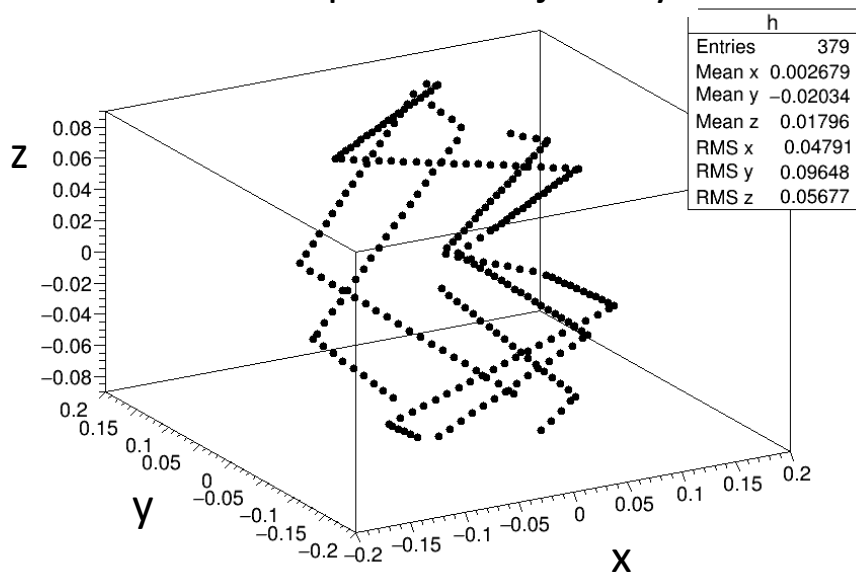


Comagnetometers (^{129}Xe)



