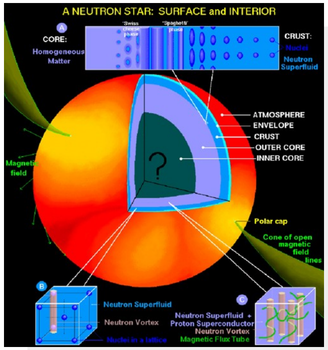
From light nuclei to neutron stars

Alex Gezerlis

UNIVERSITY OF GUELPH

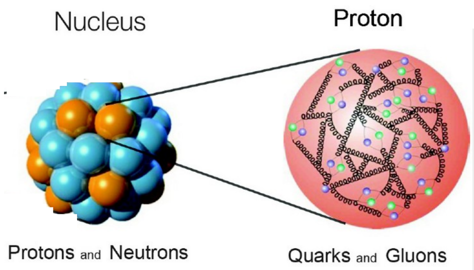
Workshop on progress in ab initio nuclear theory
TRIUMF, Vancouver BC
February 25, 2026

Outline

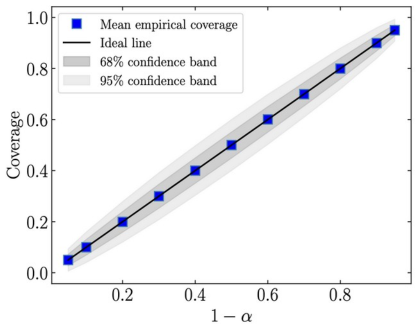


Credit: Dany Page

Motivation



Nuclear methods



Recent results

Physical systems studied

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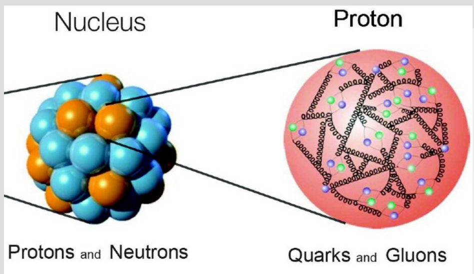
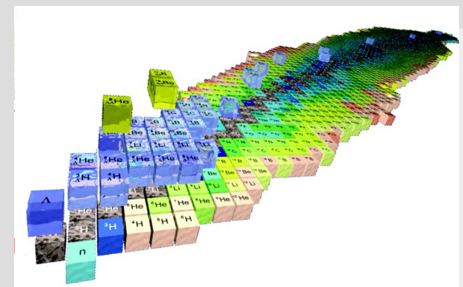
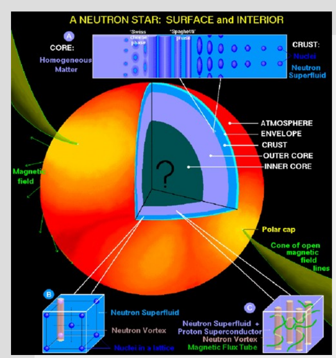
Nuclear forces



