

# Light hypernuclei based on chiral BB and 3B forces



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- Motivation
- YN interactions
- J-NCSM & SRG evolution of (hyper-)nuclear interactions
- Uncertainty of  $\Lambda$  separation energies & chiral YNN interactions
- Chiral YNN forces
- Application of YNN forces to light hypernuclei
- Conclusions & Outlook

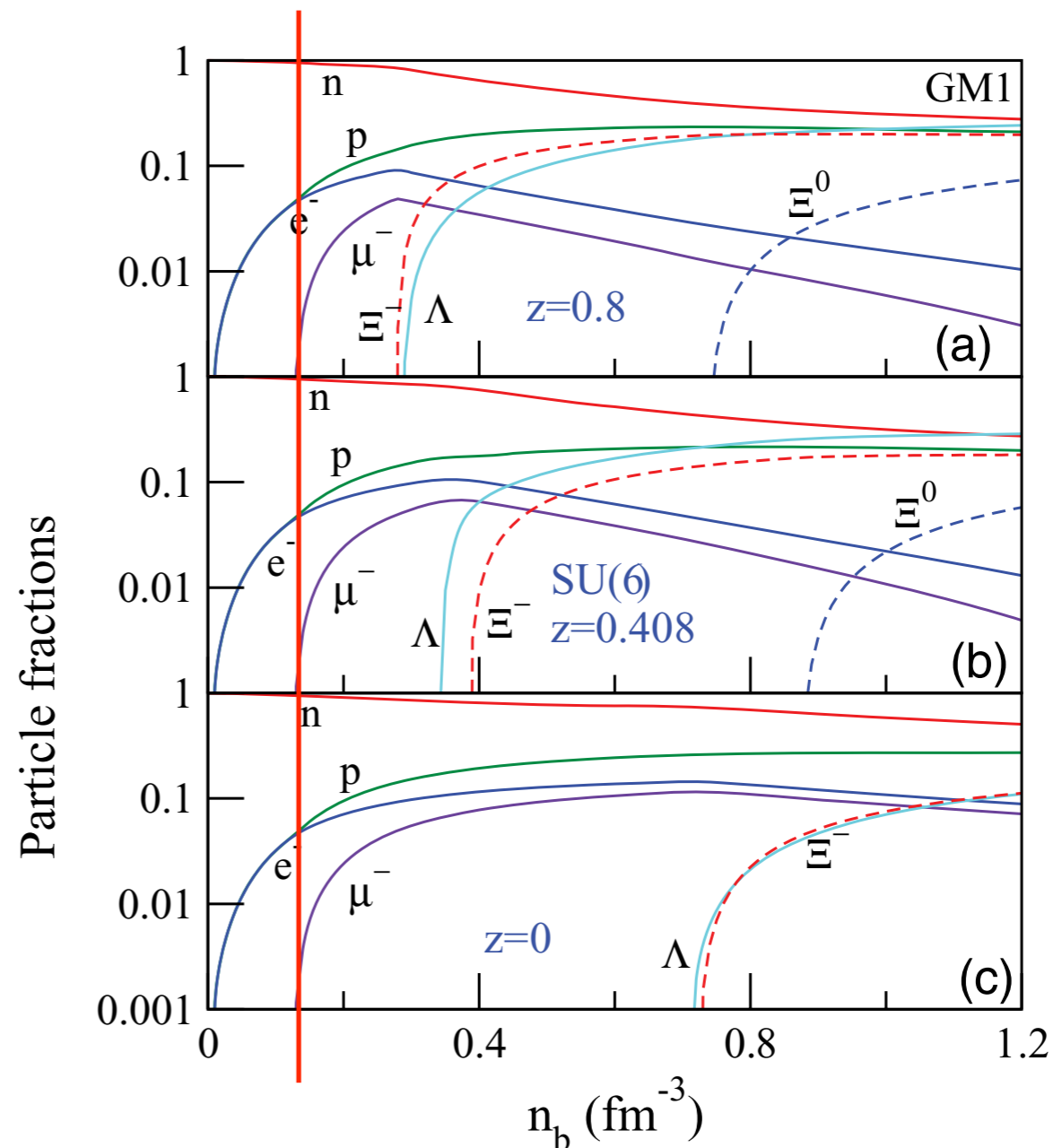
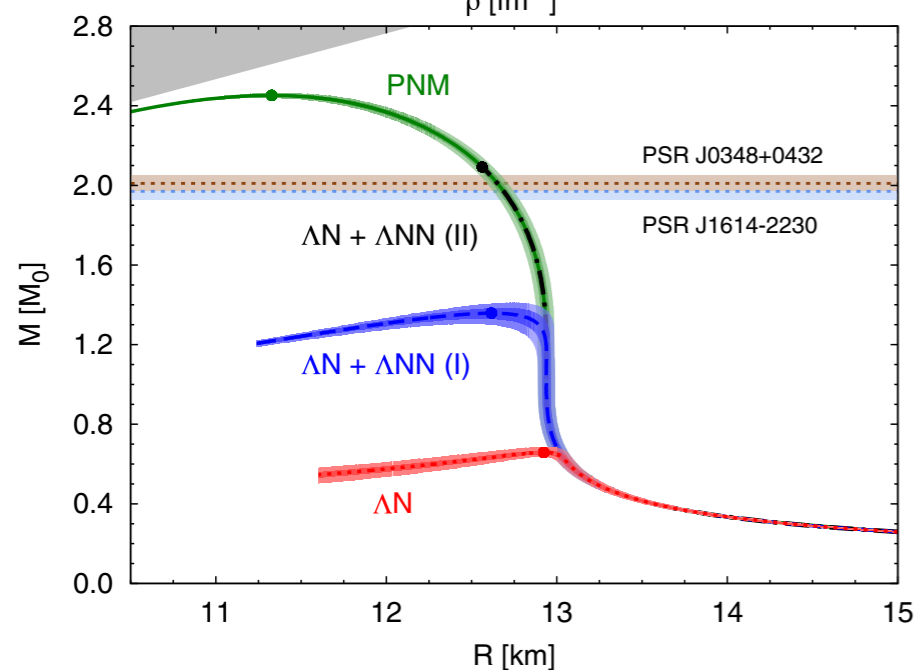
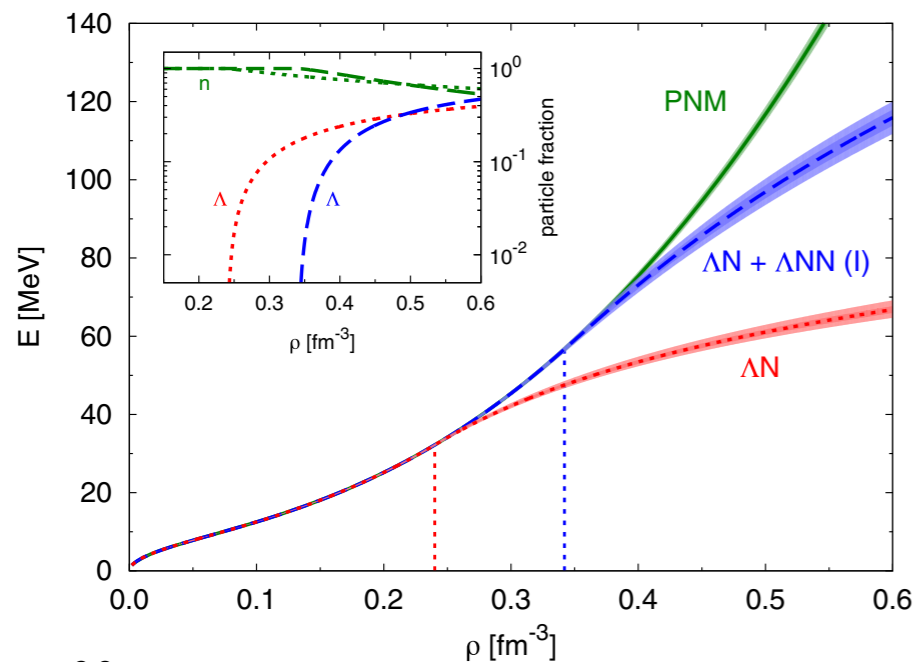
in collaboration with Johann Haidenbauer, Hoai Le, Ulf Meißner

# Hypernuclear interactions



## Why is understanding hypernuclear interactions interesting?

- *hyperon contribution to the EOS, neutron stars, supernovae*
- *"hyperon puzzle"*
- *$\Lambda$  as probe to nuclear structure*
- *flavor dependence of baryon-baryon interactions*

