Recent progress in NLEFT

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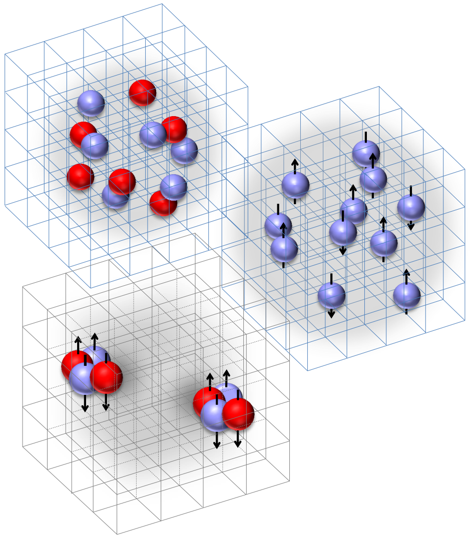
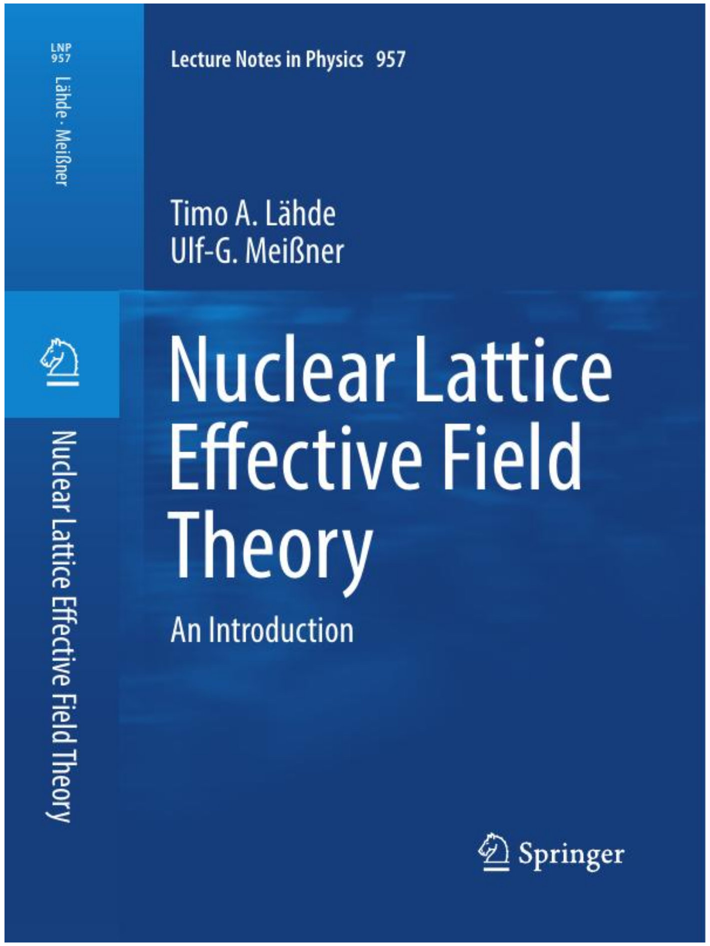
Contents

- Nuclear physics on a lattice
  - Foundations
  - The chiral Hamiltonian at N3LO
  - Applications to structure, reactions and matter
  - Towards improved 3NFs
  - Multi-strangeness matter
- Summary & outlook

Nuclear physics on a lattice





LNP 957

Lähde · Meißner

Nuclear Lattice Effective Field Theory

Lecture Notes in Physics 957

Timo A. Lähde
Ulf-G. Meißner

Nuclear Lattice Effective Field Theory

An Introduction

Springer

T. Lähde & UGM

Nuclear Lattice Effective Field Theory - An Introduction

Springer Lecture Notes in Physics **957** (2019) 1 - 396 [2nd edition in the works]

Foundations

Nuclear lattice effective field theory (NLEFT)

- *new method* to tackle the nuclear many-body problem

- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

→ see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- EFT on the lattice, maximal momentum:

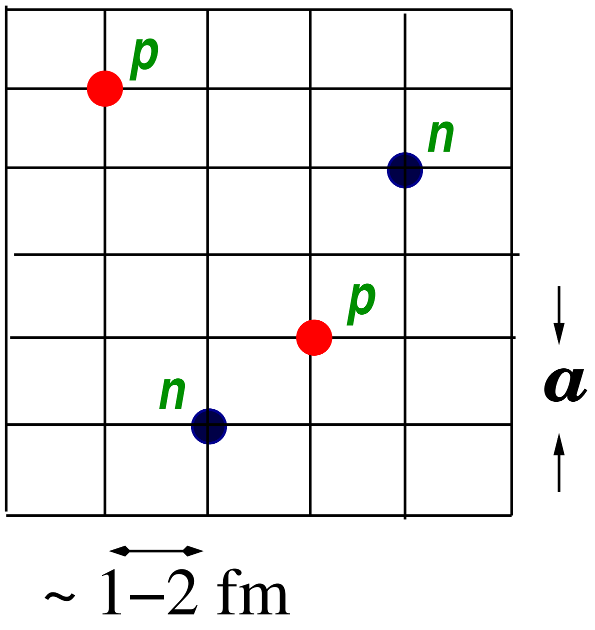
$$p_{\rm max} = \frac{\pi}{a} \simeq 315 - 630\,\text{MeV [UV cutoff]}$$

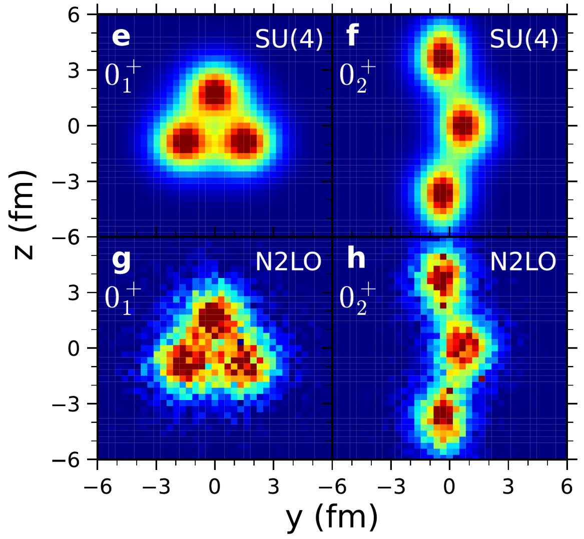
- strong suppression of sign oscillations SU(4)
  due to approximate Wigner (spin-isospin) symmetry

Wigner, Phys. Rev. **51** (1937) 106; Chen et al., Phys. Rev. Lett. **93** (2004) 242302

↪ works well for even-even nuclei

↪ we still need another method



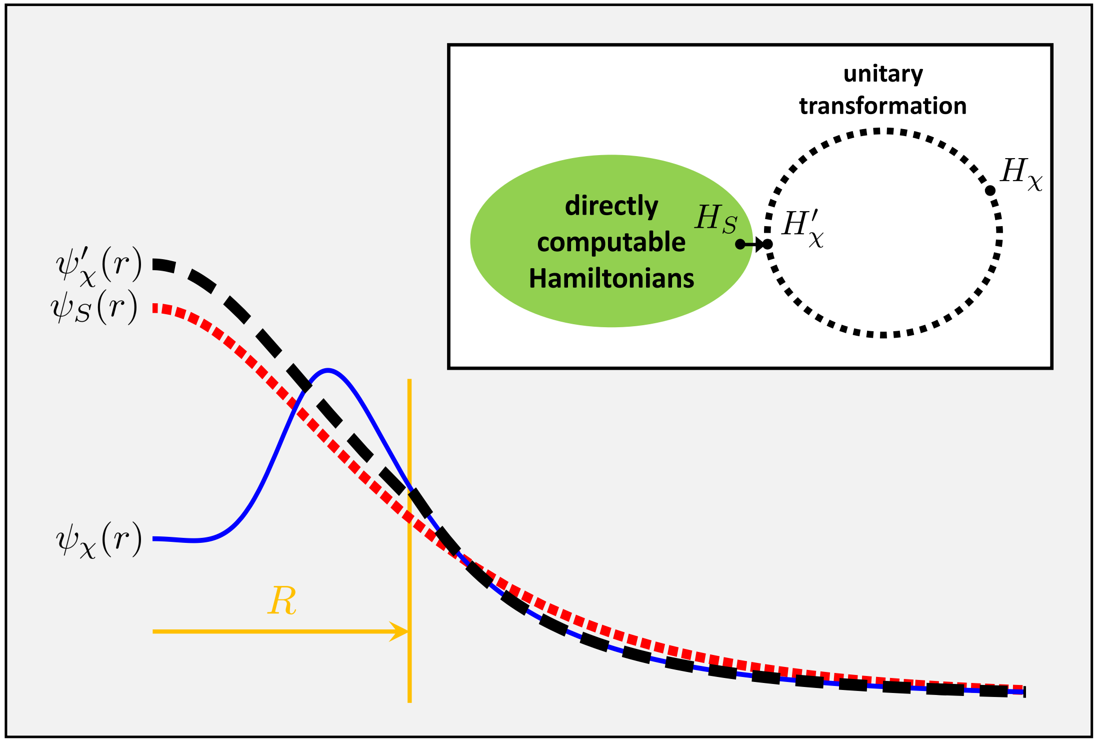


Shen et al., Nature Commun. **14** (2023) 2777

Wave function matching

Elhatisari et al., Nature **630** (2024) 59

- A new quantum many-body method: Bring a complex Hamiltonian $\boldsymbol{H_\chi}$ close to a simple one $\boldsymbol{H_S}$ (with $\boldsymbol{H_S}$ essentially free of sign oscillations)
  $\hookrightarrow$ treat $\boldsymbol{H_S}$ non-perturbatively & $\boldsymbol{H'_\chi - H_S}$ in perturbation theory
- Graphical representation of w.f. matching



$\Rightarrow$ Efficient suppression of sign oscillations, applicable in many fields!

Partial pinhole algorithm

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Nuclear radii can be expressed in terms of two-particle correlations G_{pp}, G_{pn}, G_{nn}

$$\langle r^2 \rangle = \sum_{c_1, c_2} \langle : \rho_{i_1 j_1}(\mathbf{n}_1) \rho_{i_2 j_2}(\mathbf{n}_2) : \rangle (\mathbf{n}_1 - \mathbf{n}_2)^2 \,, \quad c_k = (i_k, j_k, \mathbf{n}_k)$$

$\hookrightarrow$ the number of pinholes needed in such a calculation can be reduced:

$$\rho_M(c_1, \ldots, c_M) = \frac{(A-M)!}{A!} : \rho_{i_1 j_1}(\mathbf{n}_1) \cdots \rho_{i_M j_M}(\mathbf{n}_M) : \,,$$

with $\sum_{c_1, \ldots, c_M} \rho_M = 1$

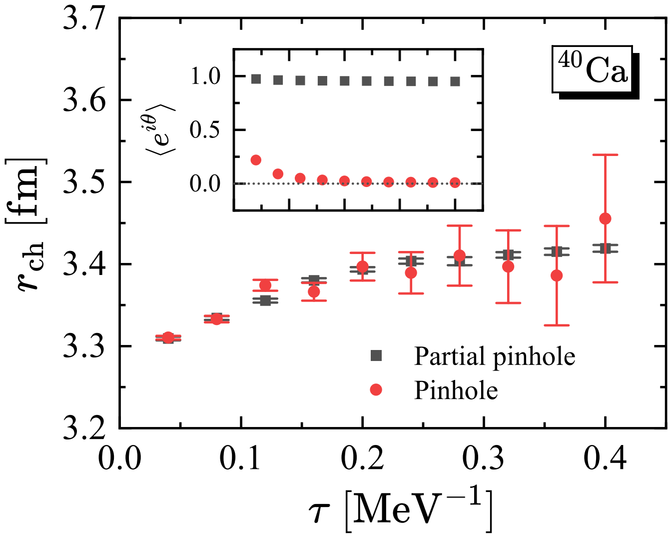
- However, $\rho_M |\Psi_A\rangle$ is not a Slater determinant, even if $|\Psi_A\rangle$ is