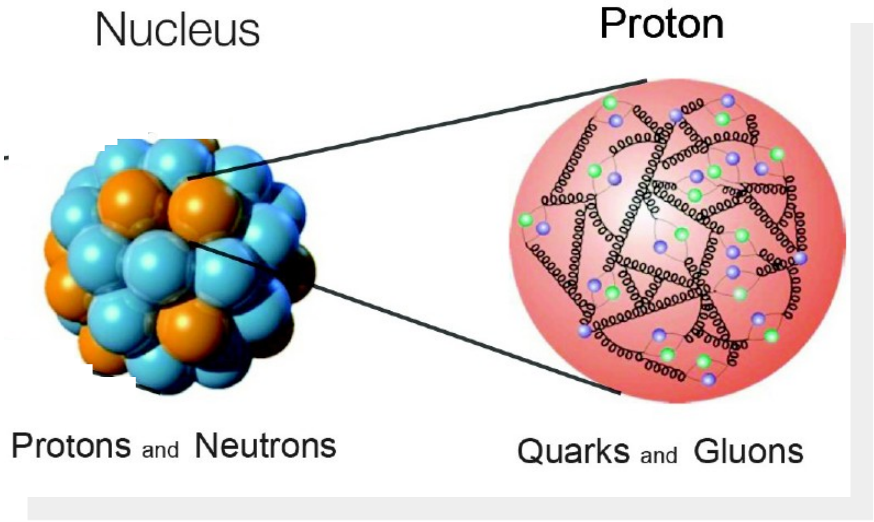
Nuclear structure



Nuclear astrophysics

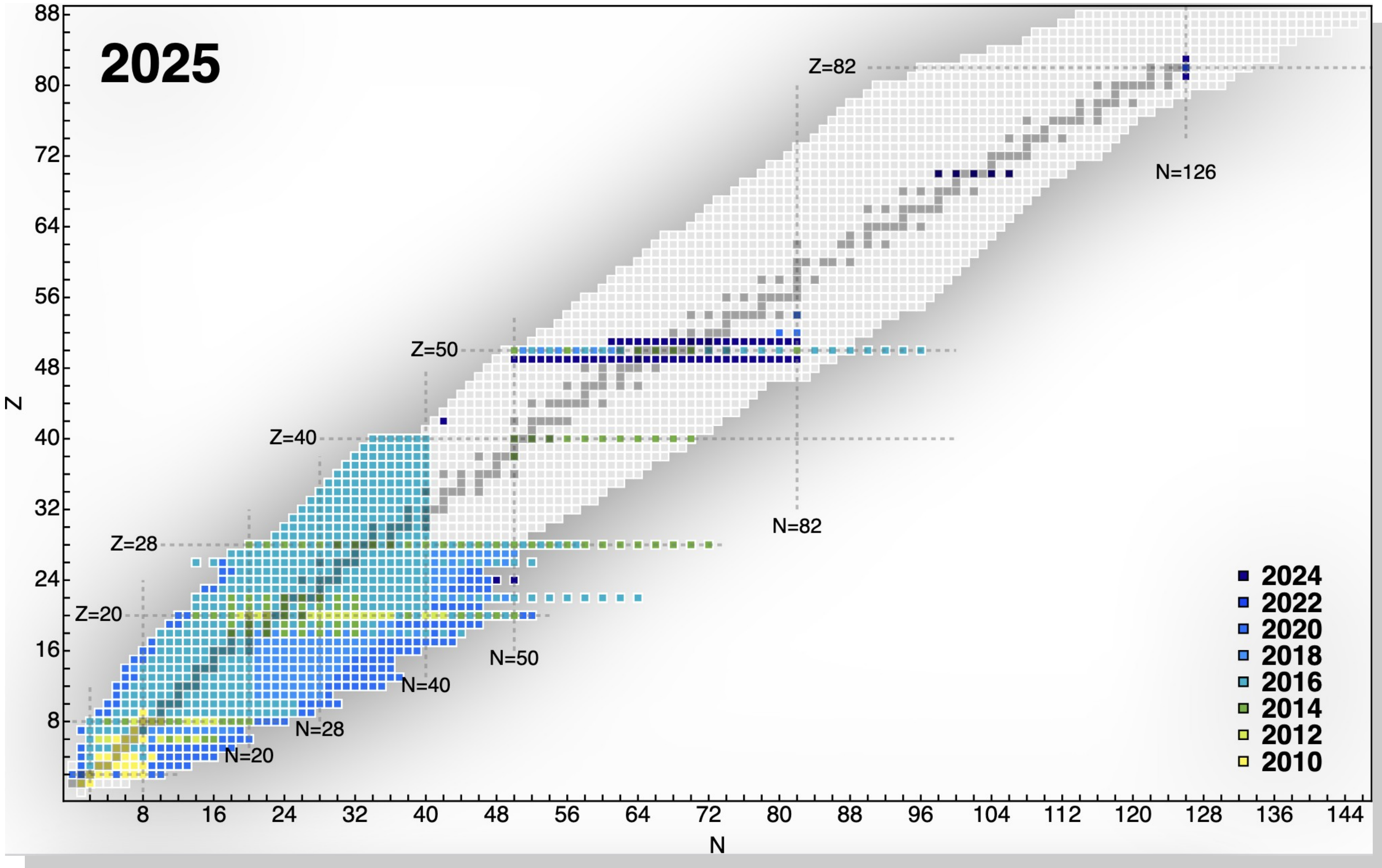


Key system: few nucleons



- No unique nuclear potential
- Preferable to use combination of phenomenological (high-quality) and more modern (conceptually clean) approach
- Desirable to make contact with underlying level
- New era, where practitioners design interactions themselves

Key system: nuclei



Heiko Hergert's propaganda plot

- Lots of recent progress
- Open-shell nuclei are the current frontier

- Goal is to study nuclei *from first principles* (when possible)