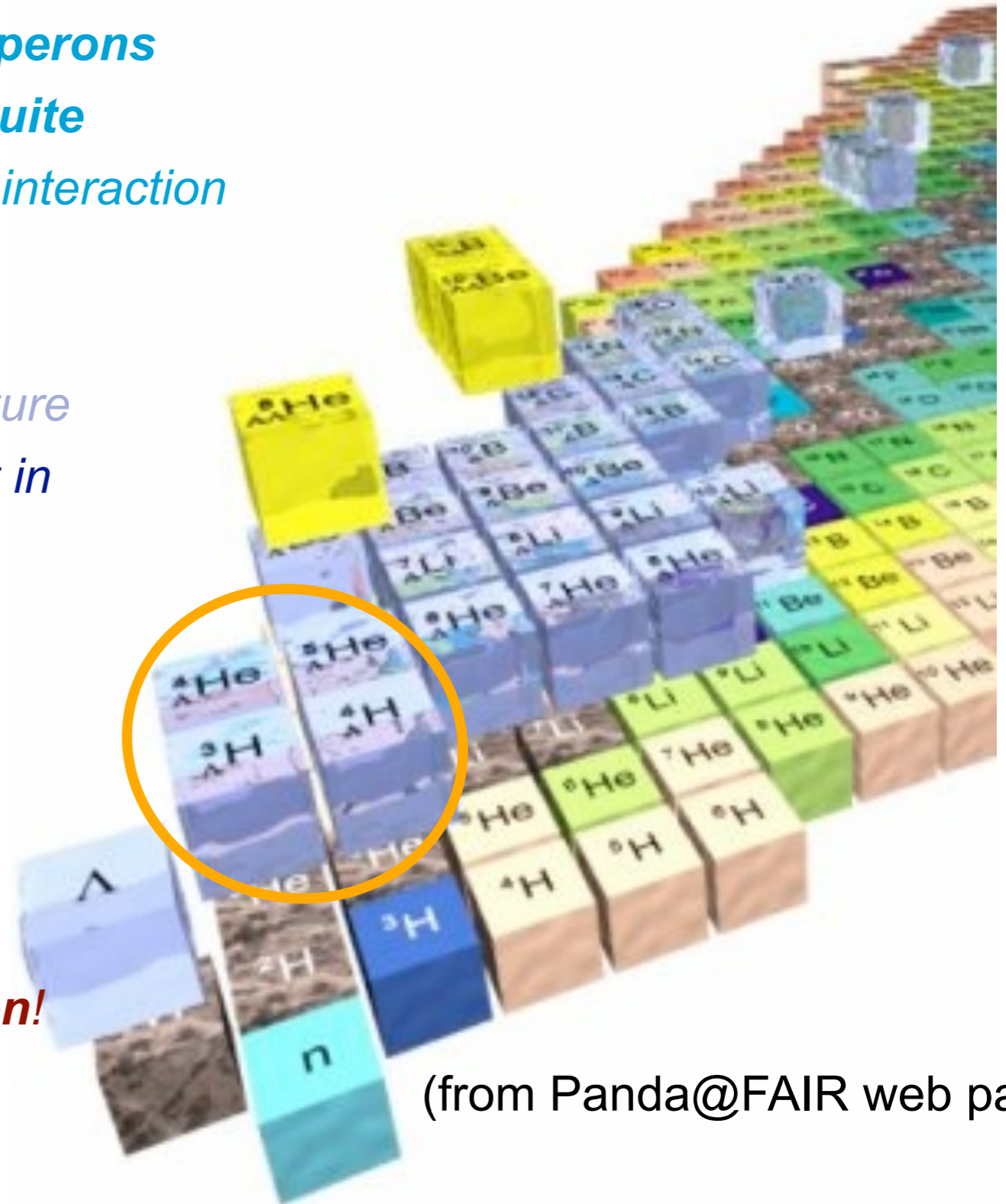


# Hypernuclei



Only few YN data. Hypernuclear data provides additional constraints.

- $\Lambda N$  interactions are generally weaker than the NN interaction
  - naively: **core nucleus + hyperons**
  - „separation energies“ are quite independent from NN(+3N) interaction
- no Pauli blocking of  $\Lambda$  in nuclei
  - good to study nuclear structure
  - even light hypernuclei exist in **several spin states**
- **non-trivial constraints** on the YN interaction even from lightest ones
- size of **YNN** interactions?  
**need to include  $\Lambda$ - $\Sigma$  conversion!**



(from Panda@FAIR web page)

# Chiral NN & YN interactions



## EFT based approaches

### Chiral EFT implements **chiral symmetry of QCD**

- symmetries constrain exchanges of Goldstone bosons
- relations of two- and three- and more-baryon interactions
- breakdown scale  $\approx 600 - 700 \text{ MeV}$
- Semi-local momentum regularization (SMS) up to N<sup>2</sup>LO (for YN)

	BB force	3B force	4B force	
LO		—	—	<b>5 NN/YN</b> short range parameters
NLO		—	—	<b>23 NN/YN</b> short range parameters
N <sup>2</sup> LO			—	no additional contact terms in NN/YN

(adapted from Epelbaum, 2008)

Retain flexibility to adjust to data due to counter terms

**Regulator required** — cutoff/different orders often used to estimate uncertainty

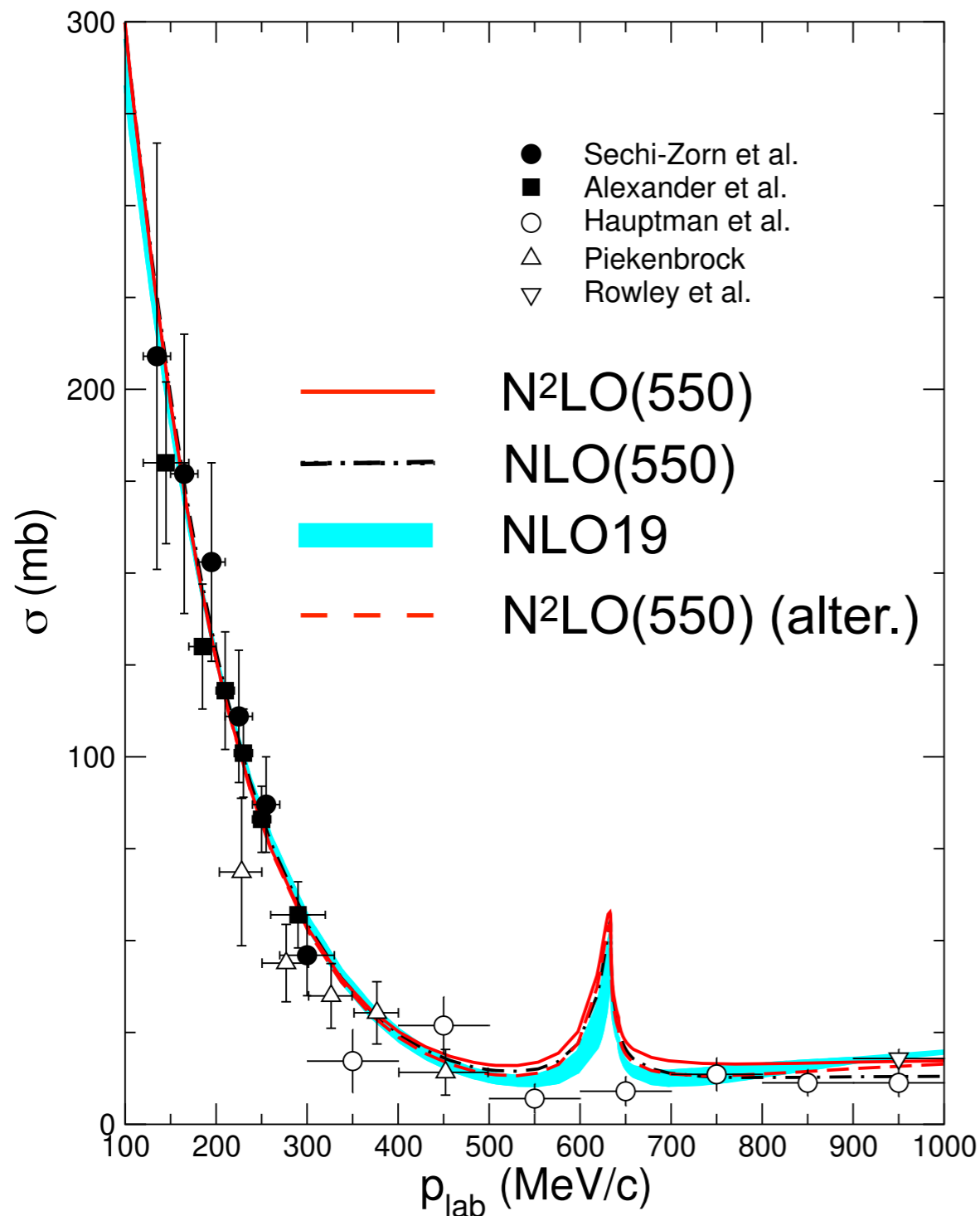
$\Lambda - \Sigma$  **conversion** is explicitly included (3BFs starting from N<sup>2</sup>LO)

# SMS NLO/N<sup>2</sup>LO interaction



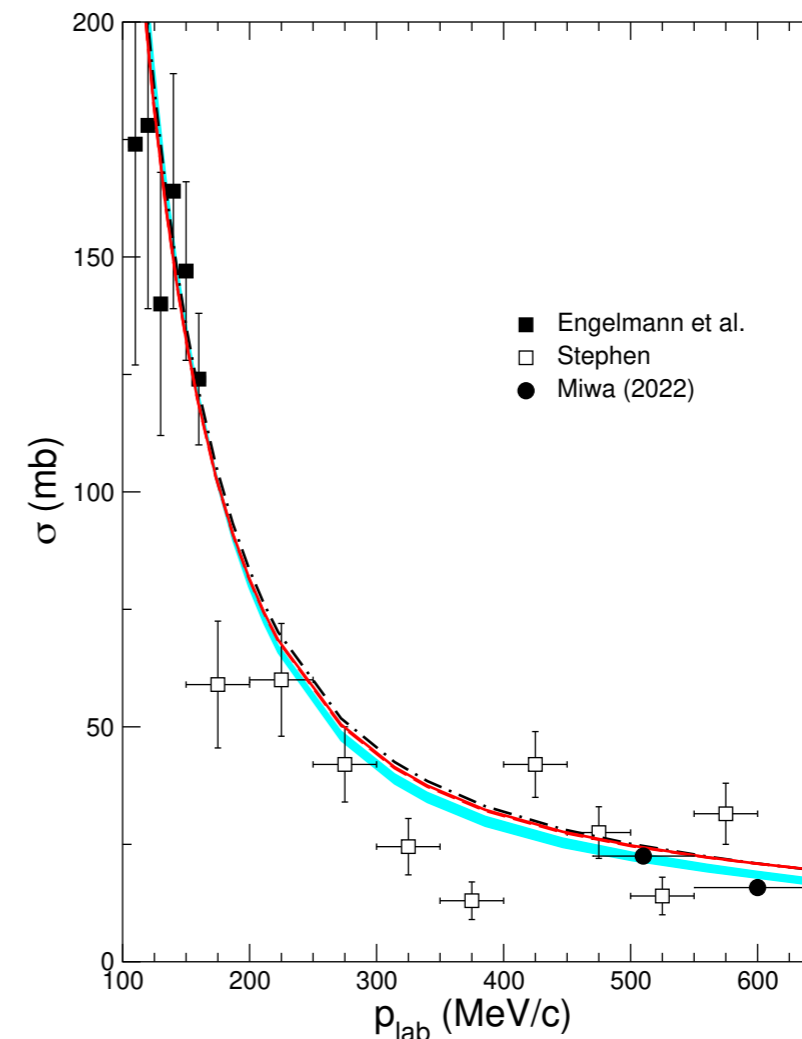
Selected results (show  $\Lambda = 550$  MeV, others are very similar in quality)