$\hookrightarrow$ use the rank-one operator method for ρ_M Ma et al. (2024)

$$: \rho_{i_1 j_1}(\mathbf{n}_1) \cdots \rho_{i_M j_M}(\mathbf{n}_M) :$$

$$= \lim_{\varepsilon \to \infty} \frac{1}{\varepsilon^M} : e^{\varepsilon [\rho_{i_1 j_1}(\mathbf{n}_1) + \cdots]} :$$

$\hookrightarrow$ much improved signal allows for a more precise extraction



The chiral Hamiltonian at N3LO

The chiral Hamiltonian

- Chiral Hamiltonian with **smeared** N3LO (2N) and N2LO (3N) interactions

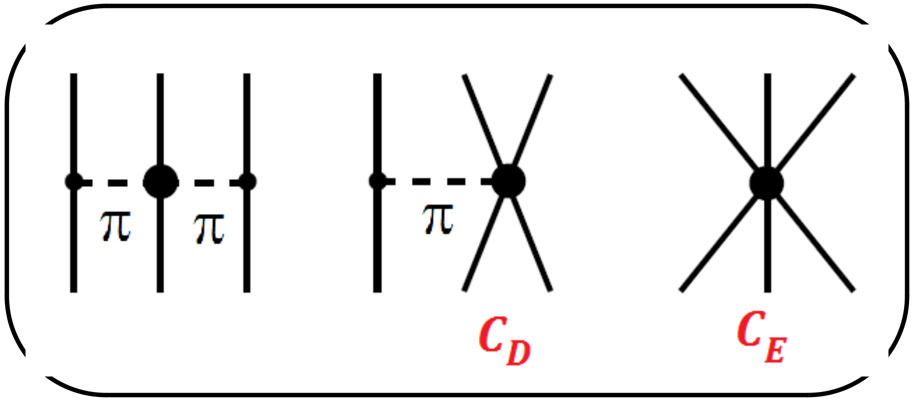
$$\boxed{H = K + V_{\rm OPE} + V_{\rm C} + V_{\rm 3N}^{\rm Q^3} + V_{\rm 2N}^{\rm Q^4} + W_{\rm 2N}^{\rm Q^4}}$$

  - K – kinetic energy term with correct nucleon dispersion relation (FFT)
  - $V_{\rm OPE}$ – regulated OPE (similar to continuum case) Reinert et al. (2018)
  - $V_{\rm C}$ – Coulomb interaction
  - $V_{\rm 3N}^{\rm Q^3}$ – 3N interaction at N2LO → to be discussed below
  - $V_{\rm 2N}^{\rm Q^4}$ – 2N short-range interaction at N3LO (2π exchange absorbed)
  - $W_{\rm 2N}^{\rm Q^4}$ – 2N Galilean invariance restoration (GIR) interaction at N3LO Li et al. (2019)

- Smearing explained in what follows
- Throughout, we use $a = 1.32$ fm, corresponding to the magic momentum cutoff $\Lambda \simeq 470$ MeV Lee et al. (2021)

Three-nucleon forces

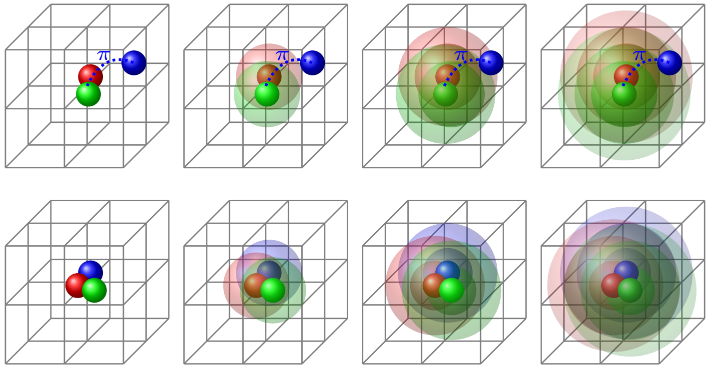
- Basic topologies at N2LO:



  - 2-pion topology entirely fixed from πN scattering
  - Smearing of the one-pion and contact term topologies

    $\hookrightarrow$ more LECs

    $\hookrightarrow$ effectively higher-order op's



  - On the market so far (more in the works!)

    $\hookrightarrow$ $\mathcal{F}_p = 8$, only local smearings Elhatisari et al. (2024)

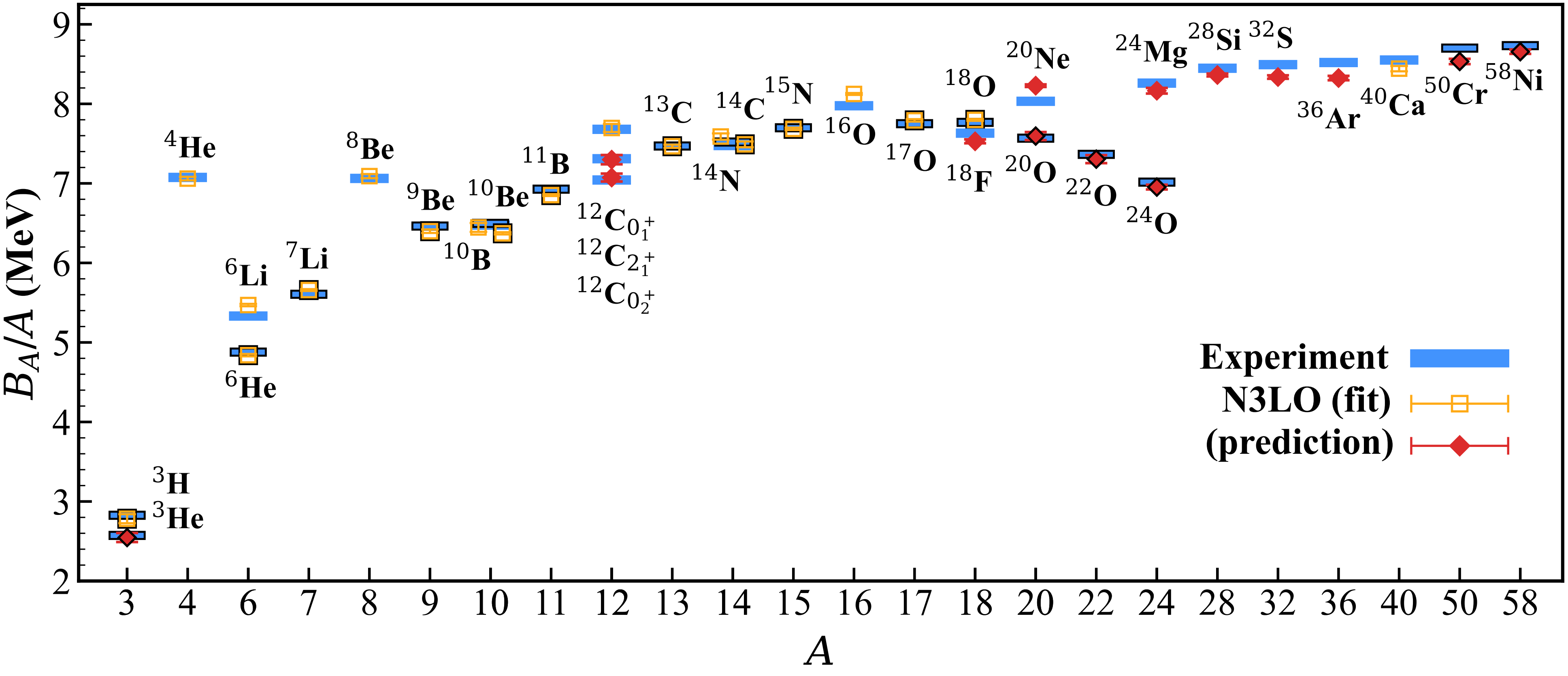
    $\hookrightarrow$ $\mathcal{F}_p = 6$, a few non-locally smeared ops Ren et al. (2025)

Fixing the 3NF LECs: Binding Energies at N3LO

Elhatisari et al., Nature **630** (2024) 59

- Quality of the fit: $\mathrm{RMSD}(\mathcal{S}) = \sqrt{\frac{1}{M_\mathcal{S}} \sum_{i\in\mathcal{S}} \left(\frac{E_i - E_i^{\mathrm{exp}}}{A_i}\right)^2}$ with size $M_\mathcal{S}$

$\hookrightarrow$ here, $\mathbf{RMSD = 0.079\ MeV}$ = 1.11% of the average BE per nucleon

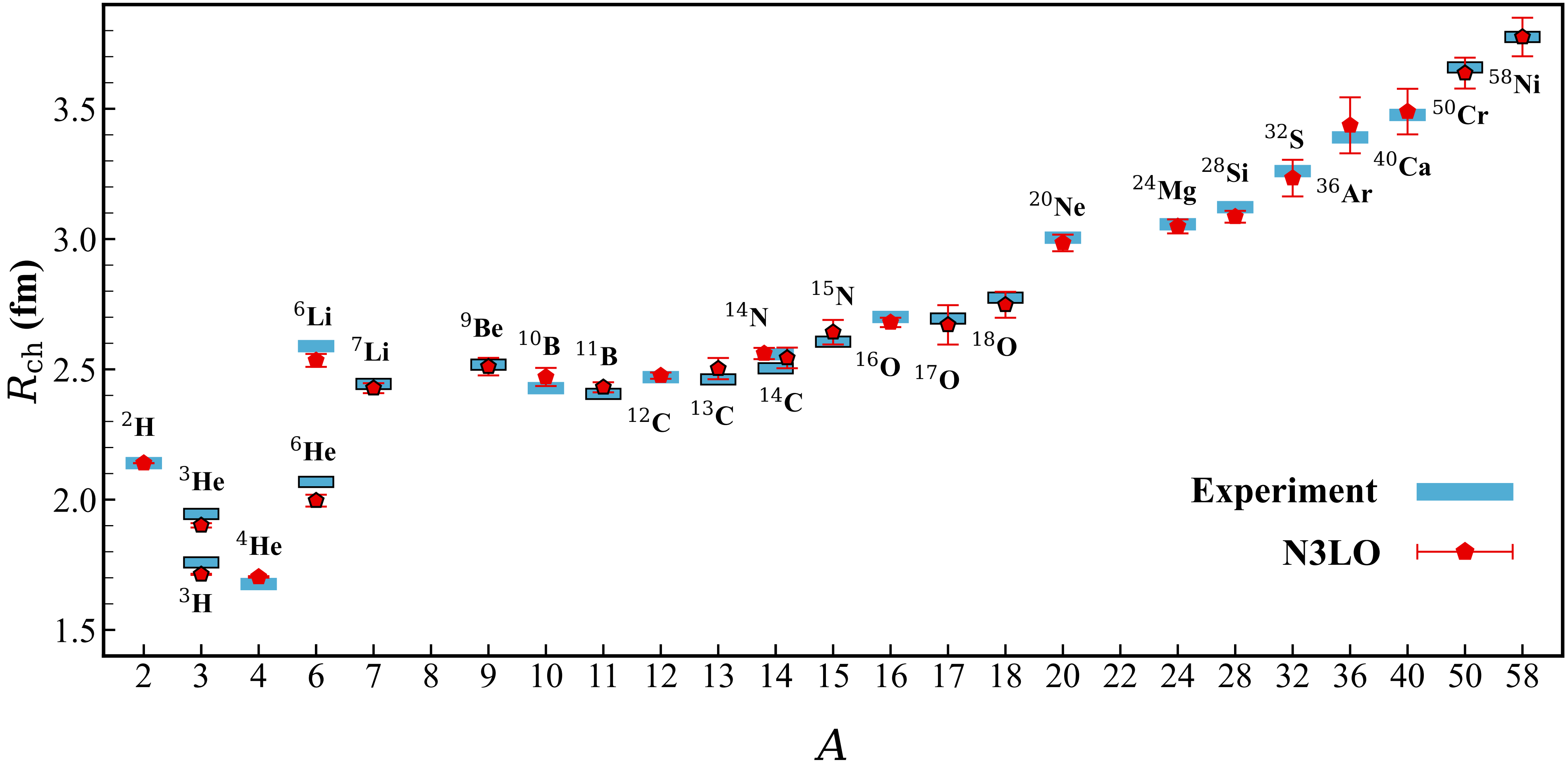


Applications to structure, reactions and matter

Prediction: Charge radii at N3LO

Elhatisari et al., Nature **630** (2024) 59

- Charge radii ($a = 1.32$ fm, statistical errors can be reduced)