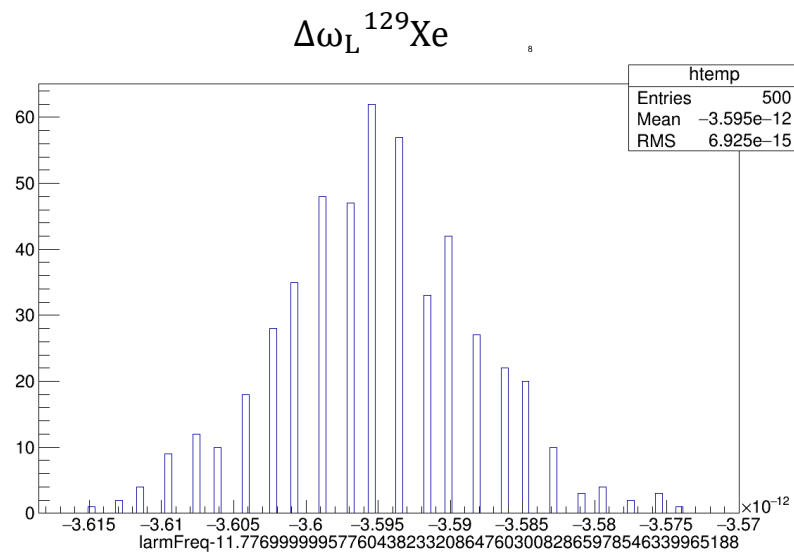
Trajectory check

^{129}Xe specular trajectory



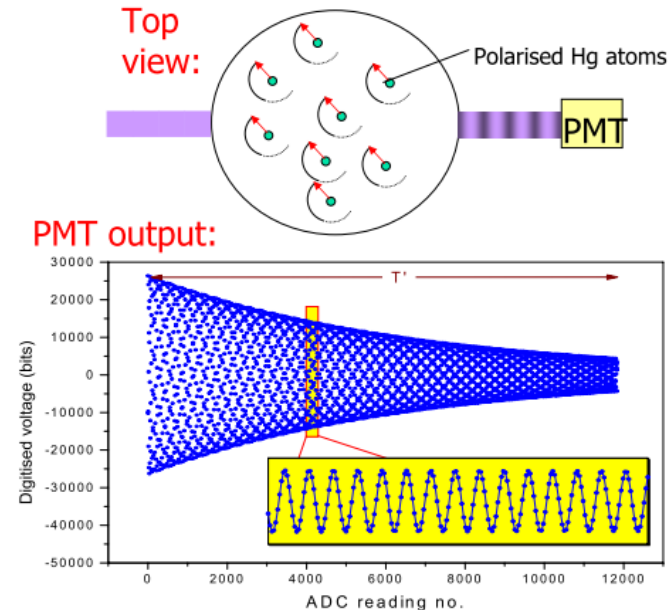
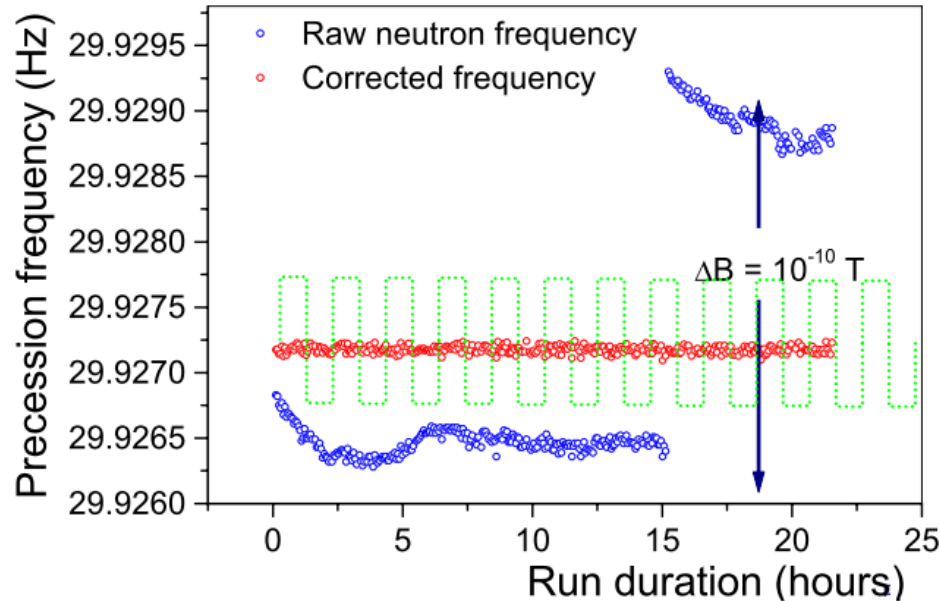
Larmor Precession Frequency Test

$\Delta\omega_{L,\text{Xe}} < 3.59 \text{ pHz}$ with **500 atoms**



Dual Comagnetometer

- Hg-199 and Xe-129 occupy cell volume
- Monitor changes in $\mathbf{B}_0 \rightarrow$ reduce systematic error



Current Status

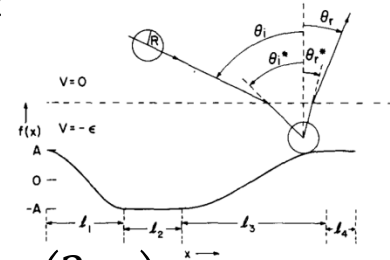
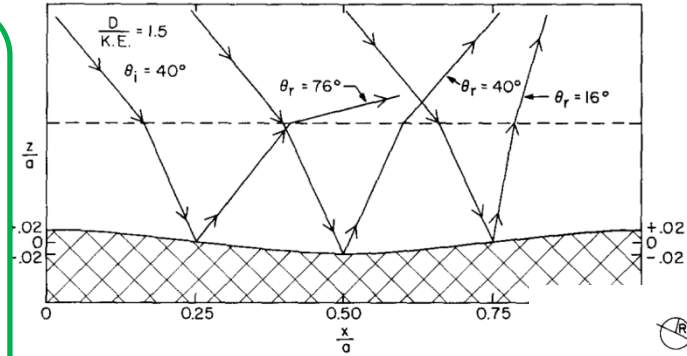
Specular and diffuse reflection model



Pendlebury: "...no dependence of the results on surface reflection law"