Key system: neutron stars

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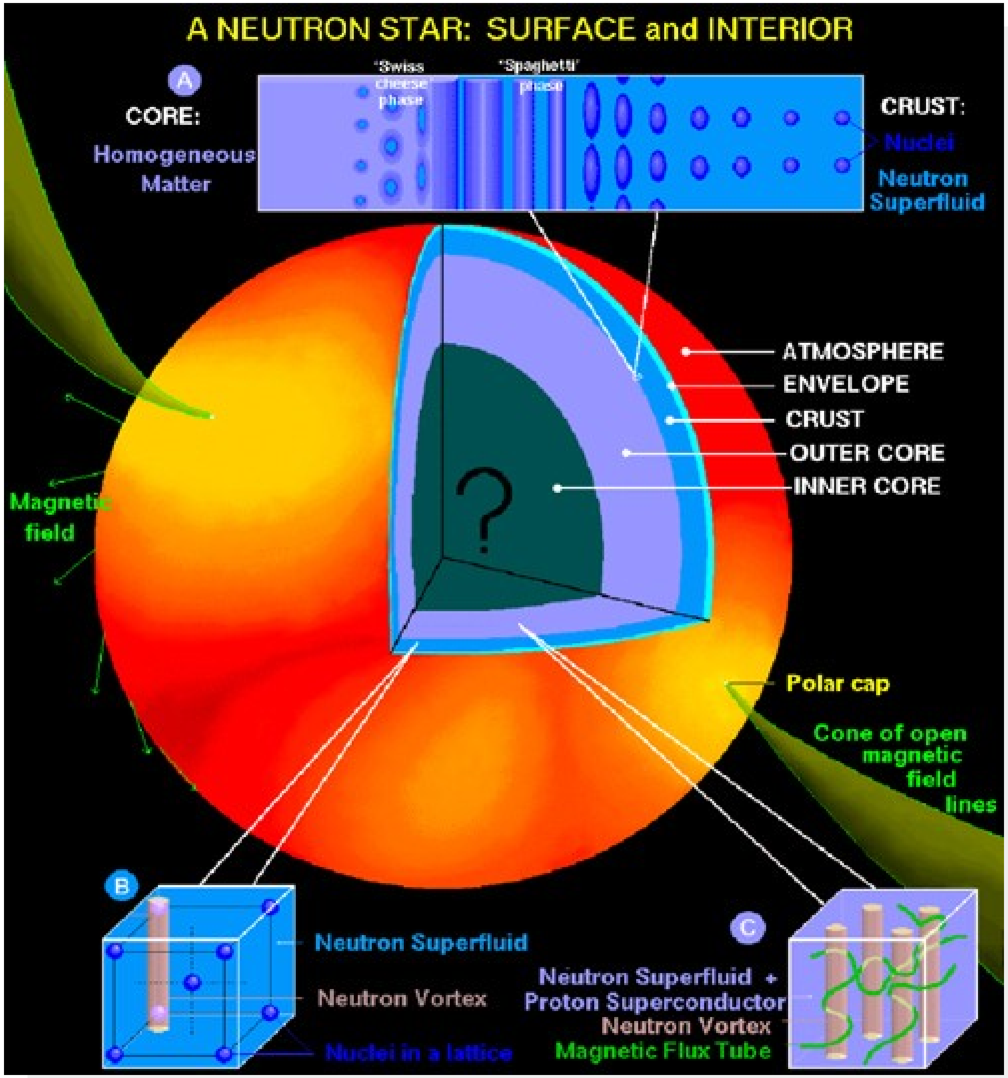
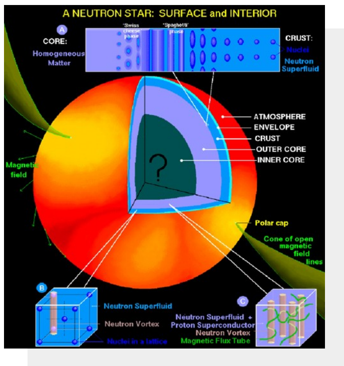


Credit: Dany Page

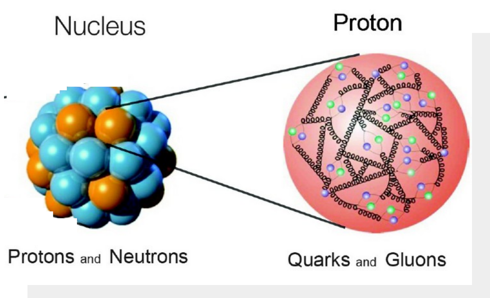
- Ultra-dense: 1.4 solar masses (or more) within a radius of 10 kilometres
- Terrestrial-like (outer layers) down to exotic (core) behaviour
- Observationally probed, i.e., not experimentally accessible
- Goal is to study neutron stars *from first principles* (when possible)