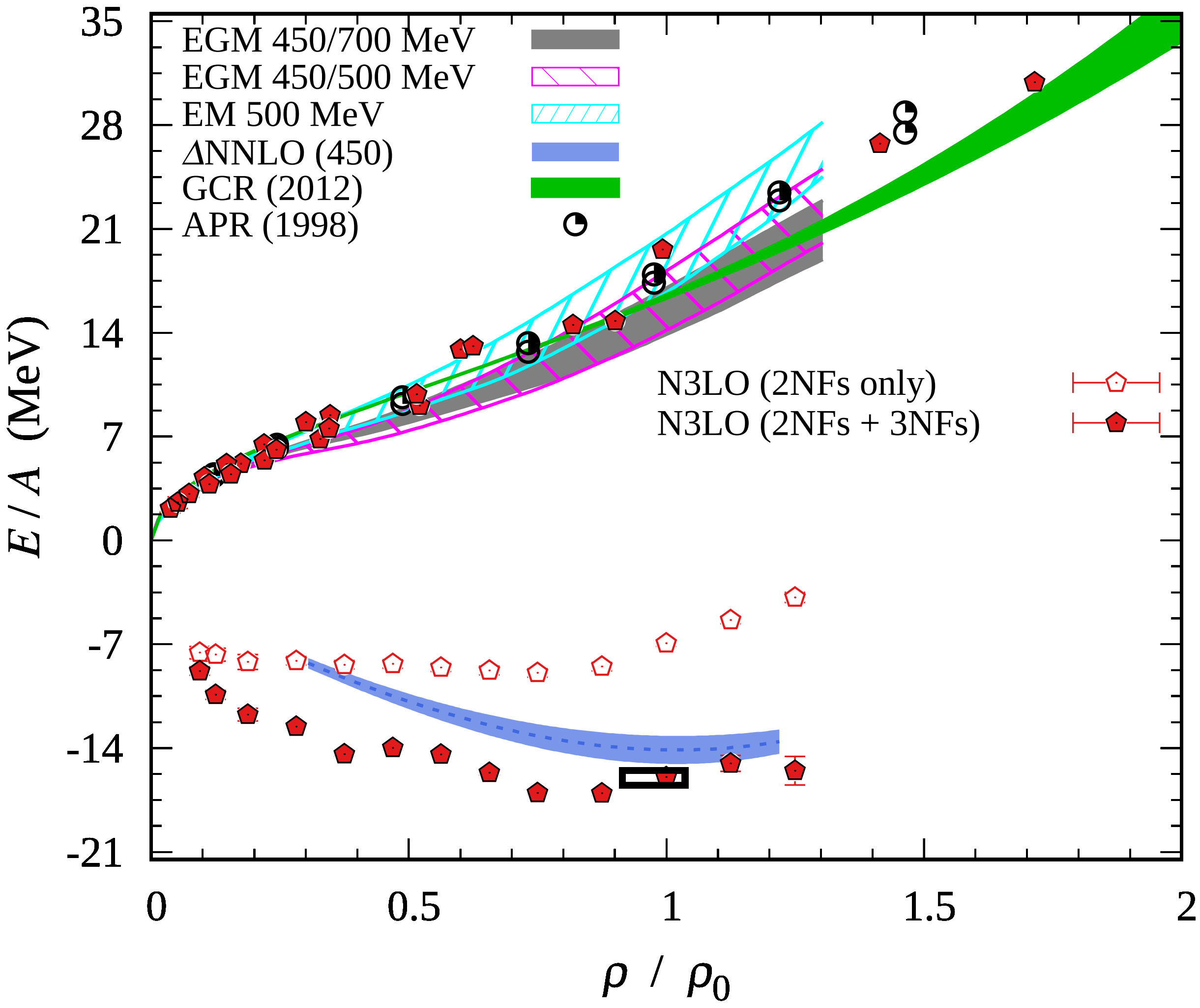


↪ no radius problem! but radii to be improved, see later

Prediction: Neutron & nuclear matter at N3LO

Elhatisari et al., Nature **630** (2024) 59

- Equation of State (EoS) of pure neutron matter & nuclear matter ($\boldsymbol{a = 1.32}$ fm)



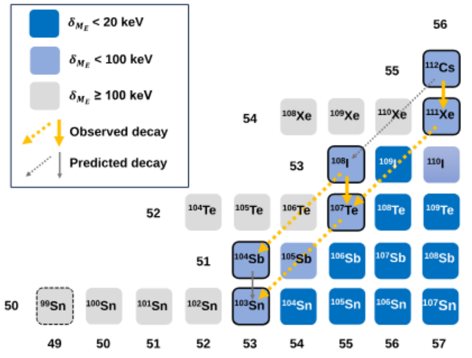
$\hookrightarrow$ can be improved using twisted b.c.'s

Is it possible to go to heavier nuclei?

- Consider the Sn isotopes close to the proton dripline:

↪ Standard candle of nuclear structure theory

↪ Even $N = Z$ nucleus close to the dripline



Ireland et al., PRC **111** (2025) 014314

↪ Inconsistencies in 2020 AME → new exp's at ISOLTRAP and LEBIT

Nies al. PRC **111** (2025) 014315; Ireland et al., 2510.11815

↪ Dripline so far based on extrapolations

- Premier playground for the newest
  generation of supercomputers (Exascale)
  here: JUPITER @ FZJ

Prediction: Tin isotopes close to the proton dripline

Hildenbrand et al., PRL **136** (2026) 062501

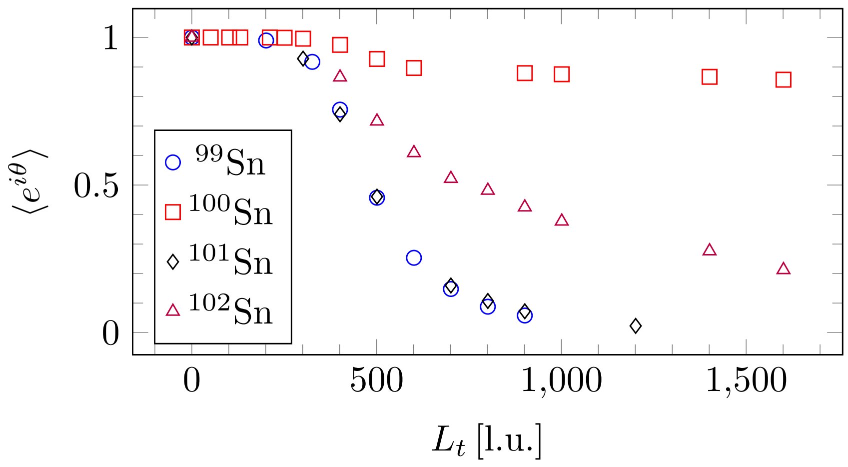
- Signal still strong enough for even-odd isotopes

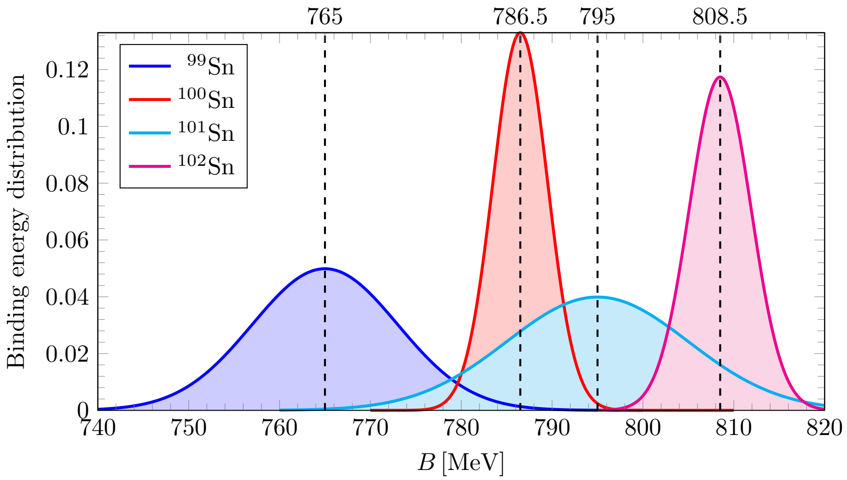
↪ can extract the BEs of ^{99}Sn to ^{102}Sn

↪ small modifications of 3NFs to describe g.s. data

Nucleus	N3LO	N3LO*	Exp.
^{99}Sn	765±8	804±8	807.9± 0.6
^{100}Sn	786.5±3.0	825.2±3.0	825.16±0.24
^{101}Sn	795±10	834±10	836.39±0.30
^{102}Sn	808.5±3.4	848.1±3.4	849.09±0.10

Isotopes	N3LO	N3LO*	Exp.
102-101	13.5±10.6	14.1±10.6	12.7±0.3
102-100*	22.5±4.6	22.9±4.6	23.9±0.3
102-99	43.5±8.7	44.1±8.7	41.2±0.6
101-100	8.5±10.4	8.9±10.4	11.2±0.4
101-99*	30.0±12.8	30.1±12.8	28.5±0.7
100-99	21.5±8.6	21.2±8.6	17.3±0.7





Prediction: Multi-neutron correlations in light nuclei I

Zhang, Elhatisari, UGM, 2512.18849 [nucl-th]

- $3n$ and $4n$ clusters much thought about

↪ breakup, knockout & double charge exchange reactions

↪ investigate multi-n correlations in light nuclei

- Use 282 chiral interactions to predict:

$$E_{\rm gs}(^6{\rm H}) = -4.94 \pm 0.14\,\text{MeV}$$

$$E_{\rm gs}(^7{\rm H}) = -5.29 \pm 0.16\,\text{MeV}$$

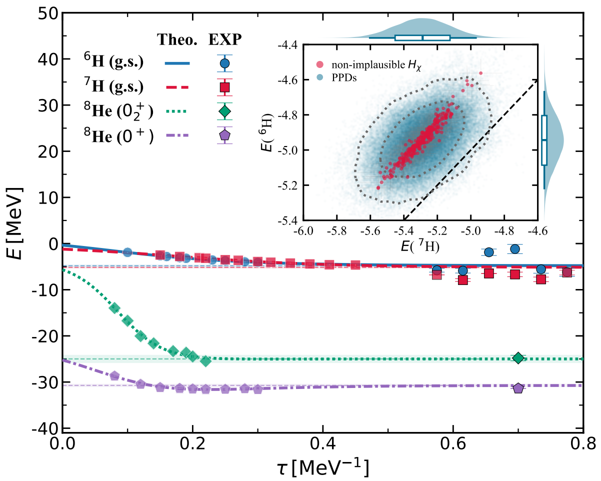
- Marginal posteriors suggest:

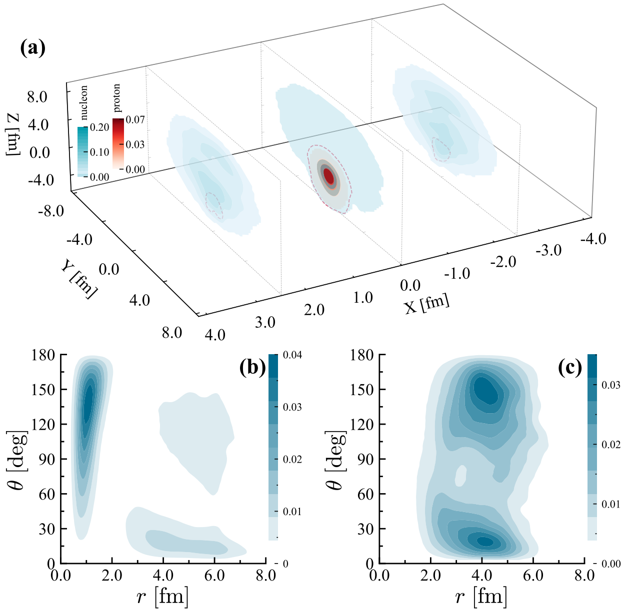
$$S_n(^7{\rm H}) = 0.35 \pm 0.32\,\text{MeV}$$