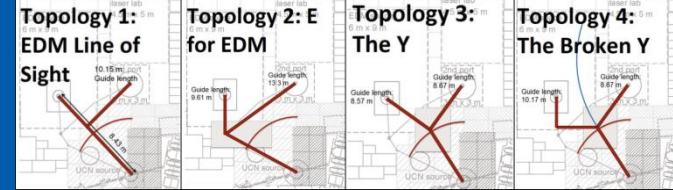


Implement **corrugated wall model** + **sticking time**



$$V = \begin{cases} 0 & z > z_0 \cos\left(\frac{2\pi x}{a}\right) \\ \infty & z \leq z_0 \cos\left(\frac{2\pi x}{a}\right) \end{cases}$$

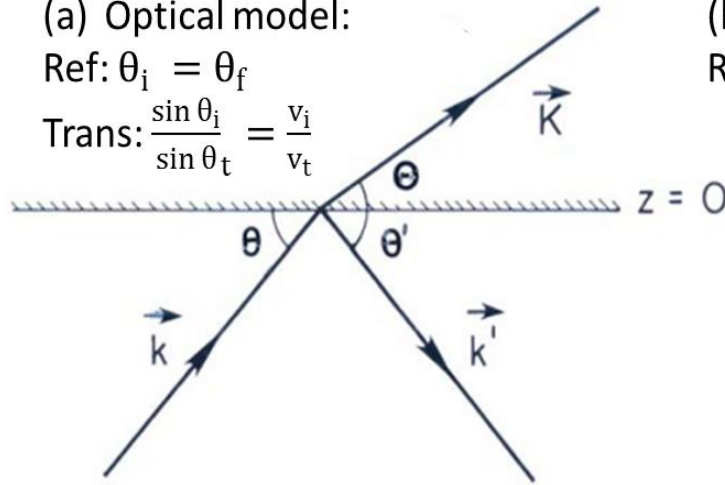
Reflection Models



(a) Optical model:

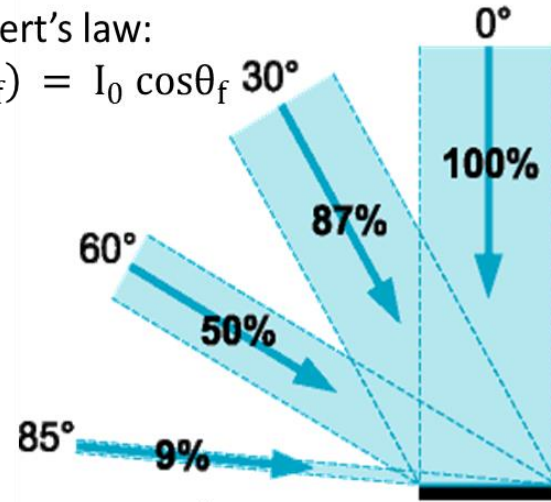
Ref: $\theta_i = \theta_f$

Trans: $\frac{\sin \theta_i}{\sin \theta_t} = \frac{v_i}{v_t}$

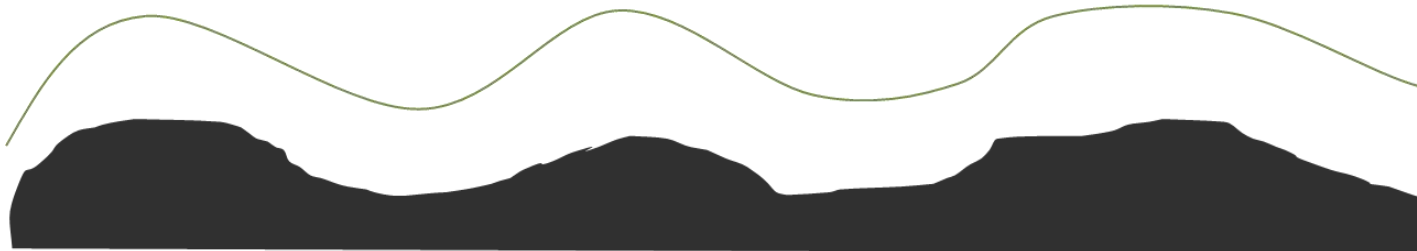


(b) Lambert's law:

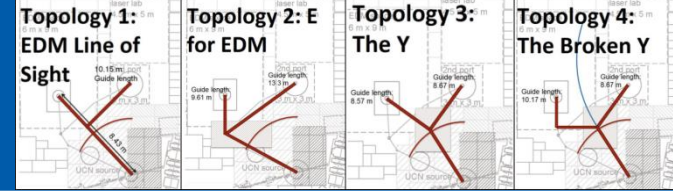
R&T: $I(\theta_f) = I_0 \cos \theta_f$



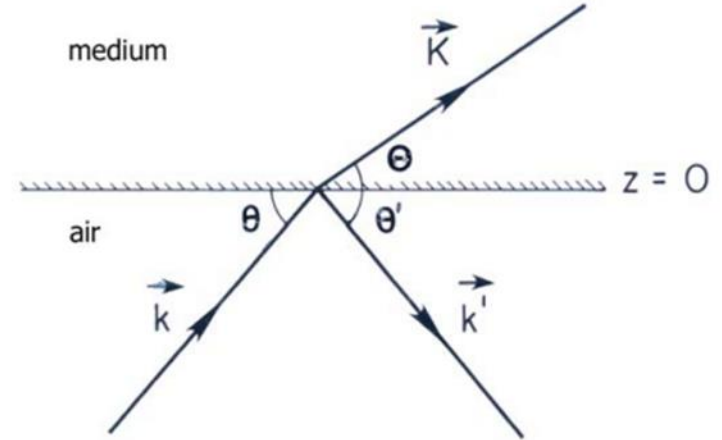
(c) MR surface parametrization: $f(\delta) = \mathbf{b}^2 \exp\left[-\frac{\delta^2}{2w^2}\right]$



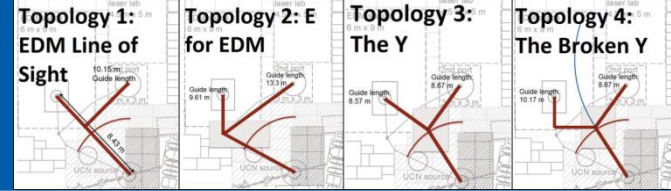
Specular Model



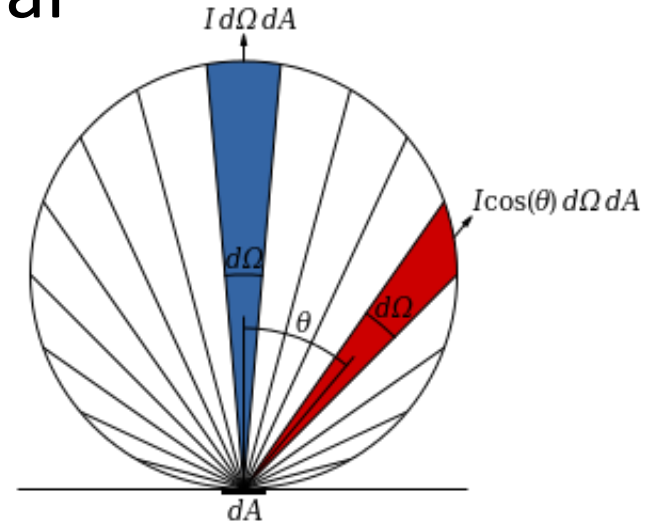
- Definition of model:
 - Ideal surface
 - Law of Reflection: $\theta_f = \theta_i$
 - Snells' law: $n \cong 1 - \frac{\lambda^2 N}{2\pi} \sqrt{b_c^2 - \left(\frac{\sigma_r}{2\lambda}\right)^2} + i \frac{\lambda N \sigma_r}{4\pi}$
 - » n = index of refraction
 - » N = nuclei number density
 - » b_c = scattering length
 - » σ_r = total loss cross section
 - » λ = neutron de Broglie wavelength



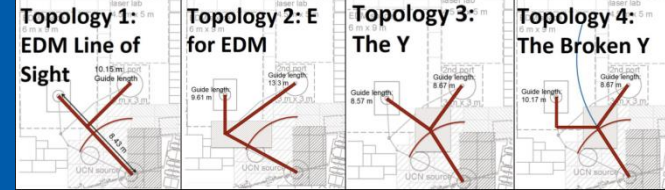
Lambert's Model



- “Radiant intensity observed any angle is directly proportional to cosine of the direction of incidence and the normal”
- $I(\theta_f) = I_0 \cdot \cos(\theta_f)$