Outline

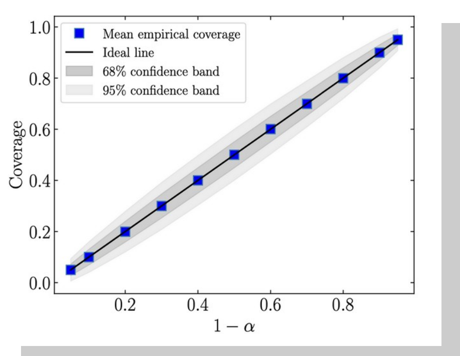


Credit: Dany Page

Motivation

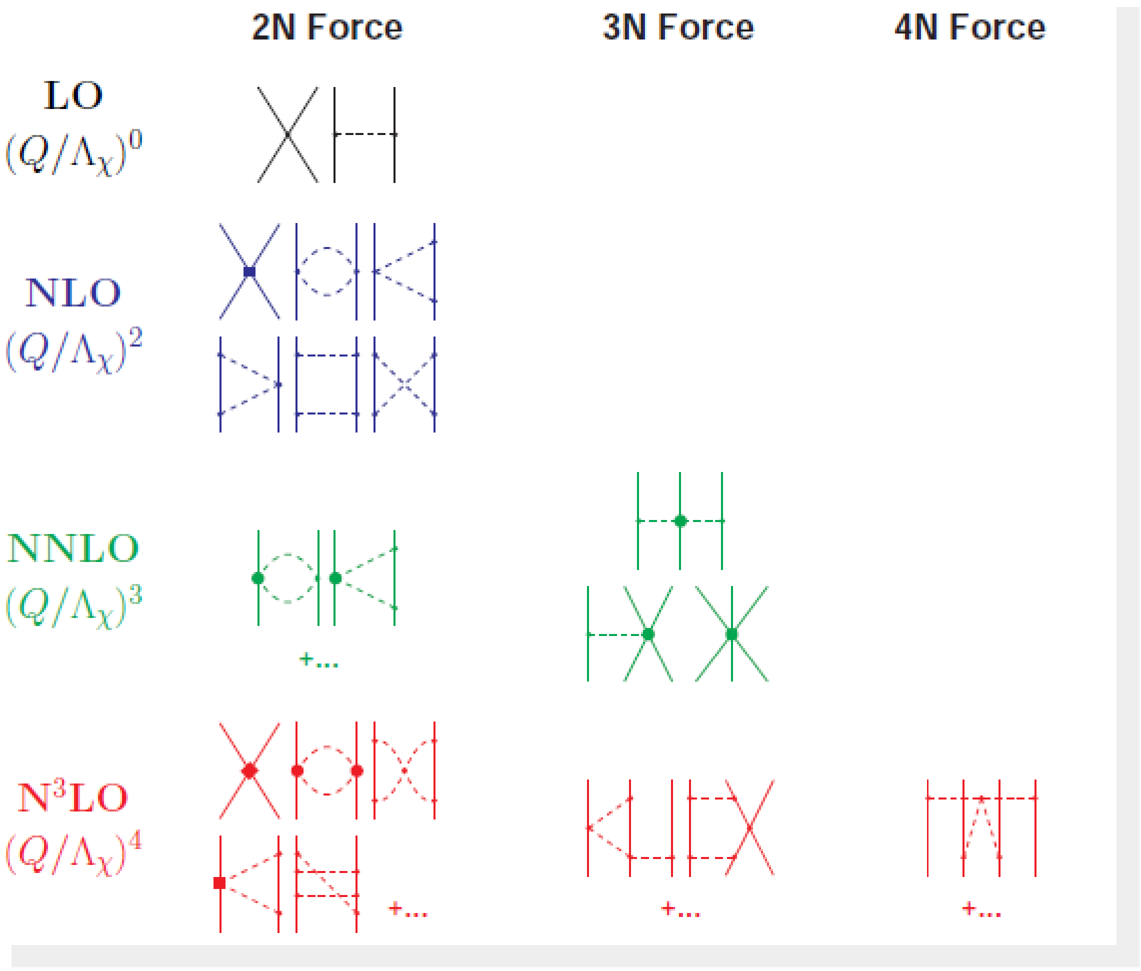


Nuclear methods



Recent results

Nuclear interactions



- Attempts to connect with underlying theory (QCD)
- Low-momentum expansion
- Naturally emerging many-body forces
- Low-energy constants from experiment or lattice QCD
- Now available in non-local, local, or semi-local varieties
- Power counting's relation to renormalization actively investigated

S. Weinberg, U. van Kolck, E. Epelbaum, N. Kaiser ...

But even with the interaction in place, how do you solve the many-body problem?