$\Lambda p \rightarrow \Lambda p$



- most relevant cross sections very similar in NLO and N<sup>2</sup>LO
- similar to NLO19
- alternative fit (see later)

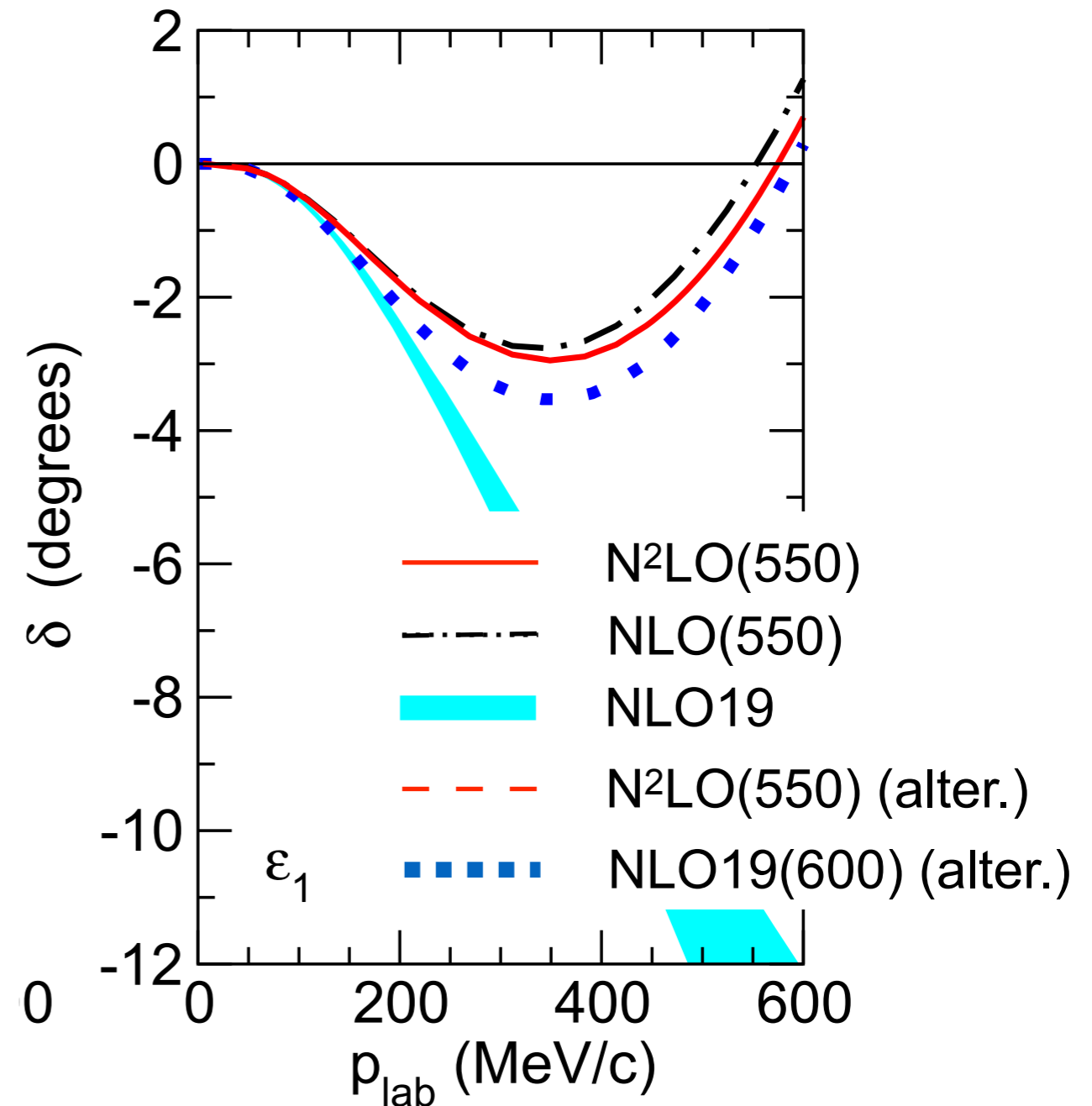
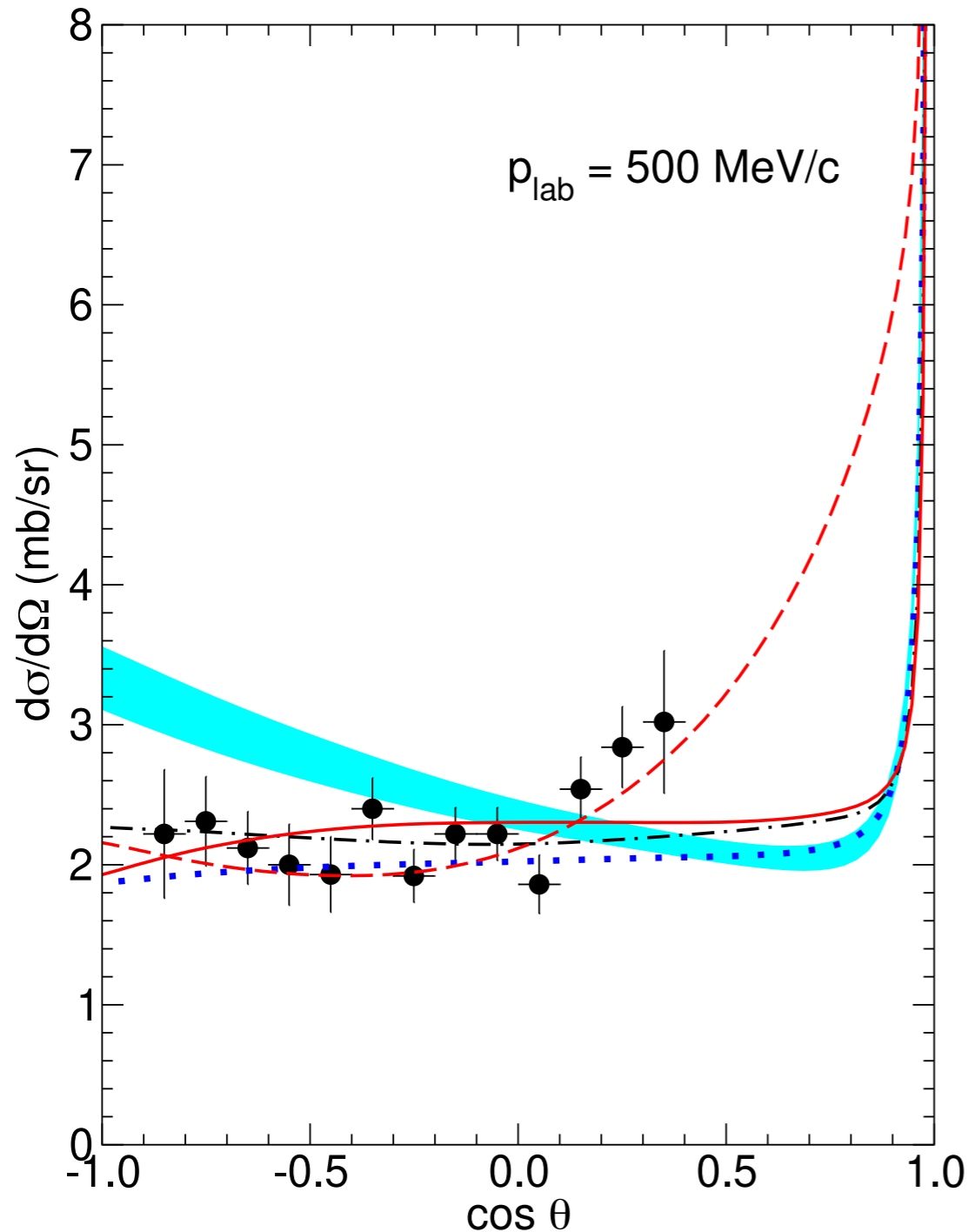
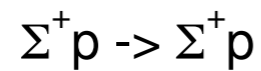
$\Sigma^- p \rightarrow \Lambda n$



# SMS NLO/N<sup>2</sup>LO interaction



new data (Miwa(2022)) at higher energies provides new constraints!





**Similarity renormalization group** is by now a **standard tool** to obtain soft effective interactions for various many-body approaches (NCSM, coupled-cluster, MBPT, ...)

