Coulomb Artifacts and $b\bar{b}$ Hyperfine Splitting in Lattice NRQCD

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Based On

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Bottomonium

Bound state of bottom quark-antiquark pair

$$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$$

- Υ: triplet state (spin 1)
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  - First determination of ground state hyperfine splitting

$$\boxed{E_{hfs} = M_{\Upsilon(1S)} - M_{\eta_b(1S)}}$$

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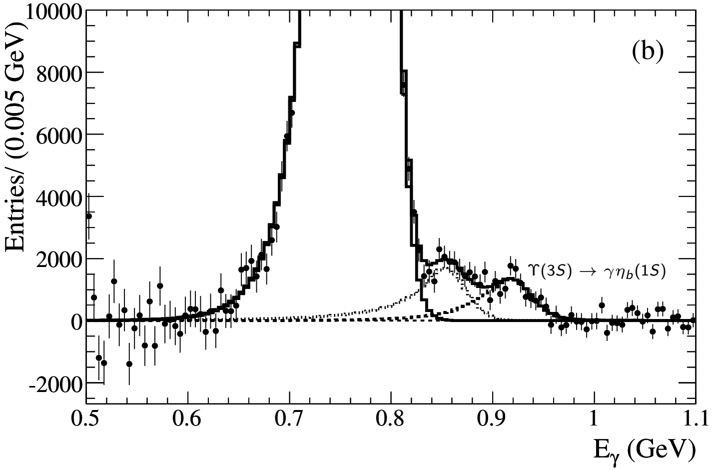
BaBar Collaboration, PRL 101, 071801 (2008)



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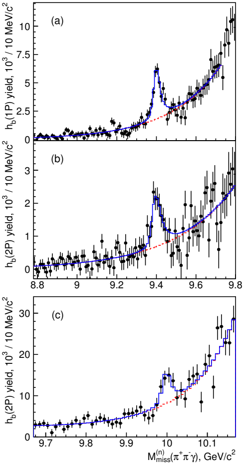

$h_b(1P) \to \gamma\eta_b(1S)$

$h_b(2P) \to \gamma\eta_b(1S)$

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$\Rightarrow$ Matching procedure should be investigated

Lattice NRQCD

Heirachy of energy scales in heavy quarkonium dynamics:

- Rest mass ($\sim m_q$)
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- Soft(er) modes simulated on the lattice

$\mathcal{O}(v^4)$ NRQCD Lagrangian (Kinetic + HFS)

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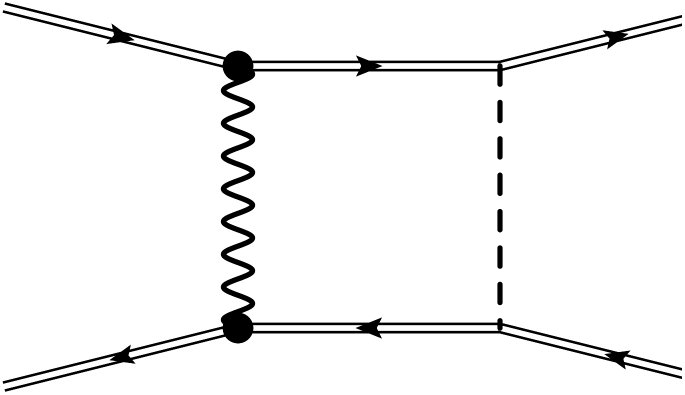
Interested in d_σ linear dependence on am_q (linear artifacts)

Coulomb Linear Artifacts

Where would they come from?

NRQCD Planar Ladder Diagram



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HPQCD result includes linear term in d_σ

Schrodinger Matching

T. Liu, A. Penin, A. Rayyan JHEP02(2017)084

Problem: For Coulomb bound states, $v \sim \alpha_s$

- n-loop planar ladder diagrams $\propto \left(\frac{\alpha_s}{v}\right)^n$
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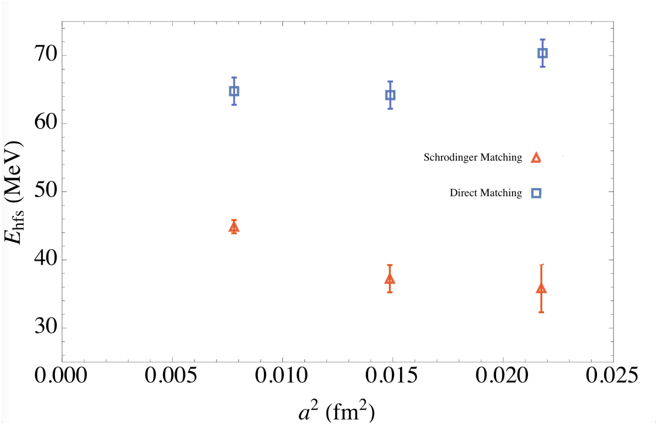
$\Rightarrow$ HPQCD result contains sprious contribution