Nuclear many-body problem

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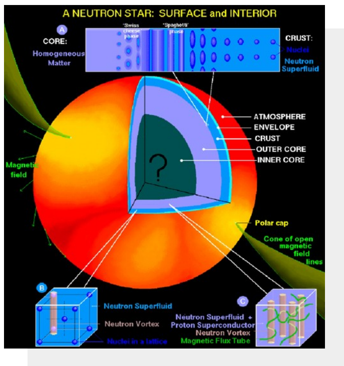
$$H\Psi = E\Psi$$

where $$H = \sum_i K_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

Wave function depends on coordinates, spin projections, and isospin projections, so we are faced with a large number of complex coupled second-order differential equations

Outline



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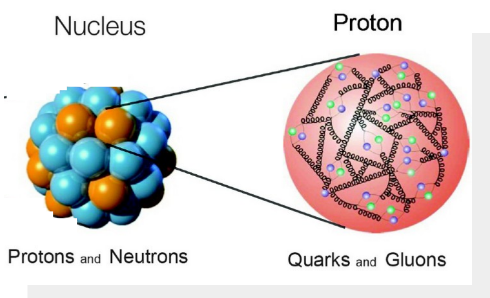
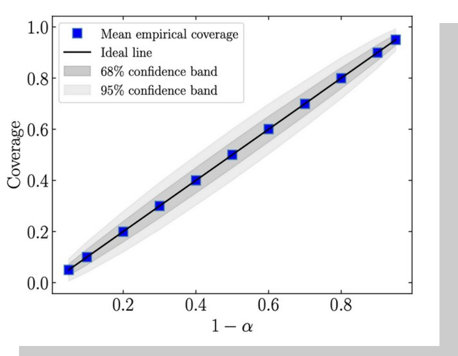
Motivation



Nuclear methods



Recent results

Recent results

- (QMC and emulators for light nuclei)
- Neural-network wave functions for light nuclei
- Conformal prediction for nucleon-nucleon scattering

(QMC and emulators for light nuclei)

R. Curry, K. Hebeler, S. Gandolfi, A. Gezerlis,
A. Schwenk, R. Somasundaram, and I. Tews,
arXiv:2510.15860

Varying the three-nucleon interaction

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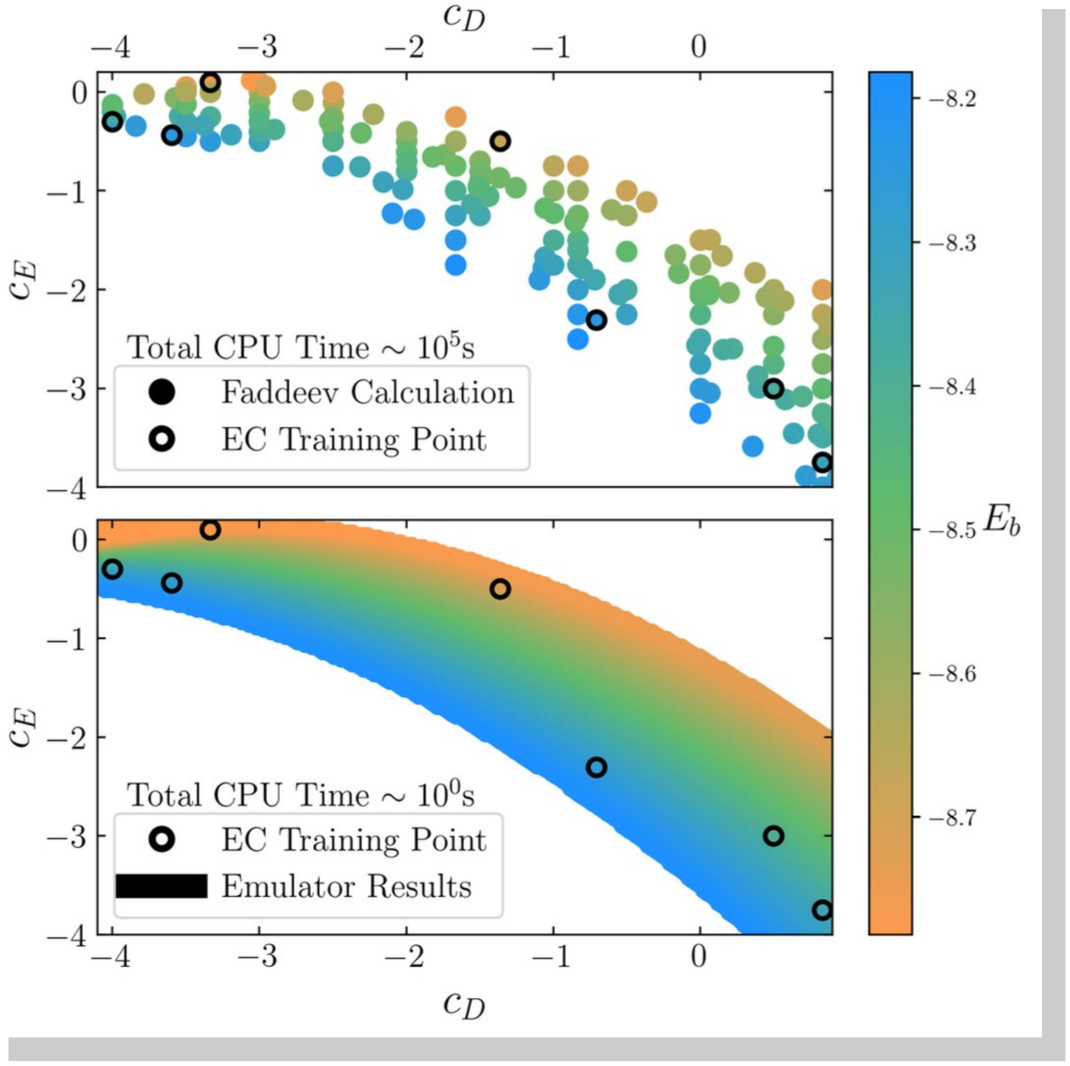
Emulators make the calculation so cheap that we can fill out our plots:



R. Curry *et al*, arXiv:2510.15860

Varying the three-nucleon interaction

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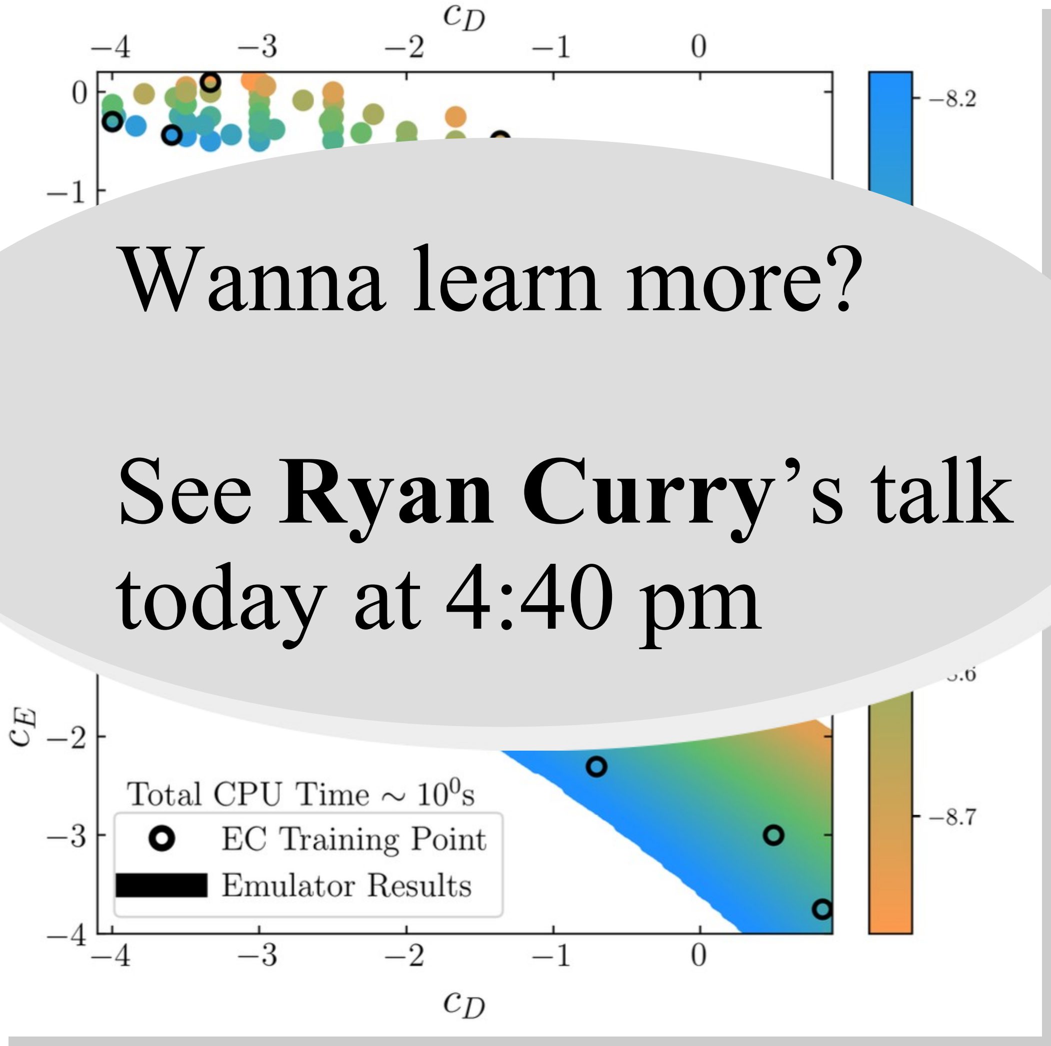