Idea: perform a unitary transformation of the NN (and YN interaction) using a cleverly defined "generator"

(Bogner et al. PRC 75,061001 (2007))

$$\frac{dH_s}{ds} = \left[ \underbrace{[T, H(s)]}_{\equiv \eta(s)}, H(s) \right] \quad H(s) = T + V(s)$$

this choice of generator drives  $V(s)$  into a diagonal form in momentum space

- $V(s)$  will be **phase equivalent** to original interaction
- short range  $V(s)$  will change towards **softer interactions**
- Evolution can be restricted to **2-,3-, ... body level** (approximation)
- $\lambda = \left( \frac{4\mu_{BN}^2}{s} \right)^{1/4}$  is a measure of the width of the interaction in momentum space
- **dependence** of results on  $\lambda$  or  $s$  is a measure for **missing terms**

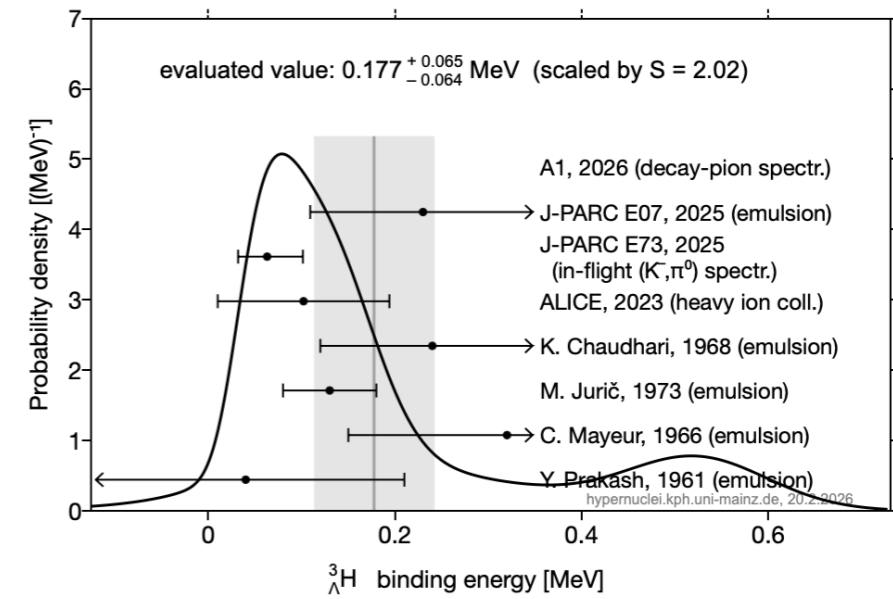
# Induced 3BF & ${}^3_{\Lambda}\text{H}$

${}^3_{\Lambda}\text{H}$  experiments are ... confusing

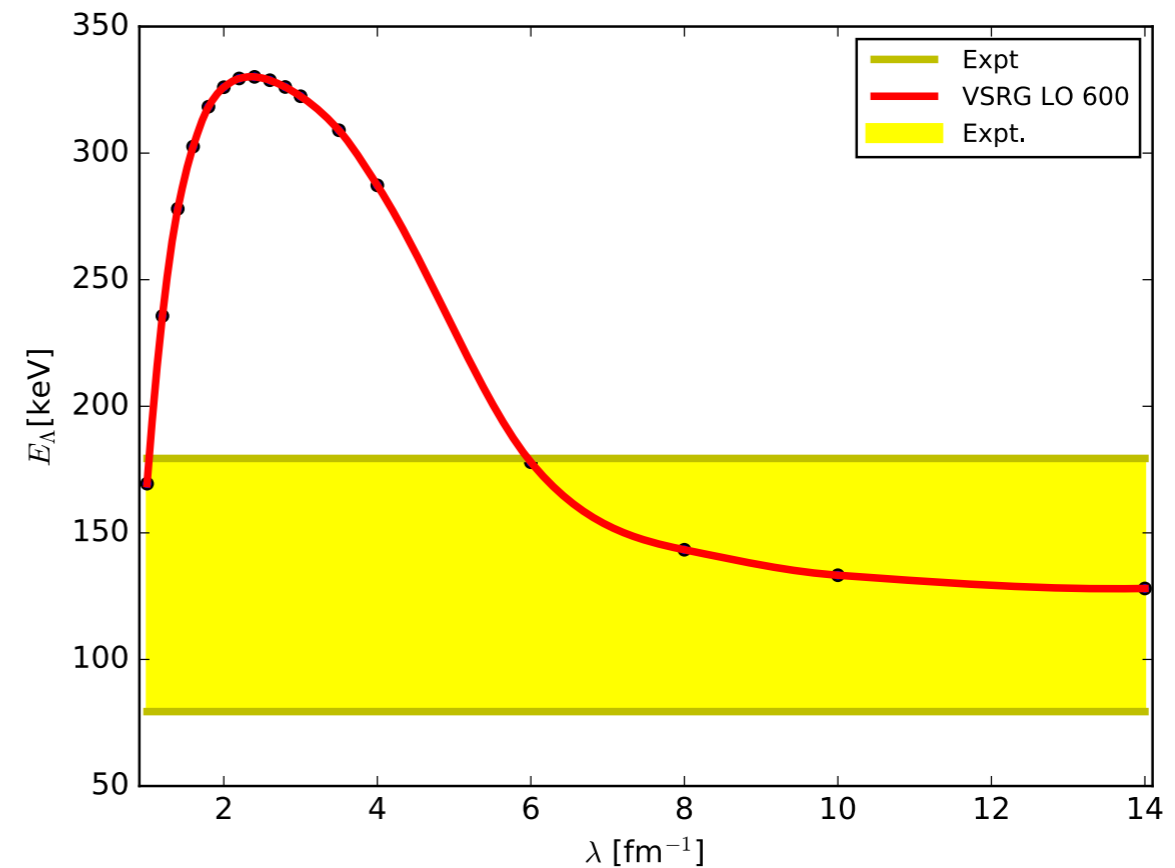
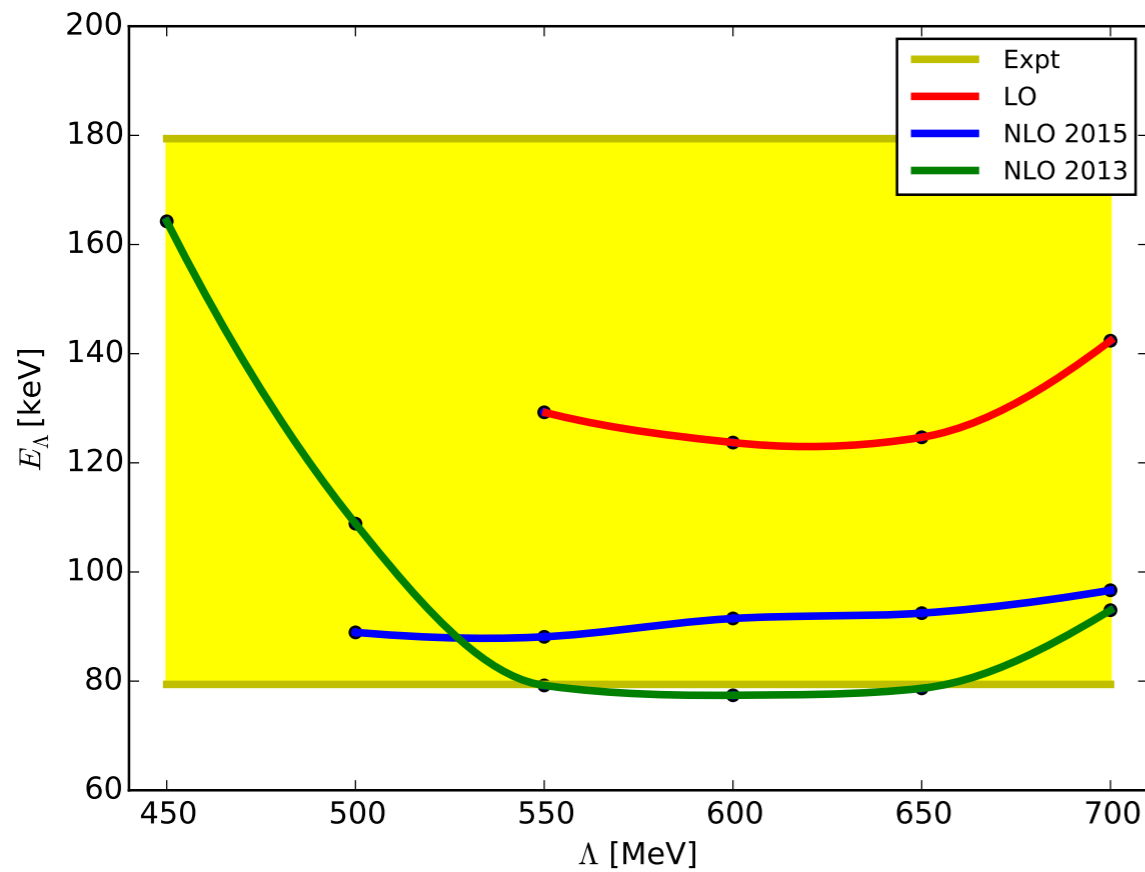
${}^3_{\Lambda}\text{H}$  used to determine spin dependence of YN

and: SRG parameter dependence is significant when NN and YN interactions are evolved