↪ sequential ^{6}H+n decay disfavored

↪ enhances the relevance of multi-n correlations

↪ two-neutron correlations clearly visible



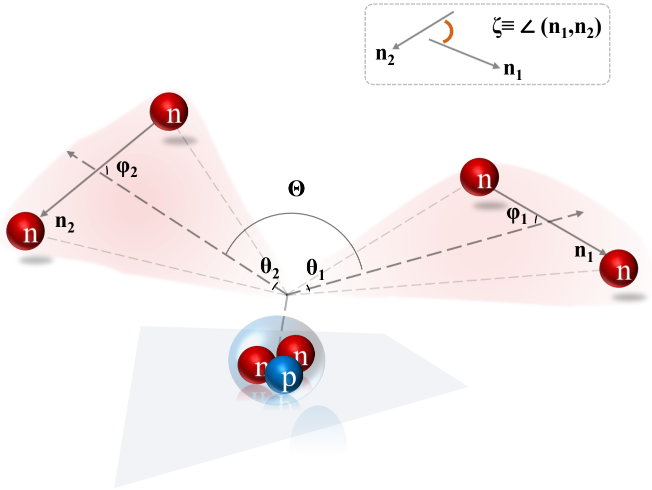


Prediction: Multi-neutron correlations in light nuclei II

Zhang, Elhatisari, UGM, 2512.18849 [nucl-th]

- Define four-neutron correlations via:

$$\rho_4(\theta_1,\varphi_1,\theta_2,\varphi_2;\Theta,\zeta) = \sum_{i<j,\,k<l}^{(ij),(kl)} \Big\langle \delta(\theta_{ij}-\theta_1)\,\delta(\varphi_{ij}-\varphi_1) \times\delta(\theta_{kl}-\theta_2)\,\delta(\varphi_{kl}-\varphi_2) \times\delta(\Theta_{ij,kl}-\Theta)\,\delta(\zeta_{ij,kl}-\zeta)\Big\rangle,$$



- The four-neutron correlations show:

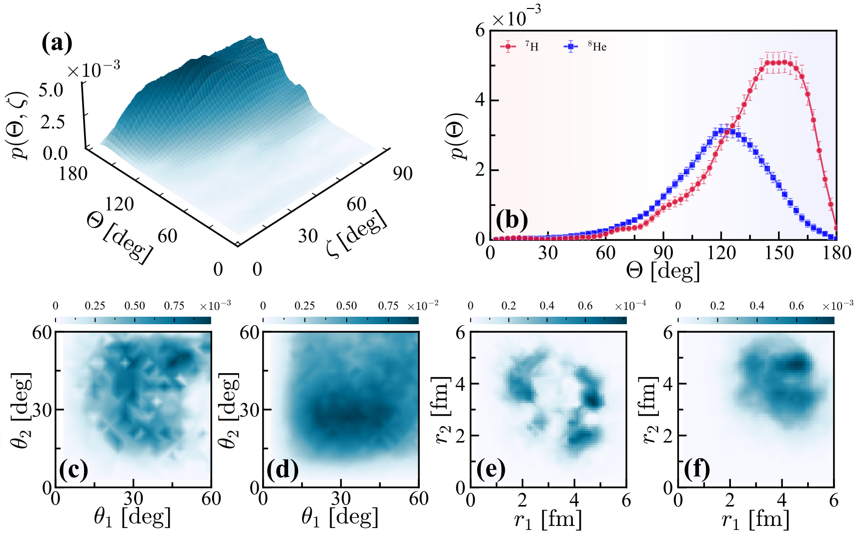
↪ symmetric $2n$-$2n$ configuration is dominant

$(\sim 95\%,\ \ \Theta \simeq 140^\circ - 160^\circ)$

↪ compact tetraneutron configuration small

$(\sim 5\%,\ \ \Theta \simeq 60^\circ - 90^\circ)$

↪ must consider the $2n$-$2n$ config. in exp analysis!



Prediction: Searching for the tetraneutron

Wu, Elhatisari, UGM, Shen, Geng, Kim, 2601.01801 [nucl-th]

- Latest tetraneutron sighting in ^{4}He(^{8}He,^{8}Be)$4n$

  Duer et al., Nature **606** (2022) 678

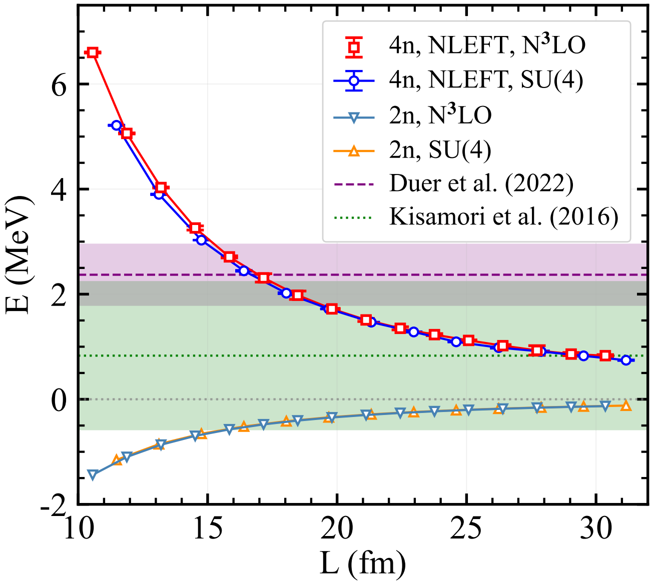
↪ analysis presumably too simplified,
various re-analyses show no resonance

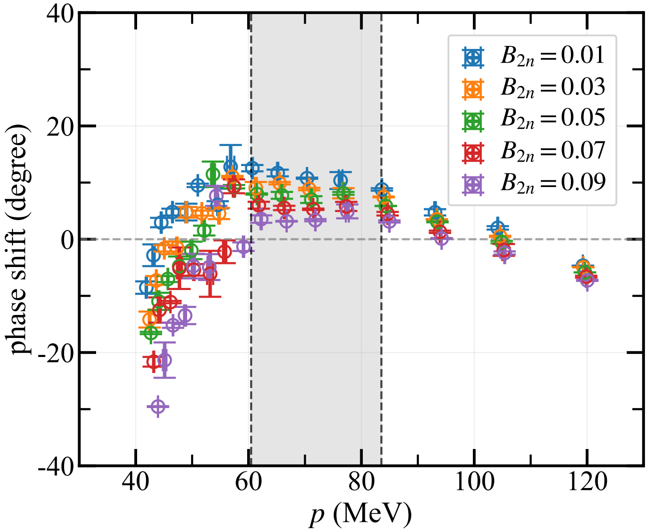
↪ most theory analyses do not give a $4n$ resonance

- Search on the lattice (SU(4) and N3LO interactions)

↪ volume-dependence shows no sign of a resonance

↪ $2n$-$2n$ scattering shows a shallow attraction
in the momentum range $p \simeq 60 - 85$ MeV
corresponds to confined $4n$ energies of $1.7 - 3.3$ MeV
similar to the $4n$ peak energies in experiment





Prediction: Neutron-alpha scattering

Elhatisari, Hildenbrand, UGM, J. Phys. **G 52** (2025) 125102

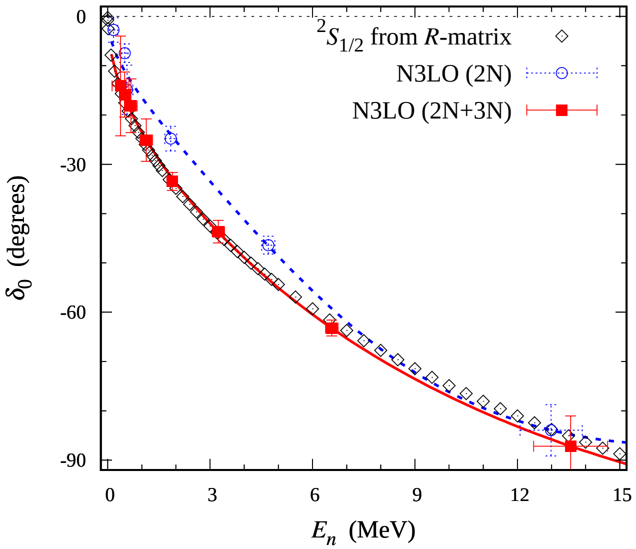
- n-α scattering know to be a good testbed of 3NFs

  Pieper, Pandharipande (1993), Quaglioni, Navratil (2008), Kravvaris et al. (2020), ...

- Use the time-honoured Lüscher approach

- Compare to R-matrix results

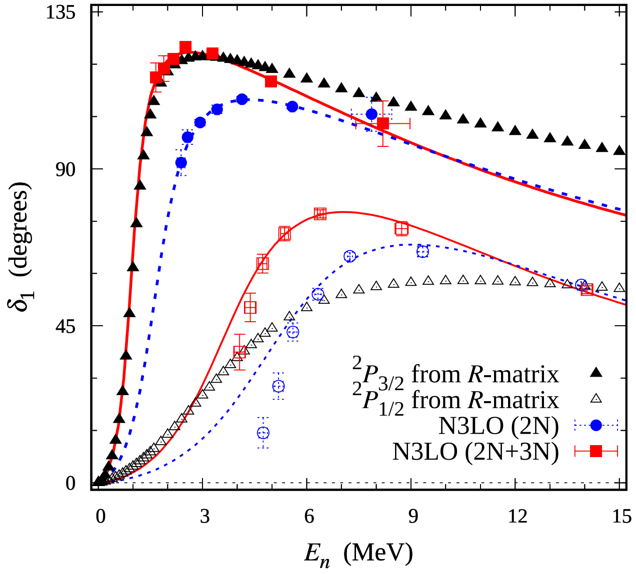
  G. M. Hale, private communication (2023)



$\hookrightarrow$ S-wave ($^2S_{1/2}$) well described when 3NFs are included

$\hookrightarrow$ Large $^2P_{3/2}$ well described when 3NFs are included

$\hookrightarrow$ Visible deviation in the $^2P_{1/2}$ wave for $E_n \geq 3$ MeV



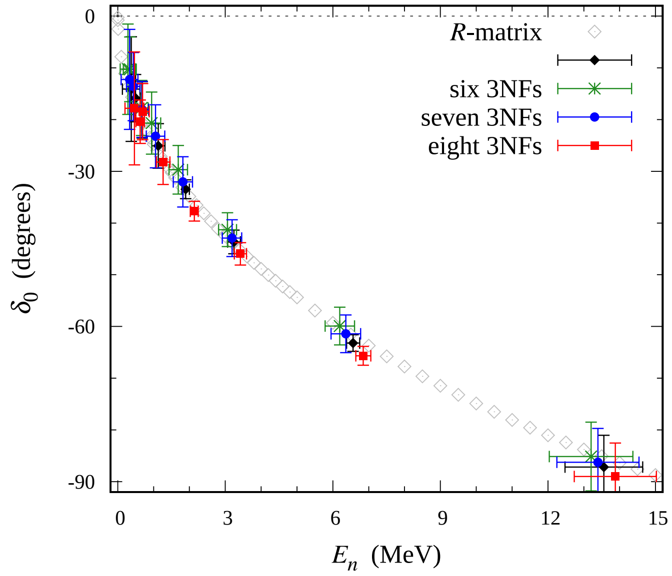
- What is the reason for the deviations in the $^2P_{3/2}$-$^2P_{1/2}$ splitting?

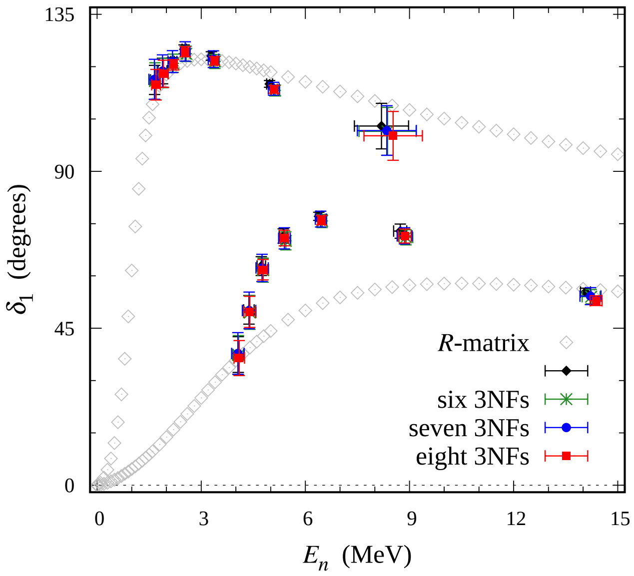
Neutron-alpha scattering revisited

Elhatisari, Hildenbrand, UGM, J. Phys. **G 52** (2025) 125102

- Investigate one possible reason: Choice of nuclei to fix the LECs?