Wanna learn more?

See **Ryan Curry**’s talk today at 4:40 pm

R. Curry *et al*, arXiv:2510.15860

Neural-network wave functions for light nuclei

P. Weng, A. Gezerlis, and J. Holt, arXiv:2505.11442

Quantum Monte Carlo in one slide

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Variational Monte Carlo

Encode parameters in trial wave function Ψ_V and use Rayleigh-Ritz principle to minimize expectation value of Hamiltonian

Diffusion Monte Carlo

Project out excited-state contributions, to reach the ground state

$$\Psi(\tau \to \infty) = \lim_{\tau \to \infty} e^{-(\mathcal{H} - E_T)\tau} \Psi_V$$

Earlier work

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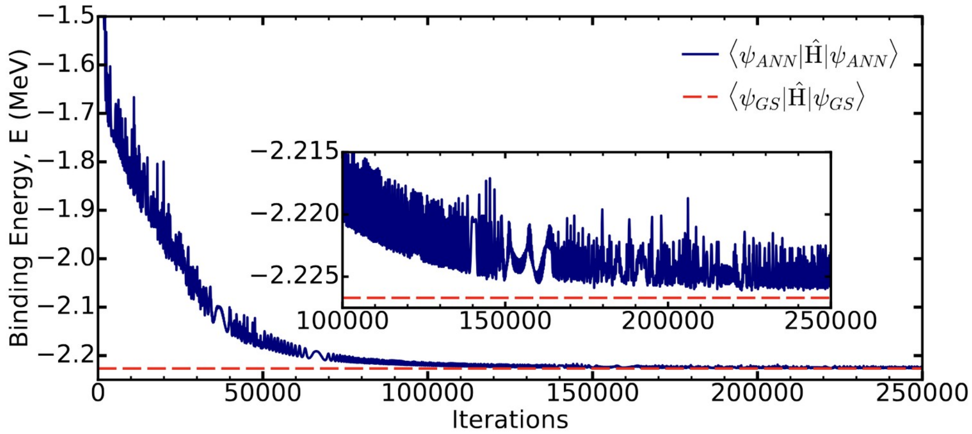
Deuteron



J. W. T. Keeble and A. Rios,
Phys. Lett. B **809**, 135743 (2020)

Light nuclei

	Λ	VMC-ANN	VMC-JS	GFMC	GFMC$_c$
^{2}H	4 fm^{-1}	−2.224(1)	−2.223(1)	−2.224(1)	-
	6 fm^{-1}	−2.224(4)	−2.220(1)	−2.225(1)	-
^{3}H	4 fm^{-1}	−8.26(1)	−7.80(1)	−8.38(2)	−7.82(1)
	6 fm^{-1}	−8.27(1)	−7.74(1)	−8.38(2)	−7.81(1)
^{4}He	4 fm^{-1}	−23.30(2)	−22.54(1)	−23.62(3)	−22.77(2)
	6 fm^{-1}	−24.47(3)	−23.44(2)	−25.06(3)	−24.10(2)