→ induced YNN interactions are important



<https://hypernuclei.kph.uni-mainz.de>

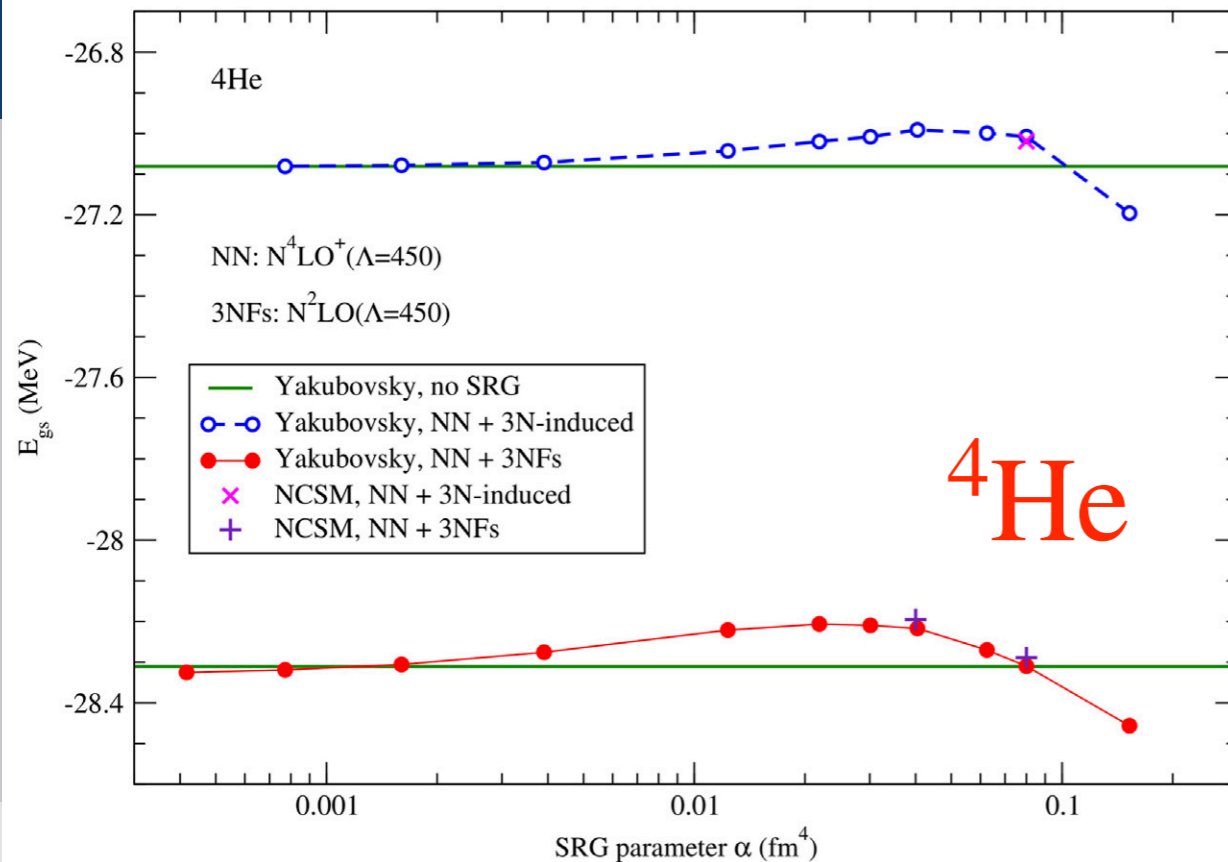


Hypertriton energy used to determine relative strength of singlet and triplet YN force

# SRG dependence of results

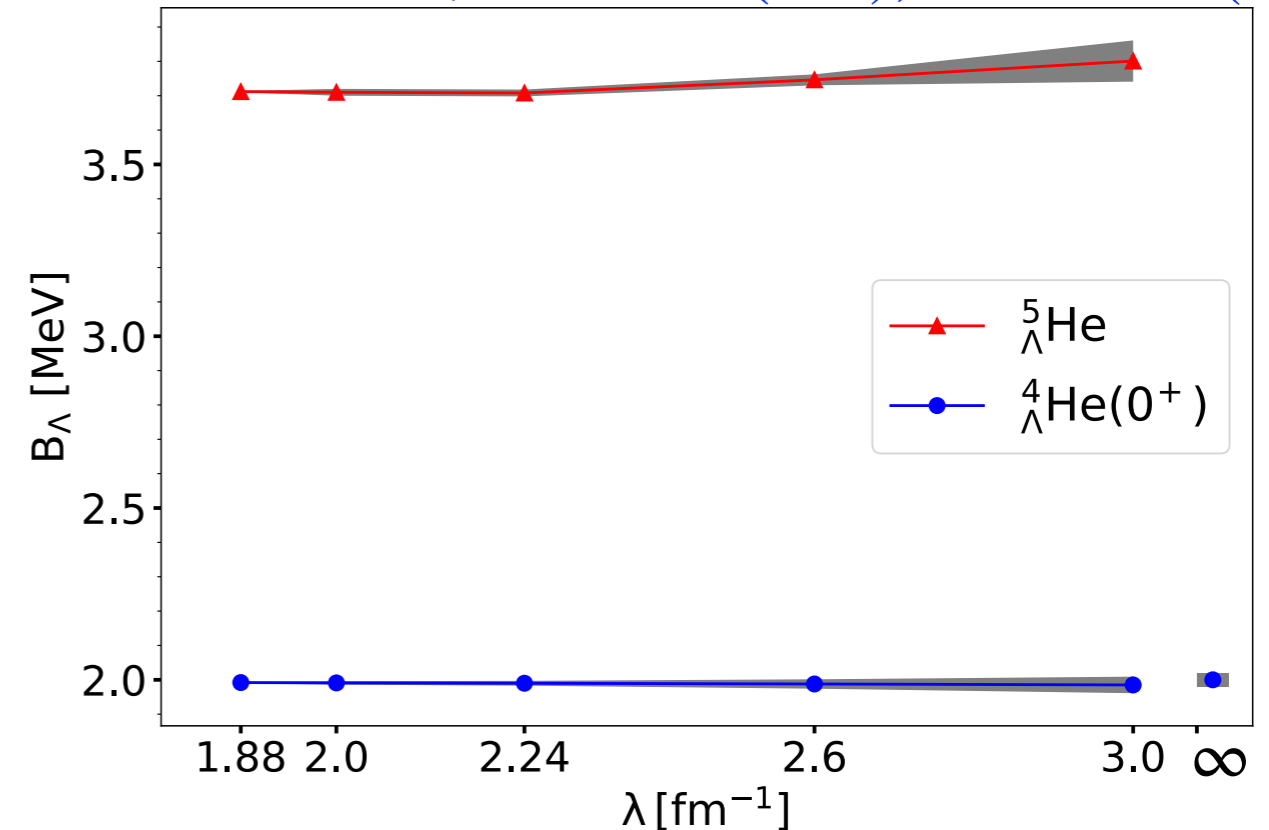


- SRG-induced 3N and YNN interactions
- ${}^4\text{He}$  binding energies varies by  $\approx 100 - 200$  keV (relevant in the future?)
- separation energies are even less dependent (YNNN forces small)



(Maris, Le, Nogga, Roth, Vary (2023))

NN:  $N^4\text{LO}^+$ , 3N:  $N^2\text{LO}(450)$ ; YN:  $N^2\text{LO}(550)$



(Le (2023))

For **hypernuclei**, calculations based on SRG induced BB and 3B interactions are sufficiently accurate!

# Uncertainty analysis to $A = 3$ to 5



Order N<sup>2</sup>LO requires combination of chiral NN, YN, 3N and **YNN** interaction

Results for **different orders** enable uncertainty estimate:

Ansatz for the order by order convergence:

$$X_K = X_{ref} \sum_{k=0}^K c_k Q^k \quad \text{where} \quad Q = M_{\pi}^{eff} / \Lambda_b \quad (X_{ref} \text{ LO, exp., max, ...})$$