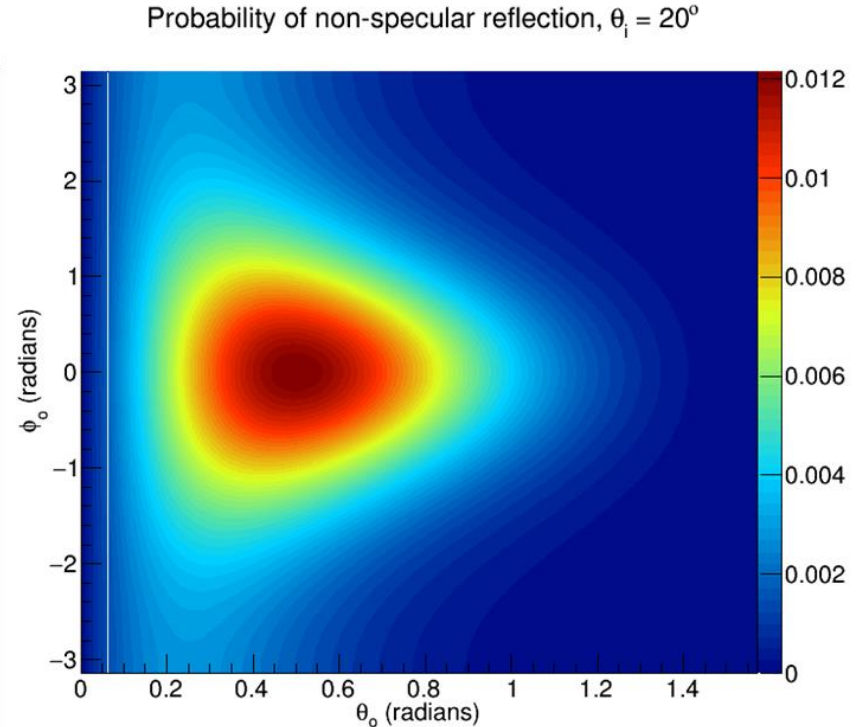
Micro-roughness Model



- Surface roughness modelled by Guassian peaks with Gaussian distribution

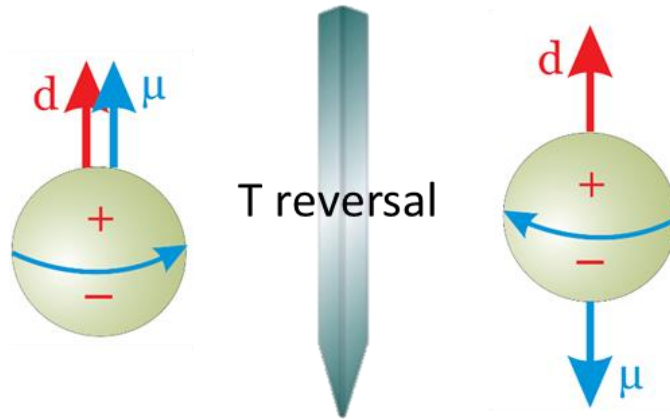
$$- f(\vec{r}) = f(r) = b^2 \exp\left[-\frac{r^2}{2w^2}\right]$$

- Dependent on θ_i
- Energy dependent
- Material dependent



Neutron and CP violation

1. Consider neutron with d_n and $\mu \rightarrow$ apply time reversal
2. d_n remains unchanged but μ reverses
3. T-symm. violated \rightarrow CP violated



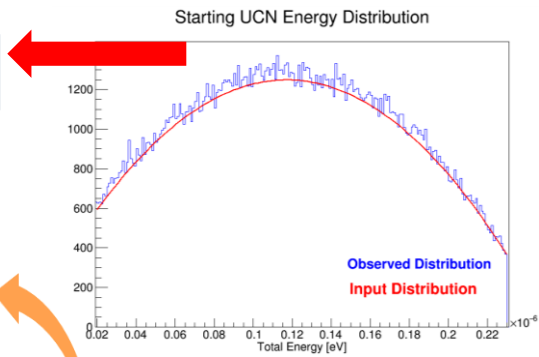
Monte Carlo (MC) Simulations



“Use **random numbers** to **sample** different **probability distributions**.”