- Perform MCMC searches to explore systematically the set of 8 3NF ops and also reduce to sets with 6 and 7 ops and quantify the uncertainty using RSMD





$\hookrightarrow$ clearly, this is not the source of the problem

$\hookrightarrow$ must improve the 3NFs!

Towards improved 3NFs

A minimal set of 3NFs

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Consider sets of 6 3NF op's (called minimal set)

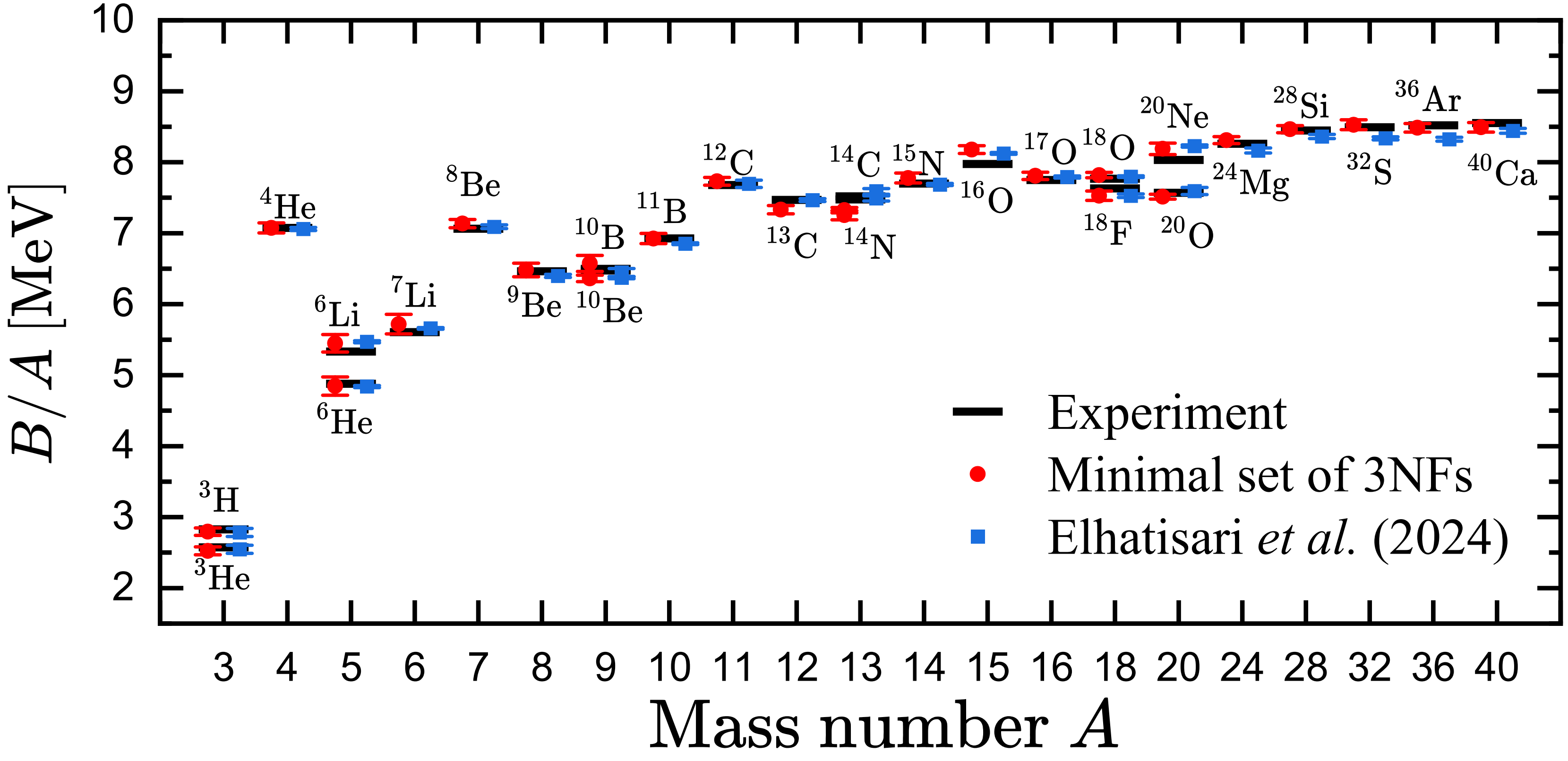
No.	V_{3N}'s
1	$V^{(0)}_{c_{E,s_{NL1}}}$, $V^{(3)}_{c_D}$, $[V^{(0)}_{c_E}]_{044}$, $[V^{(0)}_{c_E}]_{224}$, $[V^{(0)}_{c_E}]_{125}$, $[V^{(0)}_{c_E}]_{145}$
2	$[V^{(1)}_{c_E}]_{000}$, $V^{(3)}_{c_D}$, $[V^{(0)}_{c_E}]_{044}$, $[V^{(0)}_{c_E}]_{222}$, $[V^{(0)}_{c_E}]_{114}$, $[V_{c_{E,s_L}}]_{033}$
3	$V^{(0)}_{c_{E,s_{NL1}}}$, $V^{(3)}_{c_D}$, $[V^{(0)}_{c_E}]_{044}$, $[V^{(0)}_{c_E}]_{224}$, $[V^{(0)}_{c_E}]_{125}$, $[V^{(0)}_{c_E}]_{235}$
4	$V^{(0)}_{c_D}$, $V^{(2)}_{c_D}$, $[V^{(0)}_{c_E}]_{044}$, $[V^{(0)}_{c_E}]_{222}$, $[V^{(0)}_{c_E}]_{224}$, $[V^{(0)}_{c_E}]_{125}$
5	$V^{(1)}_{c_D}$, $V^{(3)}_{c_D}$, $[V^{(0)}_{c_E}]_{033}$, $[V^{(0)}_{c_E}]_{044}$, $[V^{(0)}_{c_E}]_{222}$, $[V^{(0)}_{c_E}]_{125}$
6	$V^{(0)}_{c_{E,s_{NL1}}}$, $V^{(0)}_{c_D}$, $V^{(3)}_{c_D}$, $[V^{(0)}_{c_E}]_{044}$, $[V^{(0)}_{c_E}]_{224}$, $[V^{(0)}_{c_E}]_{125}$
7	$V^{(1)}_{c_D}$, $V^{(3)}_{c_D}$, $V^{(0)}_{c_{D,s_{NL2}}}$, $[V^{(0)}_{c_E}]_{044}$, $[V^{(0)}_{c_E}]_{224}$, $[V^{(0)}_{c_E}]_{125}$
8	$V^{(0)}_{c_D}$, $V^{(2)}_{c_D}$, $[V^{(0)}_{c_E}]_{044}$, $[V^{(0)}_{c_E}]_{224}$, $[V^{(0)}_{c_E}]_{125}$, $[V^{(0)}_{c_E}]_{145}$
9	$[V^{(0)}_{c_E}]_{000}$, $V^{(3)}_{c_D}$, $[V^{(0)}_{c_E}]_{224}$, $[V^{(0)}_{c_E}]_{125}$, $[V_{c_{E,s_L}}]_{044}$, $[V^{(0)}_{c_E}]_{334}$
10	$[V^{(0)}_{c_E}]_{000}$, $V^{(0)}_{c_{E,s_{NL3}}}$, $V^{(1)}_{c_D}$, $V^{(0)}_{c_{D,s_{NL3}}}$, $[V^{(0)}_{c_E}]_{224}$, $[V^{(0)}_{c_E}]_{125}$

- contains also non-local operators (marked in blue)
- only some sets contain the SU(4) symmetric $V^{(l,t)}_{c_E}$ [here: $[V^{(0)}_{c_E}]_{144}$, $[V^{(0)}_{c_E}]_{222}$]

Determination of the 3NF LECs

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Fit to the BE of a selected number of nuclei and also include the ^{4}He charge radius



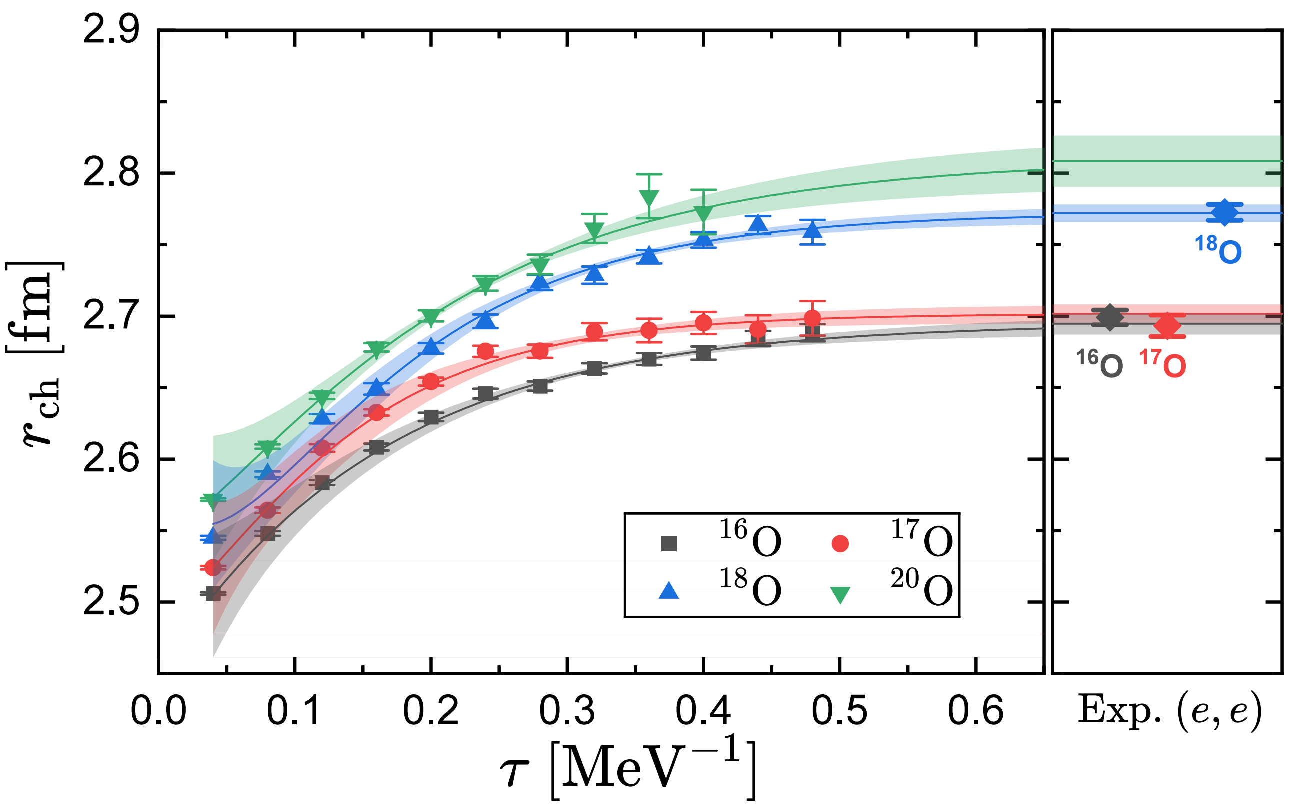
↪ Quality similar to the WFM 3NFs: RMSD = 0.101 MeV (1.4%) [0.093 MeV (1.1%)]

↪ use these now for the calculation of selected oxygen radii

Radii of selected oxygen isotopes

Ren, Elhatisari, UGM, PRL **135** (2025) 152502

- Use the partial pinhole algorithm to study the radii of ^{16,17,18,20}O with $M = 4$