**Bayesian analysis** of the uncertainty following Melendez et al. 2017,2019

**Extracting**  $c_k$  for  $k \leq K$  from calculations

→ **probability distributions** for  $c_k$

→ 
$$\delta X_K = X_{ref} \sum_{k=K+1}^{\infty} c_k Q^k$$

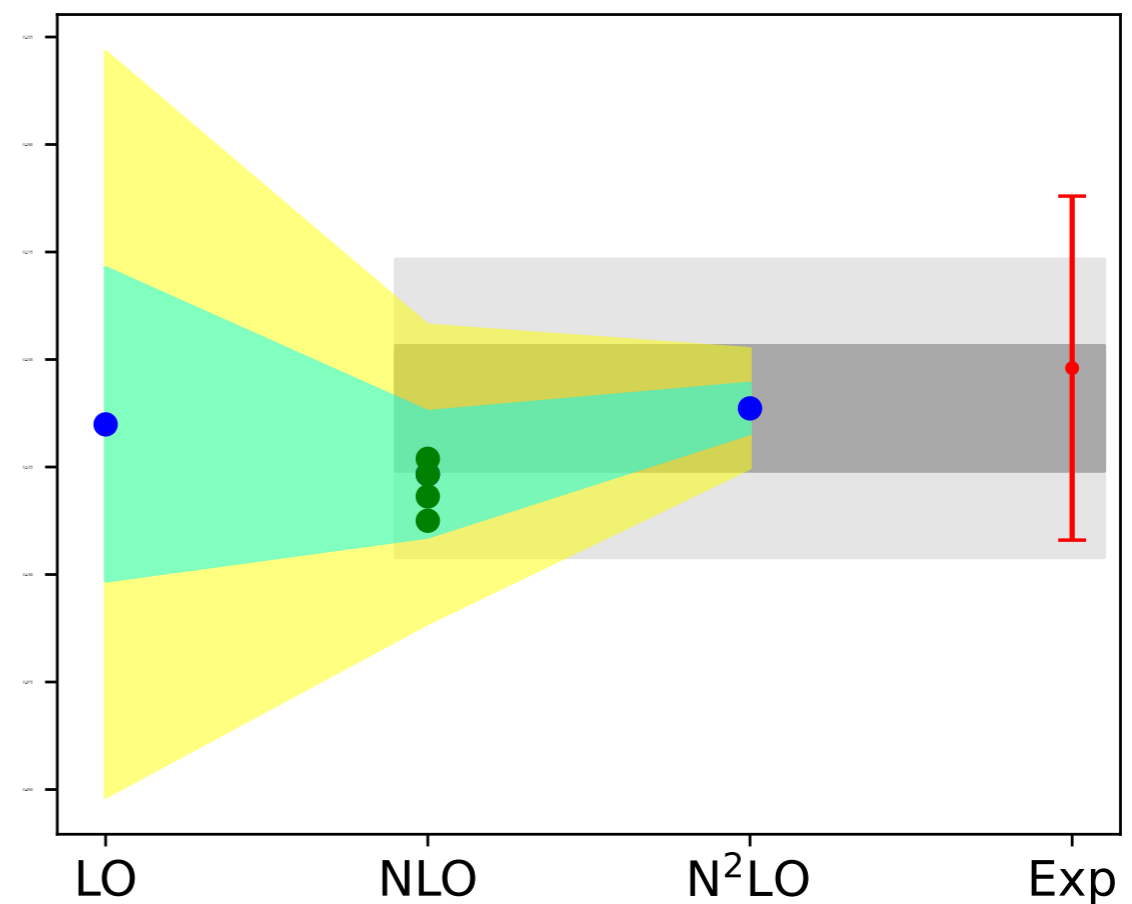
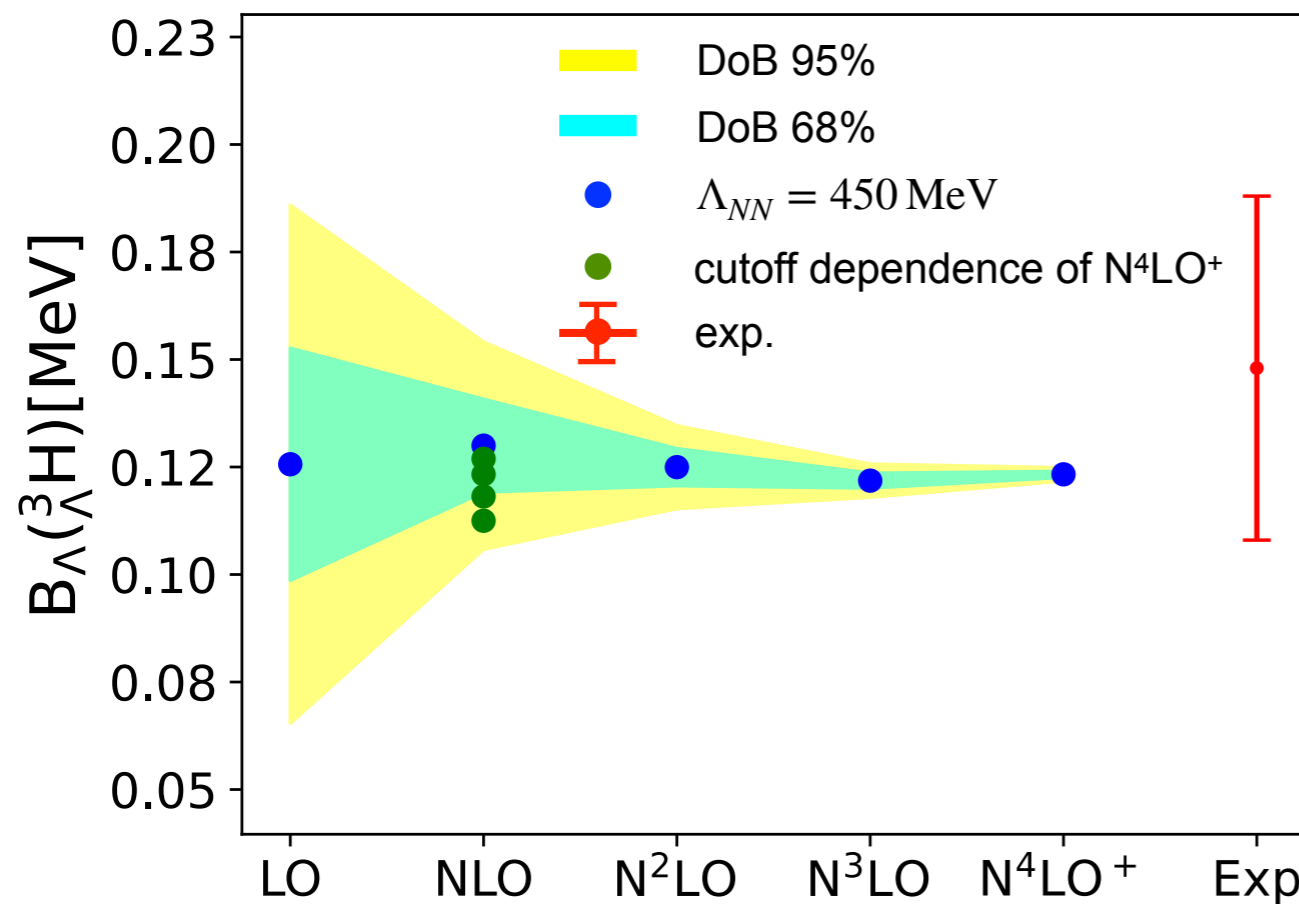
**Uncertainty due to missing higher orders is more relevant**

**than numerical uncertainty!** (for light nuclei)



- $Q$ ,  $\nu_0$  and  $\tau_0$  are chosen using all available data (NN and YN convergence)
- uncertainties are extracted using  $c_k$  for NN or YN convergence
- use  $c_k$  of individual hypernuclei

➔ individual uncertainties for NN and YN convergence for each separation energy  
 consistent with experimental data  
 cutoff dependence always at least NLO (YNN missing!)



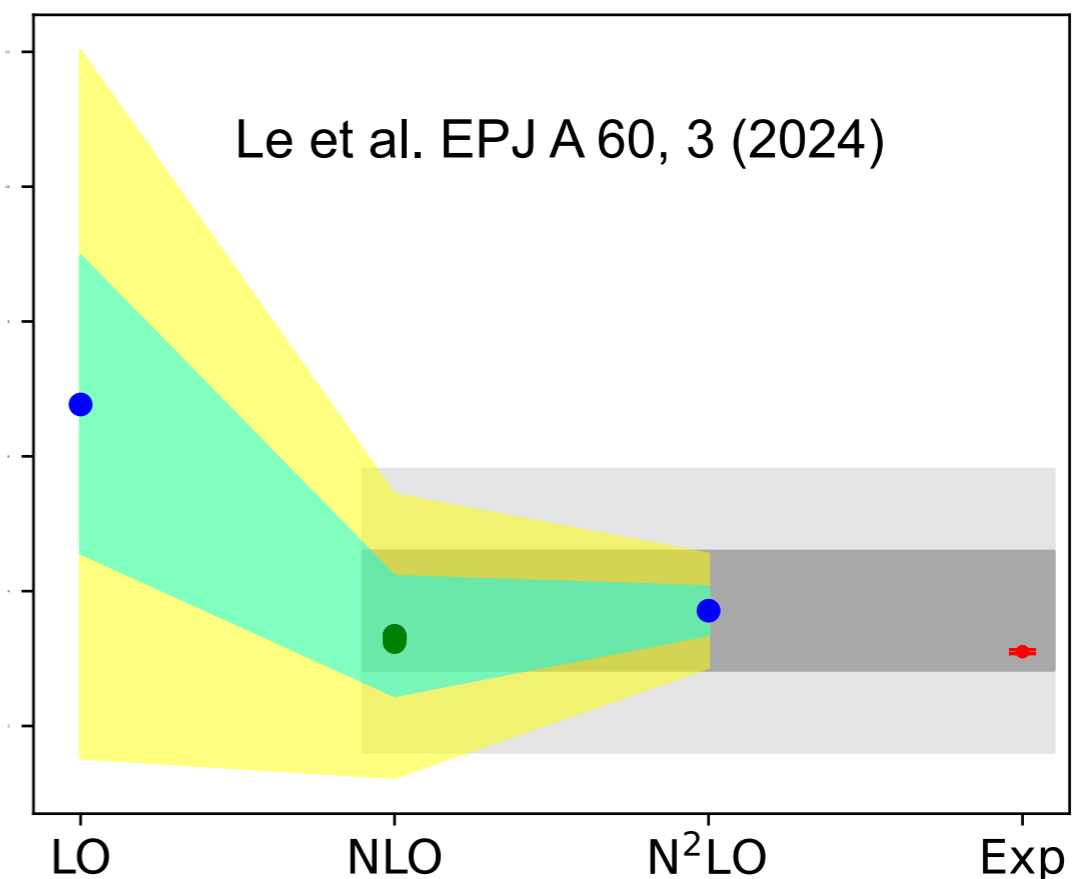
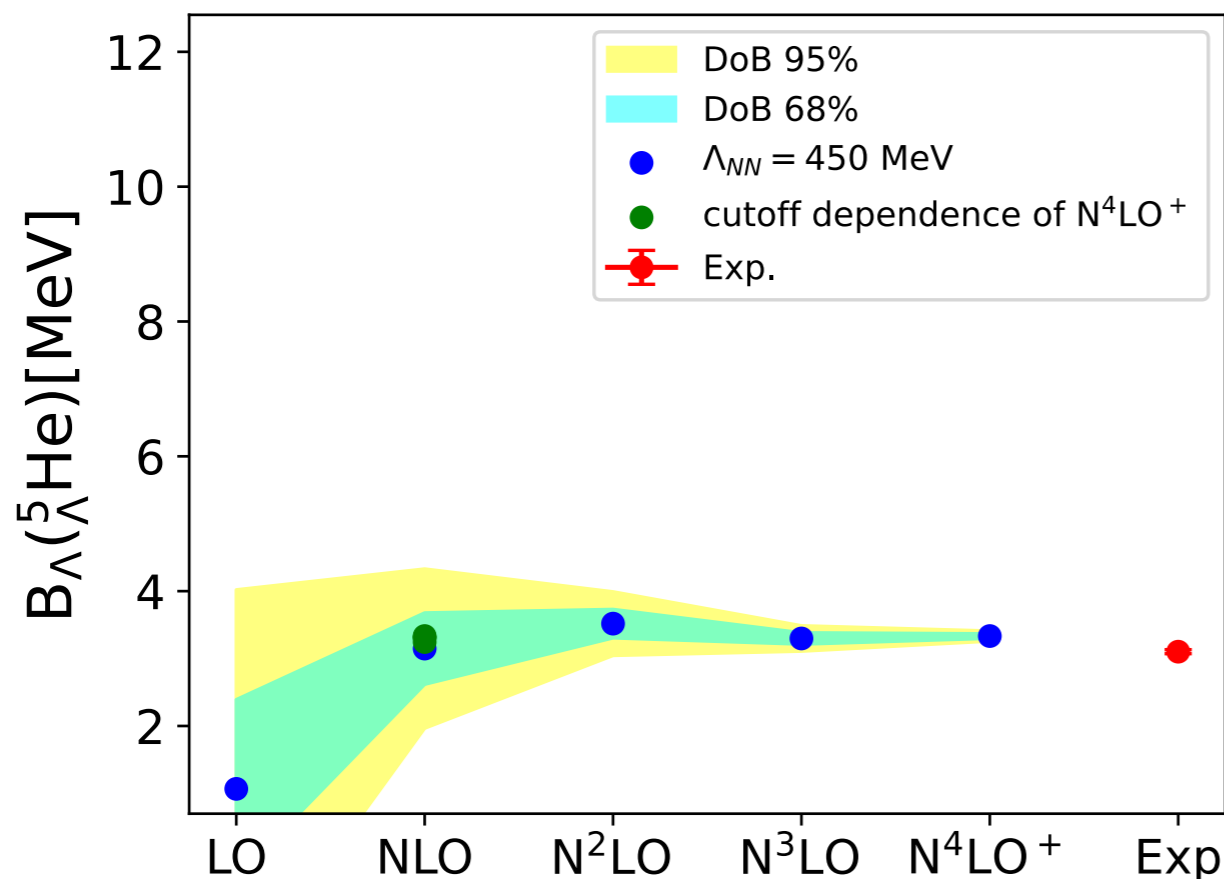
# Application to ${}^5_{\Lambda}\text{He}$ and summary