Create neutrons: **position** & **velocity***



Integrate **equation of motion**# $\times N_{\text{particle}}$

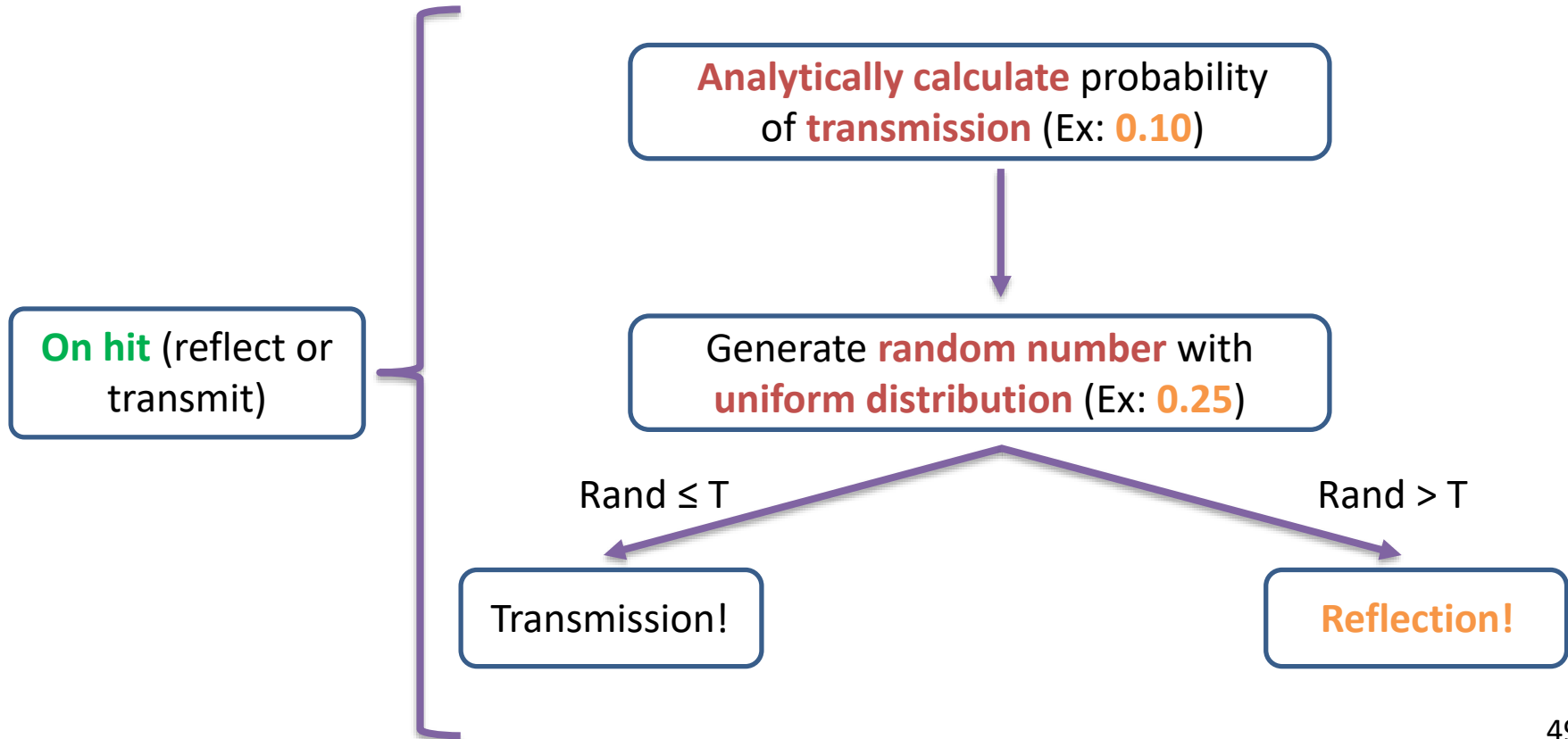
$$\ddot{\mathbf{x}} = \frac{\mathbf{F}}{m}$$

$$\ddot{\mathbf{x}} = \frac{1}{\gamma m} \left(1 - \frac{1}{c^2} \dot{\mathbf{x}} \otimes \dot{\mathbf{x}} \right) (P\mu\nabla|\mathbf{B}| - m g \mathbf{e}_z + q(\mathbf{E} + \dot{\mathbf{x}} \times \mathbf{B}))$$