Isotope	NLEFT [fm]	Exp. [fm]
^{16}O	2.704(17)	2.699(5)
^{17}O	2.709(15)	2.693(8)
^{18}O	2.768(17)	2.776(2)
^{20}O	2.810(32)	

- Trend of the data reproduced: $r_{\rm ch}(^{16}\mathrm{O}) \simeq r_{\rm ch}(^{17}\mathrm{O}) < r_{\rm ch}(^{18}\mathrm{O})$
- Prediction for $r_{\rm ch}(^{20}\mathrm{O}) \to$ nice test

Multi-strangeness matter

Multi-strangeness matter: Formalism

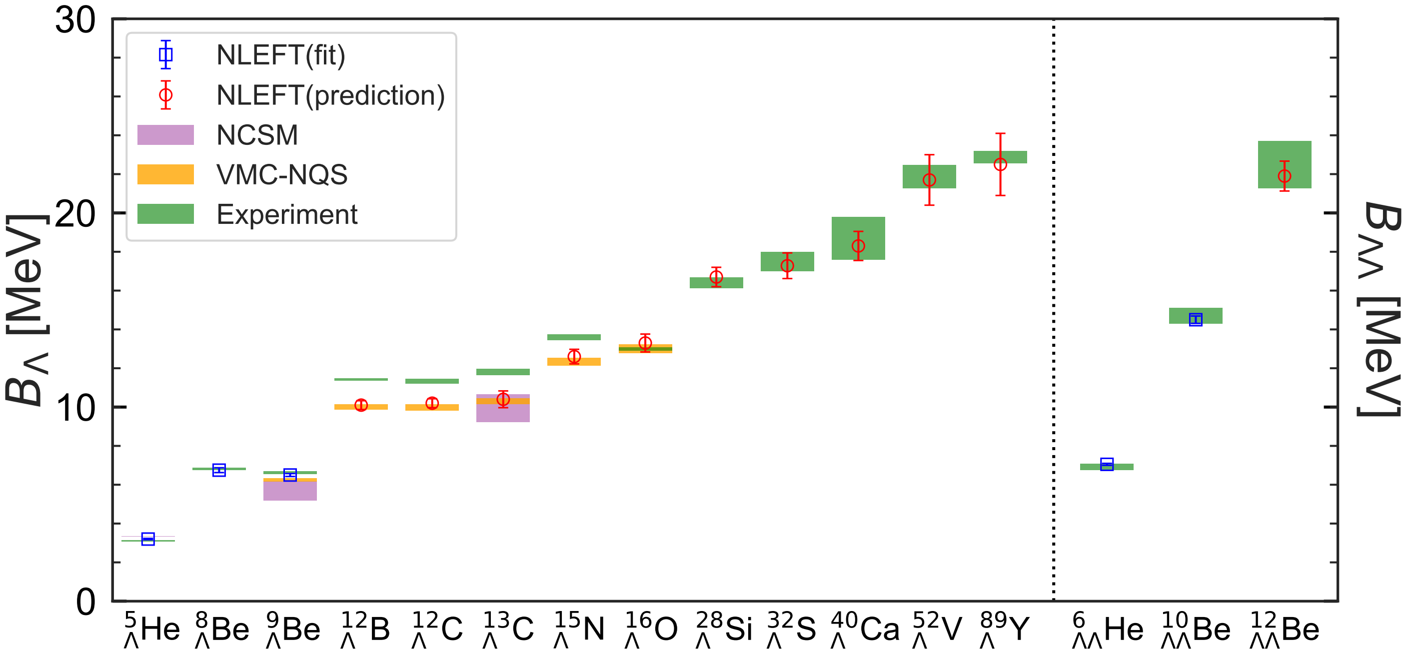
Tong, Elhatisari, UGM, Sci.Bull. **70** (2025) 825; Astrophys.J. **982** (2025) 164; +Ren, 2509.26148

- Neutron stars gained prominence in the multi-messenger area (e.g. the hyperon puzzle)

↪ construct a minimal model that captures the essential features of hypernuclei and neutron stars

- Hamiltonian: $\boldsymbol{H = H_{\rm free} + V_{NN} + V_{N\Lambda} + V_{\Lambda\Lambda} + V_{NNN} + V_{NN\Lambda} + V_{N\Lambda\Lambda}}$

↪ fix LECs from scattering data, symmetric nuclear matter and hypernuclei



NCSM: Knöll, Roth, PLB **846** (2023) 138258; VMC-NQS: DiDonna et al., PRR **8** (2026) 013160

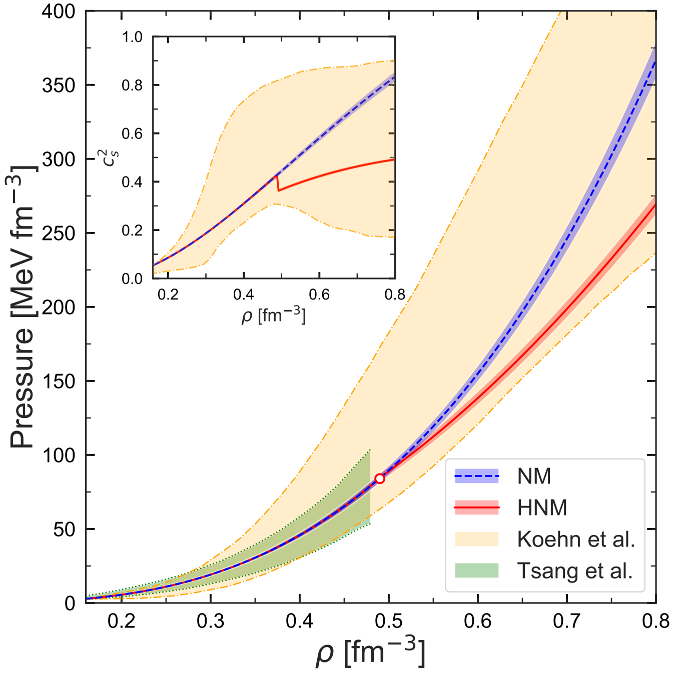
Neutron star properties

Tong, Elhatisari, UGM, Sci.Bull. **70** (2025) 825; Astrophys.J. **982** (2025) 164; + Ren, 2509.26148

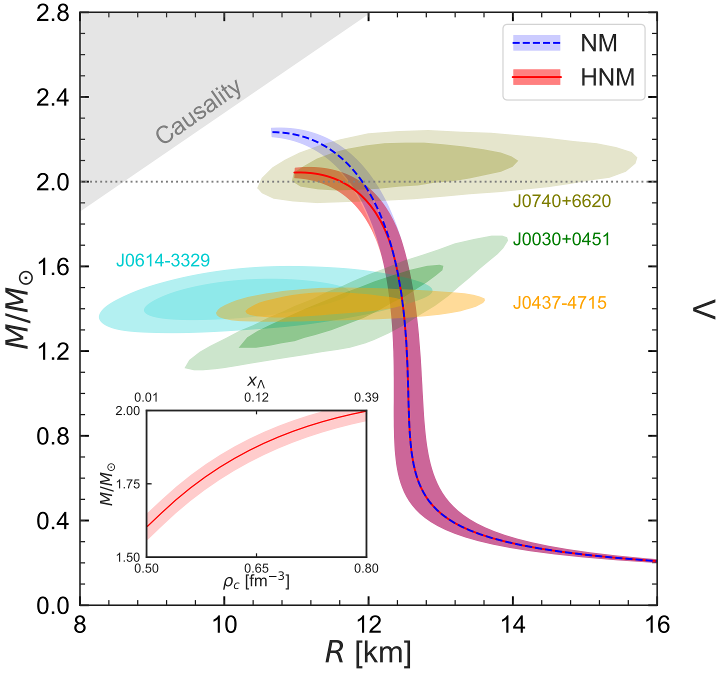
- New AFQMC algorithm allows to consider up to 232 neutrons, 25 protons and 50 Λ's

↪ parameter-free prediction of β-stable matter

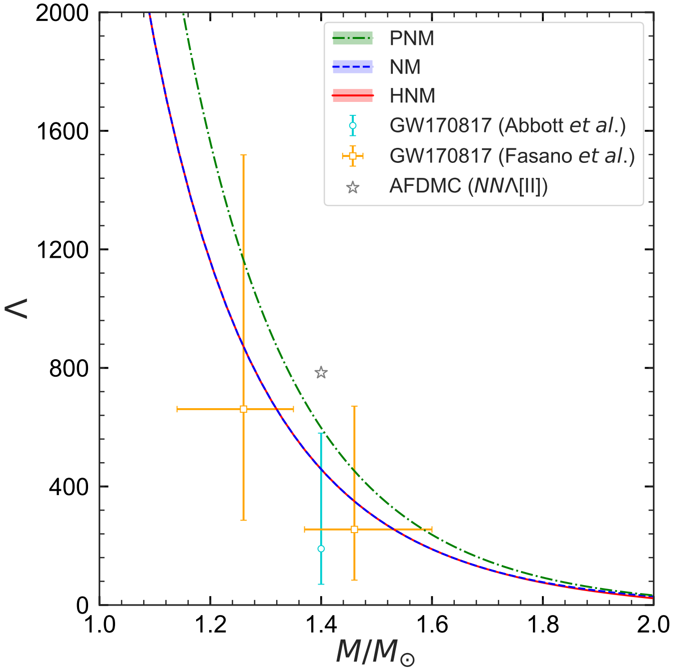
↪ consistent with all experimental and deduced constraints



Koehn et al., PRX **15** (2025) 021014;
Tsang et al., Nature Astron. **8** (2024) 328;



Vinciguerra et al., Astrophys. J. **961** (2024) 62;
Salmi et al., Astrophys. J. **974** (2024) 294;
Choudhury et al., Astrophys. J. Lett. **971** (2024) L20
Mauviard et al., Astrophys. J. **995** (2025) 60



Abbott et al., PRL **121** (2018) 161101
Fasano et al., PRL **123** (2019) 141101

Neutron star properties II

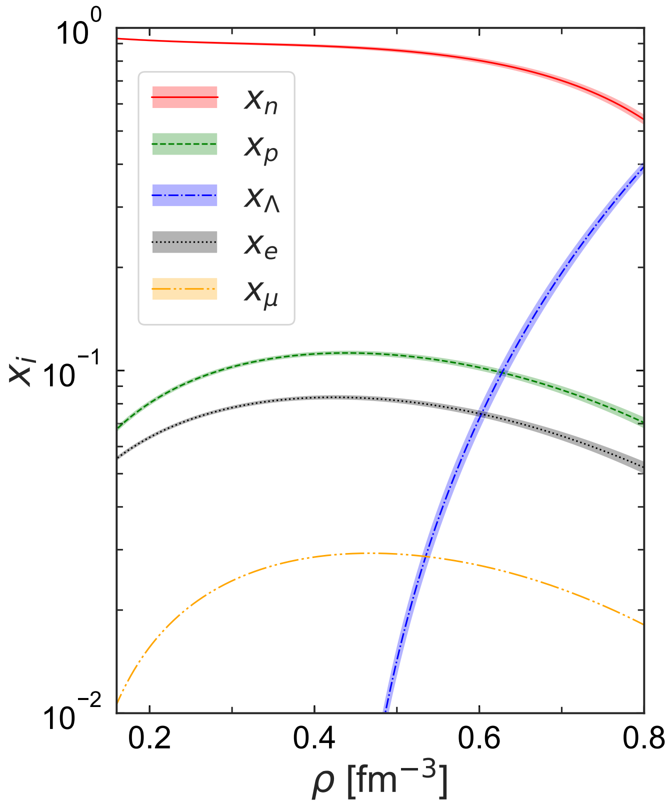
Tong, Elhatisari, UGM, Sci.Bull. **70** (2025) 825; Astrophys.J. **982** (2025) 164; + Ren, 2509.26148