- without YNN: sizable uncertainties at  $A = 4$  and 5
- $A = 3$  sufficiently accurate
- NN/YN dependence small at least for  $A = 3$

nucleus	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$
${}^3_{\Lambda}\text{H}$	0.011	0.015
${}^4_{\Lambda}\text{He} (0^+)$	0.157	0.239
${}^4_{\Lambda}\text{He} (1^+)$	0.114	0.214
${}^5_{\Lambda}\text{He}$	0.529	0.881

→ at the same time: estimate of YNN !

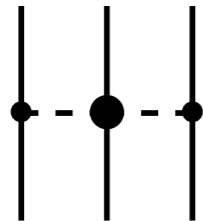




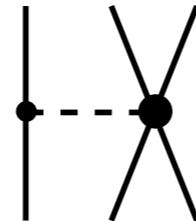
# YNN ( $\Lambda$ NN) interactions

Leading 3BF with the usual topologies (Petschauer et al. PRC 93, 014001 (2016))

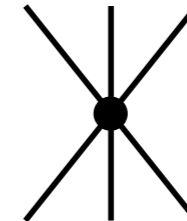
ChPT  $\longrightarrow$  all octet mesons contribute  $\longrightarrow$  **only take  $\pi$  explicitly into account**



2 LECs in  $\Lambda$ NN  
(up to 10)



2 LECs in  $\Lambda$ NN  
(up to 14)



3 LECs in  $\Lambda$ NN  
5 LECs in  $\Sigma$ NN + 1  $\Lambda$ - $\Sigma$  transition

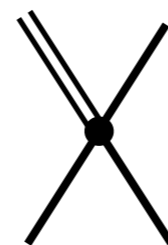
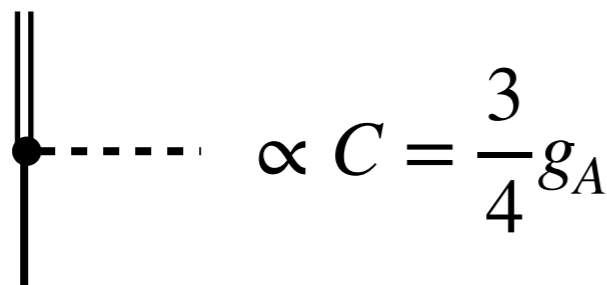
only few data  $\longrightarrow$  need to keep the **# of LECs** small

Decuplet baryons ( $\Sigma^*$ ...) might enhance YNN partly to NLO

(Petschauer et al., NPA 957, 347 (2017))

By decuplet saturation all LECs can be related to the following

leading octet-decuplet transitions (Petschauer et al. Front. Phys. 8,12 (2020))

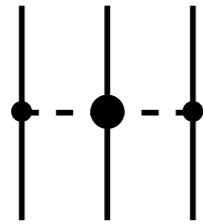


$\propto G_1, G_2 \longrightarrow$  **reduction to 2 LECs**

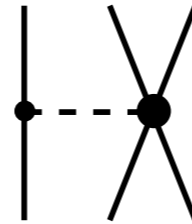
# YNN ( $\Lambda$ NN) interactions



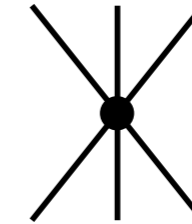
Decuplet saturation relates all LECs to  $G_1$  and  $G_2$



$$\propto C^2$$



$$\propto CG_1, CG_2$$



$$\propto (G_1)^2, (G_2)^2, G_1G_2$$

For  $\Lambda$ NN:  $\propto C^2$

$$\propto C(G_1 + 3G_2)$$

$$\propto (G_1 + 3G_2)^2 \quad \mathbf{1 \text{ LEC}}$$

➔ density dependent BB interactions (Petschauer et al., NPA 957, 347 (2017))

➔ application to nuclear matter (Haidenbauer et al., EPJ A 53, 121 (2017))

neutron stars (Logoteta et al., EJA 55, 207 (2019))

- contribution on the single particle potentials can be large
- realistic results seem to require partly cancelations of  $2\pi$  and  $1\pi$  exchange

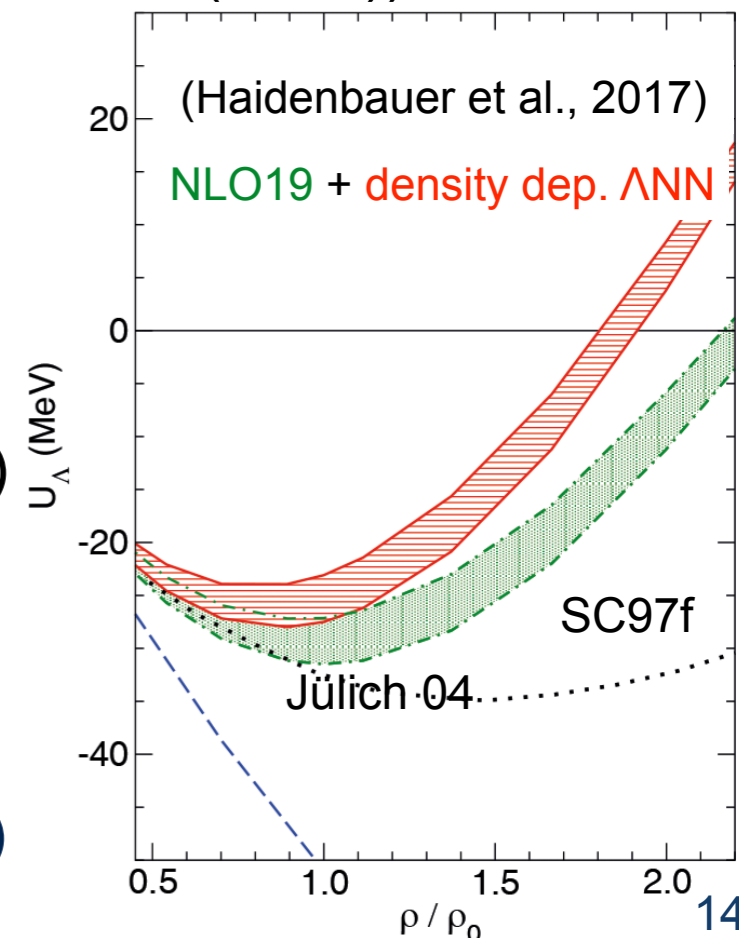
(fixes sign of  $G_1 + 3G_2$ !)