Ground State Hyperfine Splitting



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$$E_{hfs}^{lattice} = E_{hfs}\left[1 - (\Lambda a)^2 + \mathcal{O}(a^4)\right], \quad \Lambda = \frac{C_F \alpha_s m_q}{2\sqrt{2}} \sim 530 \text{ MeV}$$

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Resolved ambiguity in the lattice data!

Conclusion

- Revised matching procedure for lattice NRQCD
- Lattice data does *not* contain Coulomb linear artifacts
- Final lattice prediction:

$$\boxed{E_{hfs} = 52.9 \pm 5.5 \text{ MeV}}$$

- Agrees with Belle: 57.9 ± 2.3 MeV

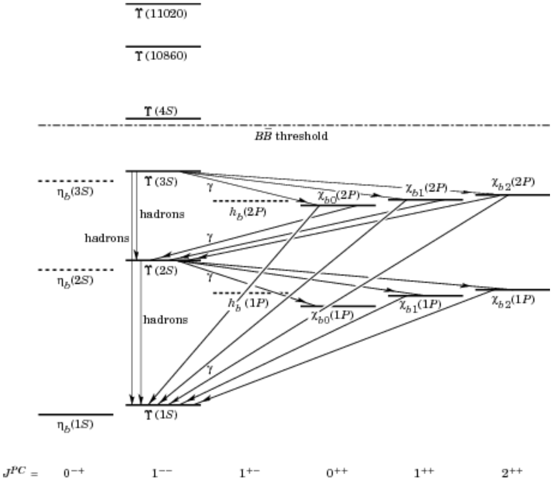
References

- B. A. Kniehl, A. Penin, A. Pineda, V. Smirnov, M. Steinhauser, PRL 92, 242001 (2004).
- R. J. Dowdall et al. [HPQCD Collaboration], PRD 85, 054509 (2012) [Erratum-ibid. 104, 199901 (2010)]
- R. J. Dowdall et al. [HPQCD Collaboration], PRD 89, 031502 (2014) [Erratum-ibid. 92, 039904 (2015)]
- M. Baker, A. A. Penin, D. Seidel and N. Zerf, PRD 92, 054502 (2015)

Why not Lattice QCD?

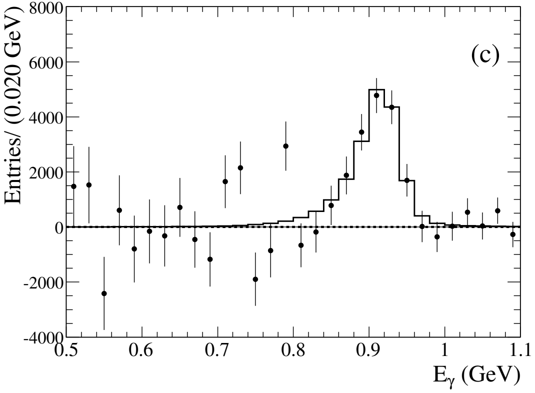
- To accomodate short-distance effects: $a \ll \frac{1}{m_q}$
- To include NP effects: $\frac{1}{\Lambda_{QCD}} \ll L$
- Number of points: $\left(\frac{L}{a}\right)^4 \gg \left(\frac{m_q}{\Lambda_{QCD}}\right)^4 \sim 20^4$ for $m_b \sim 5$ GeV
- Lattice NRQCD: $\left(\frac{L}{a}\right)^4 \gg \left(\frac{m_q v}{\Lambda_{QCD}}\right)^4 \sim 6^4$ for $m_b \sim 5$ GeV

$b\bar{b}$ Spectrum



J-M Richard, arXiv:1205.4326 (2012)

Babar decay, all background subtracted



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