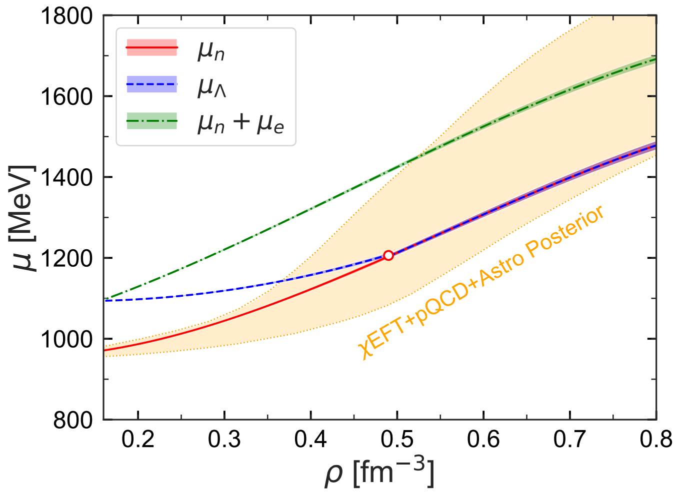
- $\boldsymbol{\Lambda}$'s appear at $\rho_\Lambda^{\rm th} \simeq \mathbf{0.490(0.002)(0.005)}$ fm^{-3}

↪ so hyperons can exist peacefully inside neutron stars

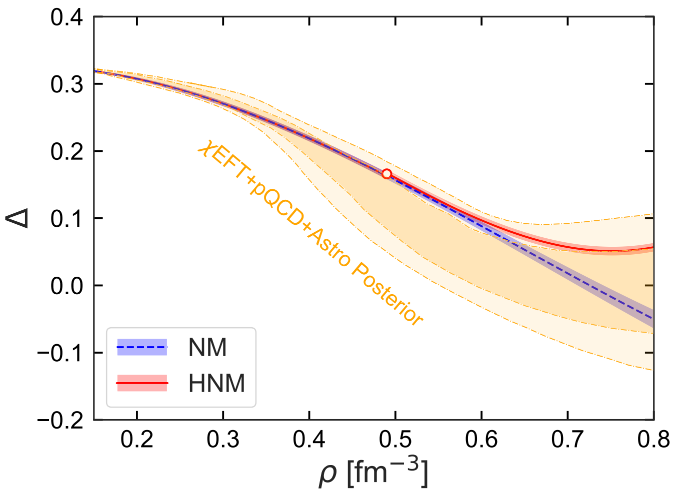
↪ proton fraction barely exceeds 10%

- Chemical potential $\boldsymbol{\mu_n}$ and trace anomaly $\boldsymbol{\Delta}$ also consistent





Brandes, Weise, PRD **111** (2025) 034005 (2025);



Annala et al. Nature Commun. **14** (2023) 8451

⇒ Very promisng results, now extend interactions to high-fidelity forces

Summary & outlook

- Nuclear lattice simulations: a new quantum many-body approach
  - → based on the successful continuum nuclear chiral EFT
  - → a number of highly visible results already obtained

- Recent developments
  - → NN(N) interaction at N3LO w/ wave function matching
    - ↪ first promising results for nuclear structure, matter and scattering
    - ↪ first results for β-decays [ulitimately $\mathbf{0\nu2\beta}$ decays]

Elhatisari, Hildenbrand, UGM, Phys. Lett. B **859** (2024) 139086 + PKU group

    - ↪ hypernuclei at N3LO are under investigation

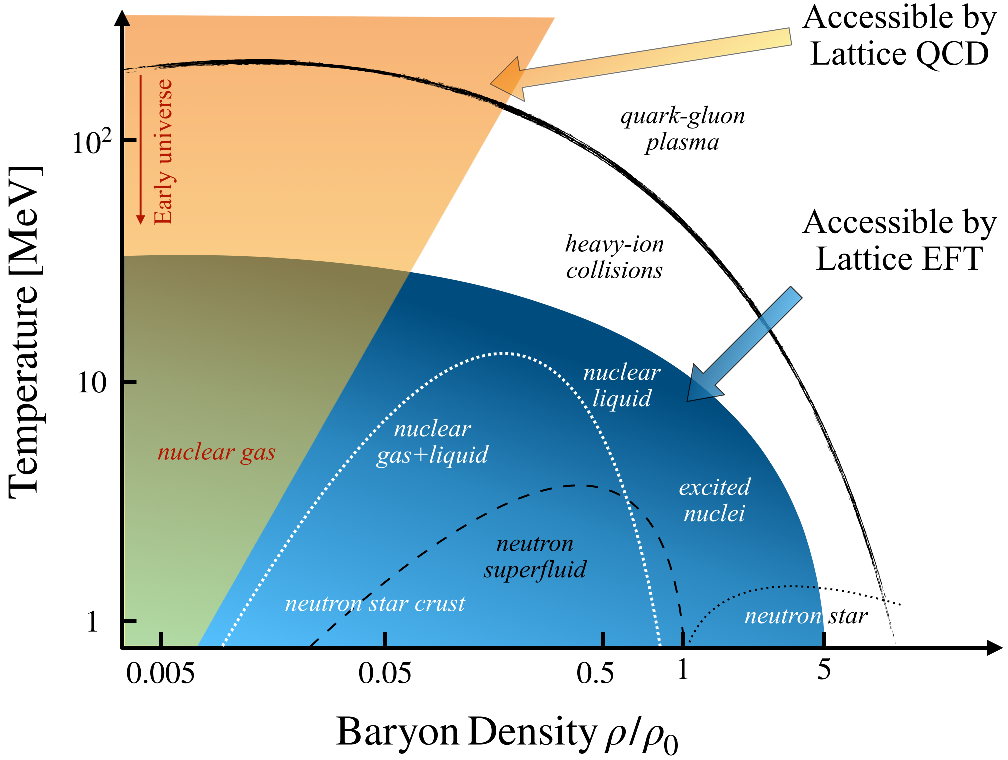
Hildenbrand et al., Eur. Phys. J. A **60** (2024) 215

- Improved 3NFs are being worked out ↪ stay tuned!

SPARES

Comparison to lattice QCD

LQCD (quarks & gluons)	NLEFT (nucleons & pions)
relativistic fermions	non-relativistic fermions
renormalizable th'y	EFT
continuum limit	no continuum limit
(un)physical masses	physical masses
Coulomb - difficult	Coulomb - easy
high T/small ρ	small T/nuclear densities
sign problem severe	sign problem moderate



- For nuclear physics, NLEFT is the far better methodology!

Transfer matrix method

- Correlation–function for A nucleons: $\boldsymbol{Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle}$

  Euclidean time

  with $\boldsymbol{\Psi_A}$ a Slater determinant for A free nucleons
  [or a more sophisticated (correlated) initial/final state]

- Transient energy

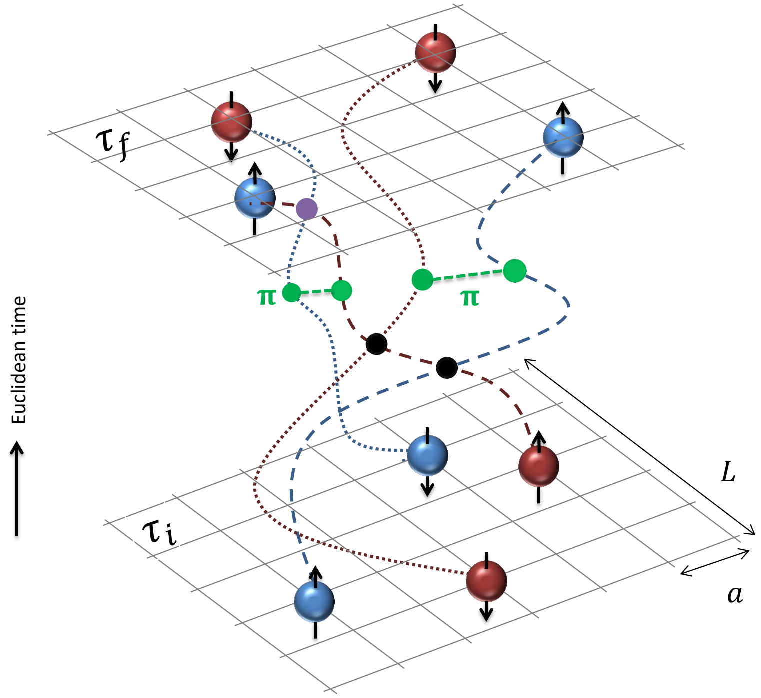
$$\boldsymbol{E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)}$$

→ ground state: $\boldsymbol{E_A^0 = \lim_{\tau\to\infty} E_A(\tau)}$

- Exp. value of any normal–ordered operator $\mathcal{O}$

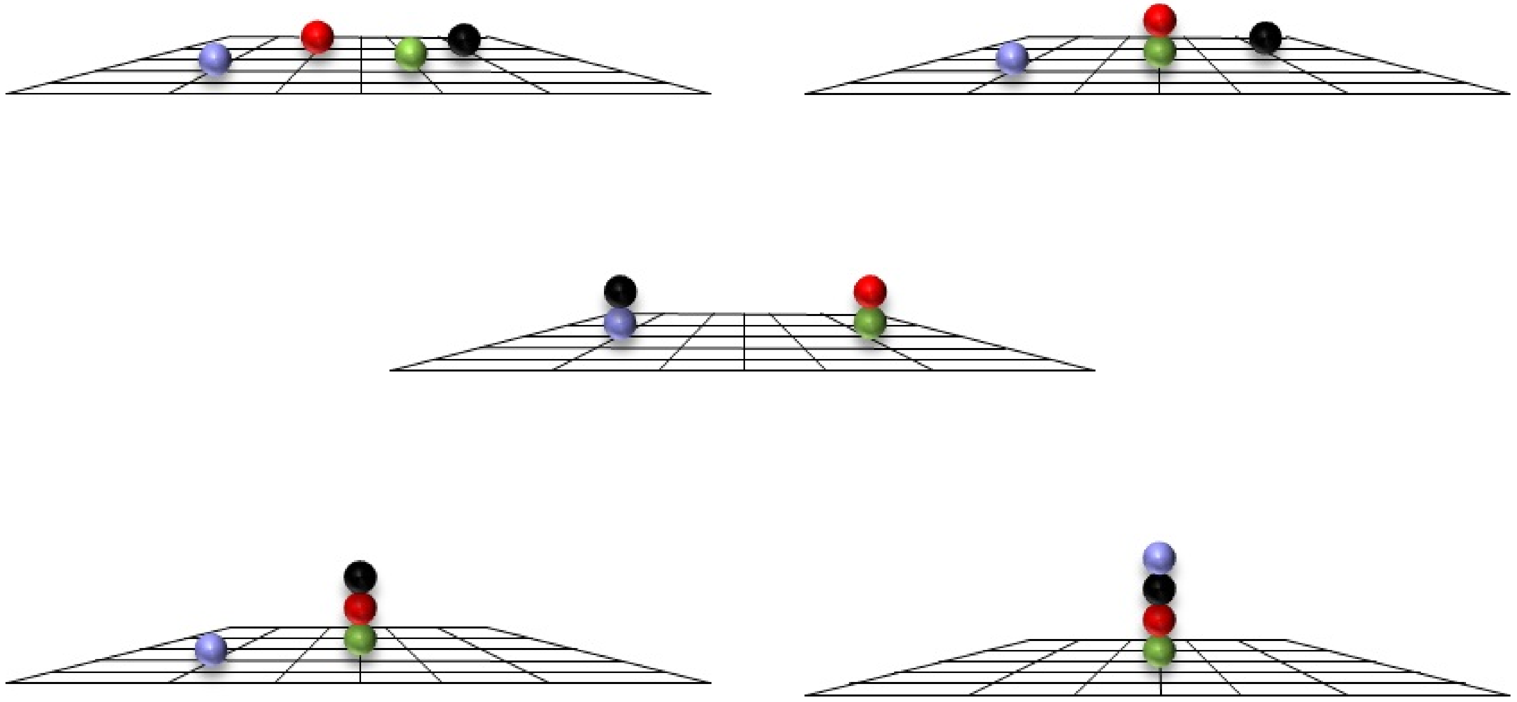
$$\boldsymbol{Z_A^{\mathcal{O}} = \langle \Psi_A | \exp(-\tau H/2)\, \mathcal{O}\, \exp(-\tau H/2)\, | \Psi_A \rangle}$$

$$\boldsymbol{\lim_{\tau\to\infty} \frac{Z_A^{\mathcal{O}}(\tau)}{Z_A(\tau)} = \langle \Psi_A | \mathcal{O} \, | \Psi_A \rangle}$$