On hit (reflect, transmit, absorb) and **decay***

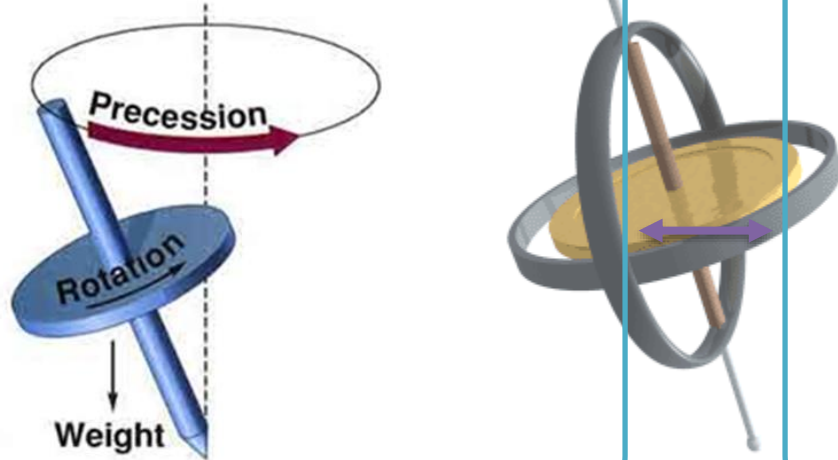
*** = MC step**
= Deterministic

Example of Monte Carlo Step

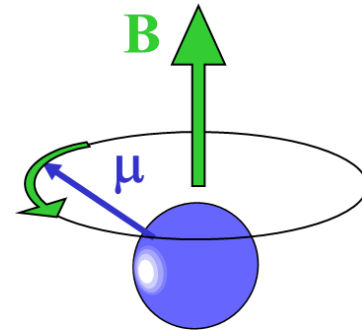


“Rotation of the axis of rotation”

Gyroscope – **gravity** field



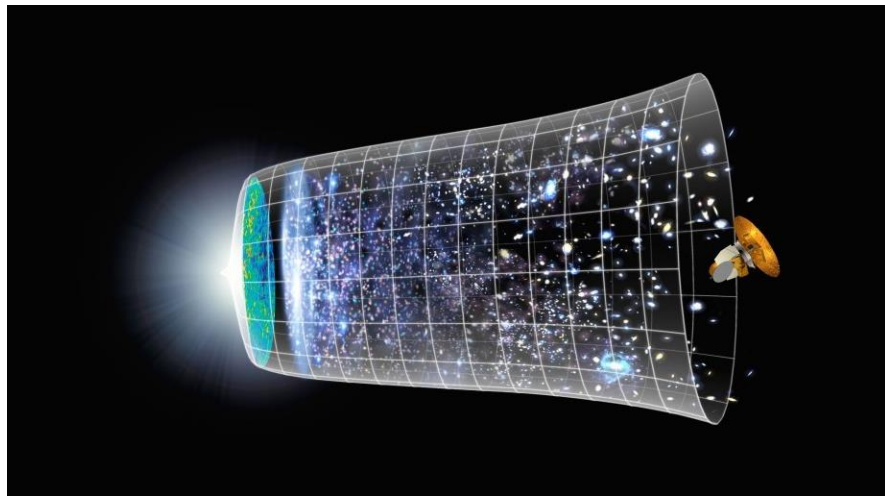
neutron **spin** – **magnetic field**



“Larmor” Precession: $\omega_L = -\gamma B$

Stronger field \rightarrow **Faster** precession (ν)

Explanation for matter/antimatter asymmetry



Evidence for physics **beyond** the **Standard model**