Recently: successful benchmark of matrix elements

(Hoai Le et al. EPJ A 61,21 (2025))

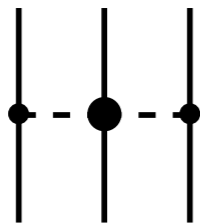
and first direct application to light hypernuclei including  $\Sigma$ 's

(Hoai Le et al. PRL 134, 072502 (2025))

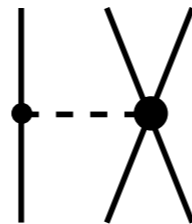




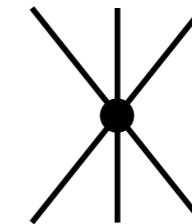
## Decuplet approximation in YNN



$$\propto C^2$$



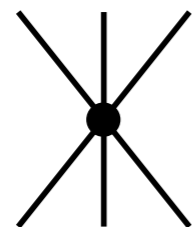
$$\propto CG_1, CG_2$$



$$\propto (G_1)^2, (G_2)^2, G_1G_2$$

is **not** sufficient to fix spin dependence

➔ +  $\Lambda$ NN contact terms **without decuplet constraints**



$$\Lambda\text{NN} \propto C'_1, C'_2, C'_3$$

*ad hoc* choice: alter  $C_2$ :

$$C'_1 = C'_3 = \frac{(G_1 + 3G_2)^2}{72\Delta}$$

$$C'_2 = 0$$



$$V_{\Lambda\text{NN}} = C'_2 \vec{\sigma}_1 \cdot (\vec{\sigma}_2 + \vec{\sigma}_3) (1 - \vec{\tau}_2 \cdot \vec{\tau}_3)$$

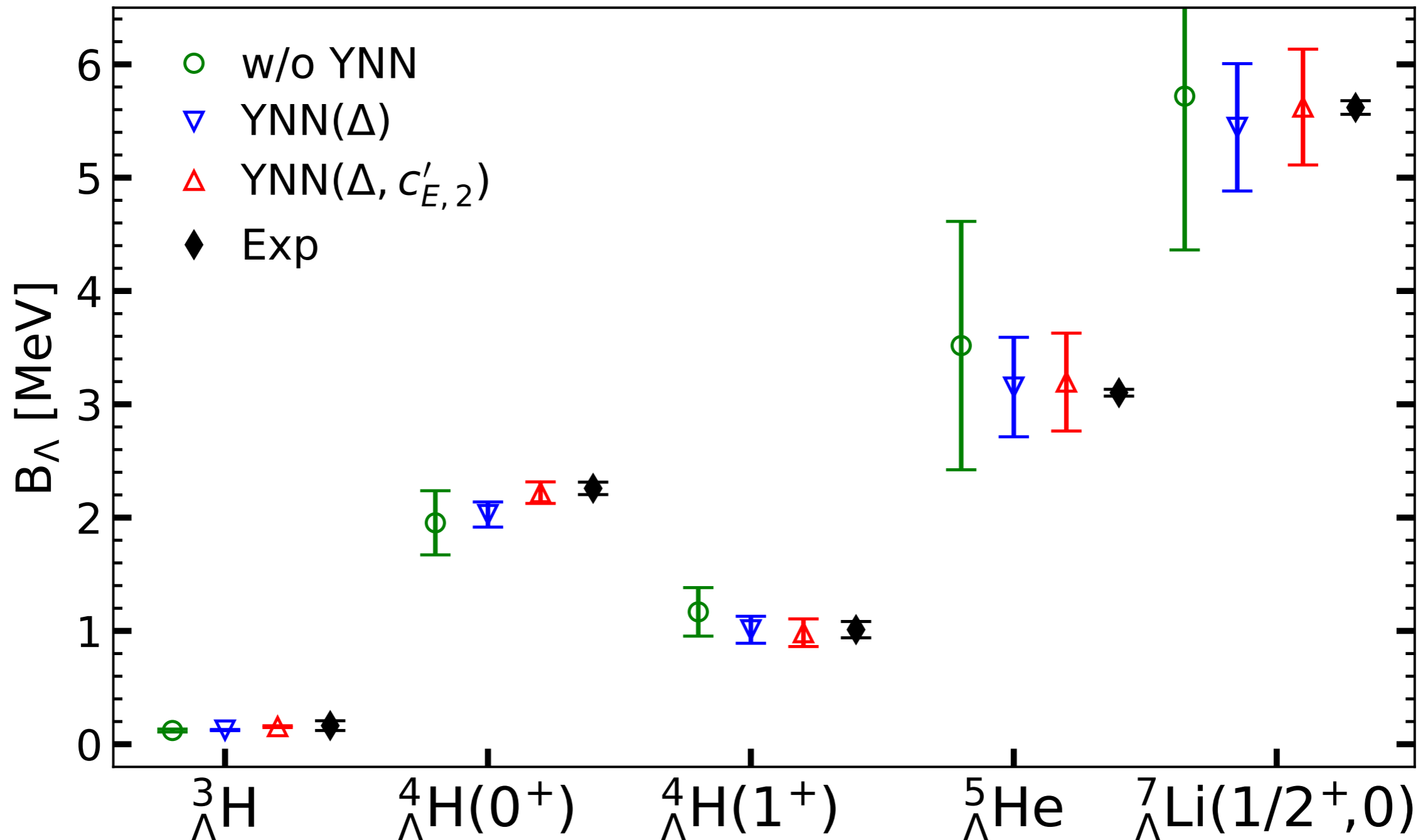
$$C'_2 = G_3$$

$C'_2$  introduces a spin dependent interaction in the most relevant particle channel

(see also Hildenbrand et al. arXiv:2406.17638)



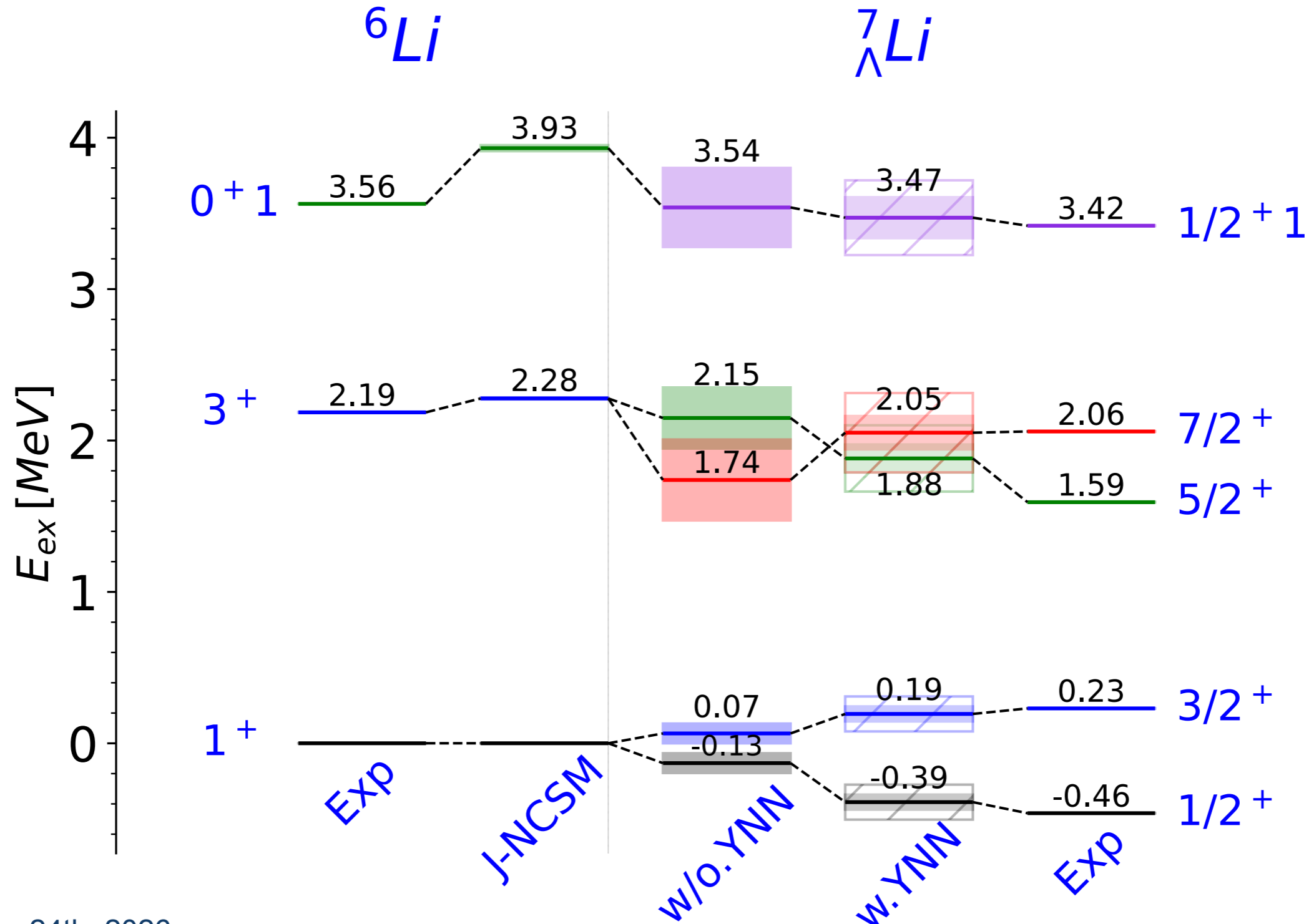
- Fit to  $0^+$  and  $1^+$  state of  ${}^4_{\Lambda}\text{He}$  and/or  ${}^5_{\Lambda}\text{He}$
- spin-dependence in  $A=4$  not well explained by decuplet saturation
- $C'_2$  term improves  $0^+$  of  ${}^4_{\Lambda}\text{He}$  and  $1/2^+$  of  ${}^7_{\Lambda}\text{Li}$
- agreement generally much better than  $N^2\text{LO}$  uncertainty



# YNN prediction for ${}^7_{\Lambda}\text{Li}$



- good agreement
- $C'_2$  term included, but not very important (not shown)
- higher states have significant uncertainty





- **YN interactions not well understood**
  - *scarce YN data*
  - *more information necessary to solve "hyperon puzzle"*
- **Hypernuclei provide important constraints**
  - $^1S_0$   $\Lambda N$  scattering length &  $^3_\Lambda\text{H}$
  - CSB of  $\Lambda N$  scattering &  $^4_\Lambda\text{He}$  /  $^4_\Lambda\text{H}$
- **SMS YN interactions up to  $N^2\text{LO}$** 
  - *order LO, NLO and  $N^2\text{LO}$  allow uncertainty quantification*
  - *have a **non-unique** determination of contact interactions (more data necessary)*
- **Chiral 3BF**
  - *decuplet saturation alone does not improve spin dependence*
  - *spin-dependent  $\Lambda\text{NN}$  leads to further improvement*
  - *study cutoff dependence / application to more p-shell hypernuclei*
  - *extension to  $\Lambda d$  scattering: probably more insight for higher densities*
  - *extension  $\Lambda d/\Lambda pp$  correlations: info on different spin/isospin states*