- Excited states: $\boldsymbol{Z_A(\tau) \to Z_A^{ij}(\tau)}$, diagonalize, e.g. $\boldsymbol{0_1^+, 0_2^+, 0_3^+, \ldots}$ in ^{12}C

Configurations



⇒ all *possible* configurations are sampled

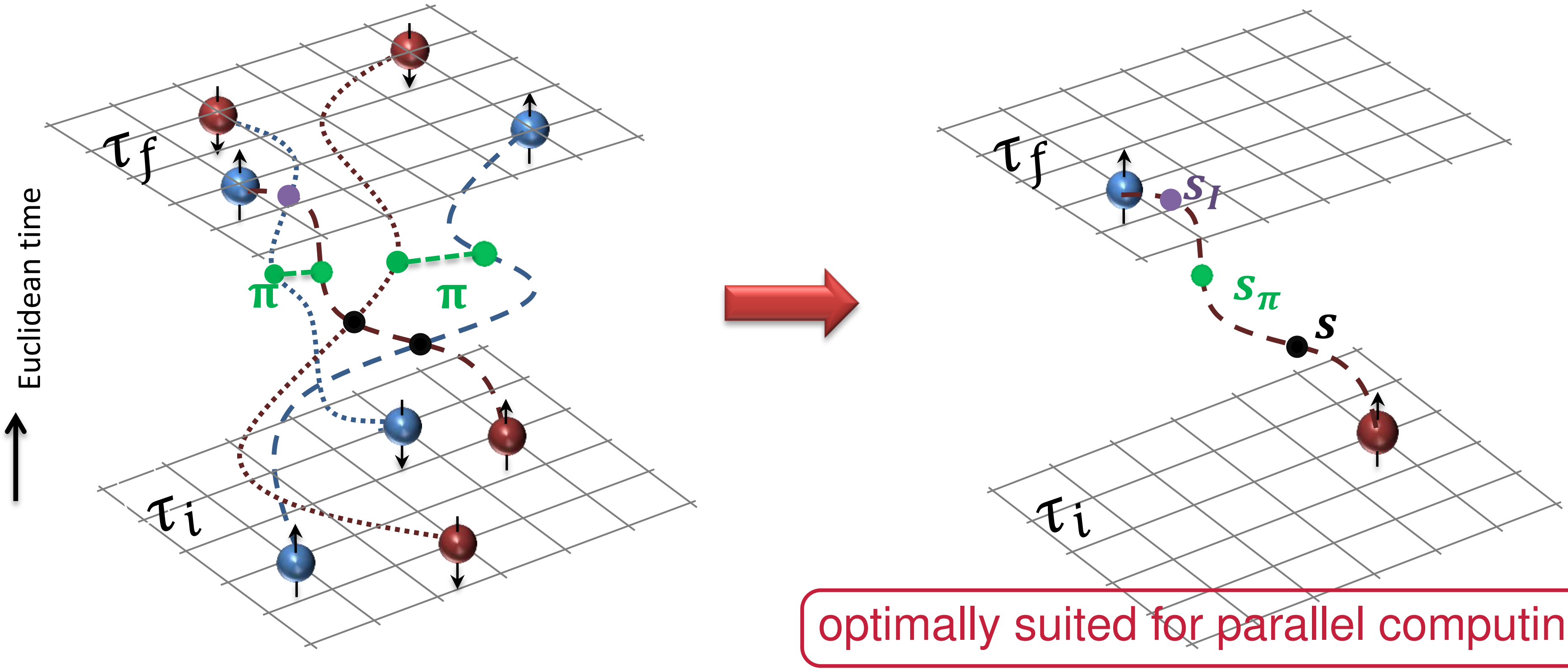
⇒ preparation of *all possible* initial/final states

⇒ *clustering* emerges *naturally*

Auxiliary field method

- Represent interactions by auxiliary fields (Gaussian completion):

$$\boxed{\exp\left[-\frac{C}{2}\left(N^\dagger N\right)^2\right] = \sqrt{\frac{1}{2\pi}}\int ds \exp\left[-\frac{s^2}{2} + \sqrt{C}\, s\left(N^\dagger N\right)\right]}$$



optimally suited for parallel computing!

Computational equipment

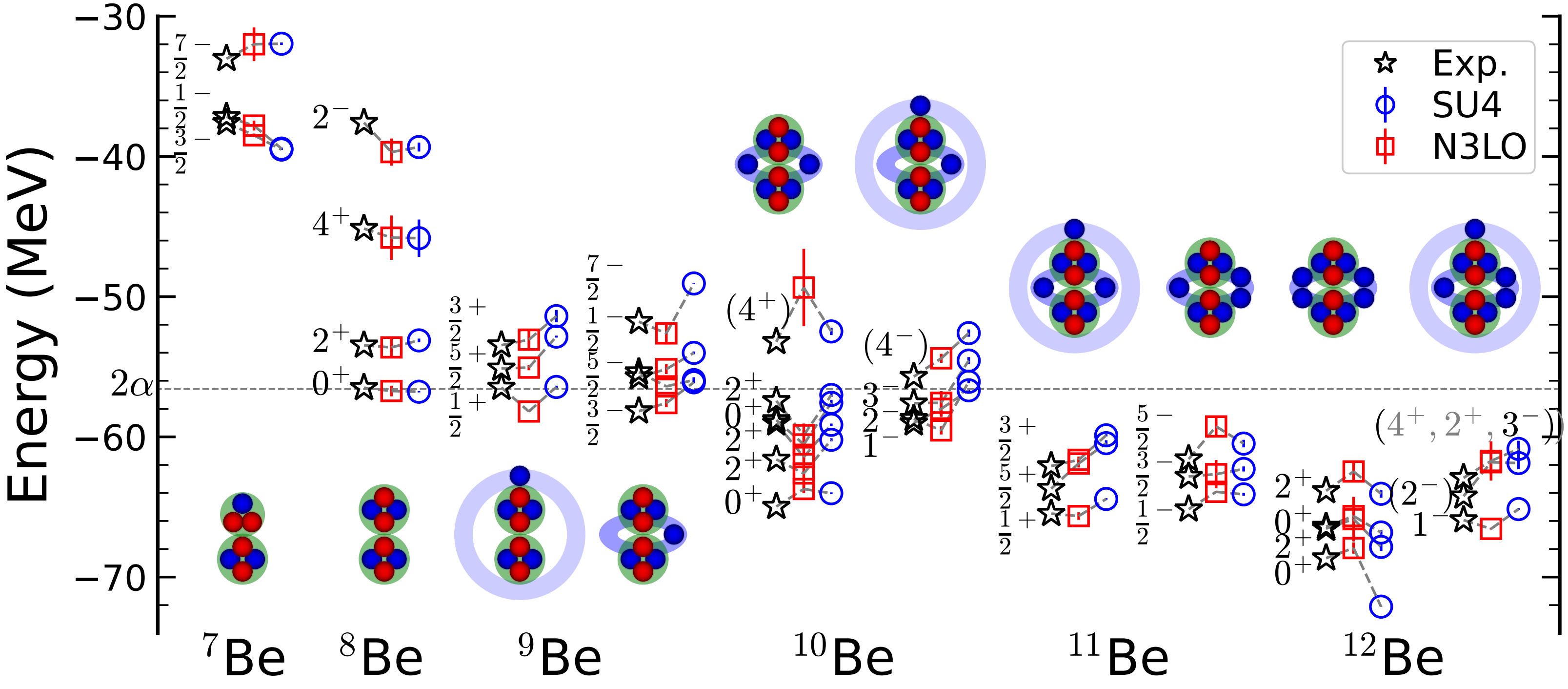
- Present = JUPITER + FRONTIER + ...

1 Exaflop

Prediction: Be isotopes

Shen et al., Phys. Rev. Lett. **134** (2025) 162503

- Systematic study of the Be isotopes & their radii & their em transitions:



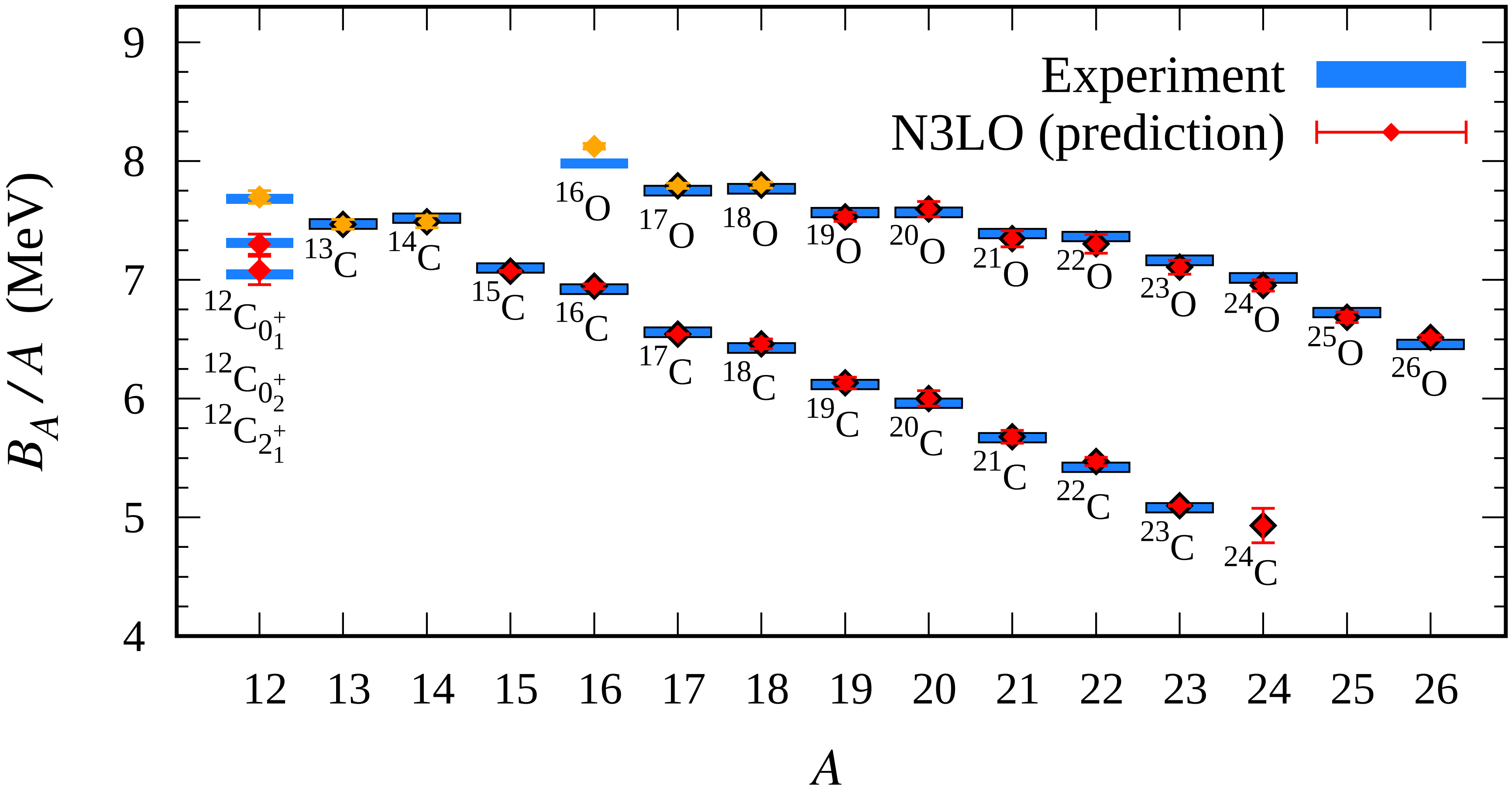
↪ new method to quantify nuclear shapes

↪ clusters, halos, molecular orbitals in **one shot**

Prediction: Isotope chains of carbon & oxyen

Song et al., Phys. Lett. **B 872** (2026) 140086

- Towards the neutron drip-line in carbon and oxygen:



↪ 3NFs of utmost importance for the n-rich isotopes!

↪ universal features of neutron correlations