C. Adams, G. Carleo, A. Lovato, N. Rocco,
Phys. Rev. Lett. **127**, 022502 (2021)

N.B. Limited to pionless Hamiltonian

Earlier work

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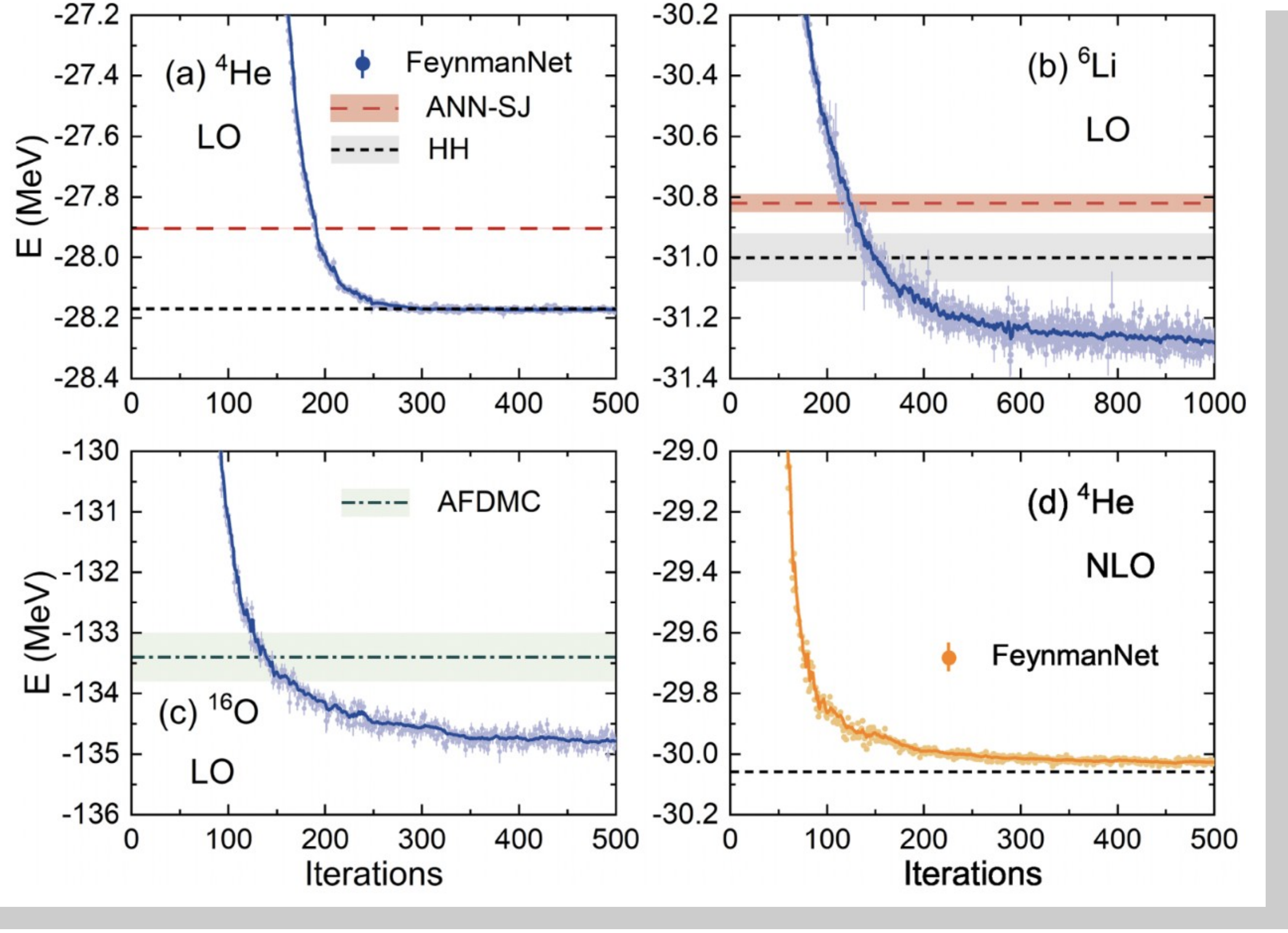
More on light nuclei



Y. L. Yang and P. W. Zhao, Phys. Rev. C **107**, 034320 (2023)

N.B. Also limited to pionless Hamiltonian

Neural networks for light nuclei

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Spin-isospin correlations

$$|\psi\rangle = \mathcal{S}\prod_{i<j}\left(1+\sum_{\mathrm{x}} u_{ij}^{(\mathrm{x})}\hat{O}_{ij}^{(\mathrm{x})}\right) f_{ij}^{(c)}|\Phi\rangle$$

$$|\psi\rangle \to \left(1+\sum_{i<j<k}\sum_{\mathrm{cyc}}\sum_{\mathrm{x}} \epsilon^{(\mathrm{x})}\hat{V}_{ijk}^{(\mathrm{x})}\right)|\psi\rangle$$

for N^2LO chiral Hamiltonian

P. Weng, A. Gezerlis, and J. Holt, arXiv:2505.11442

Neural networks for light nuclei

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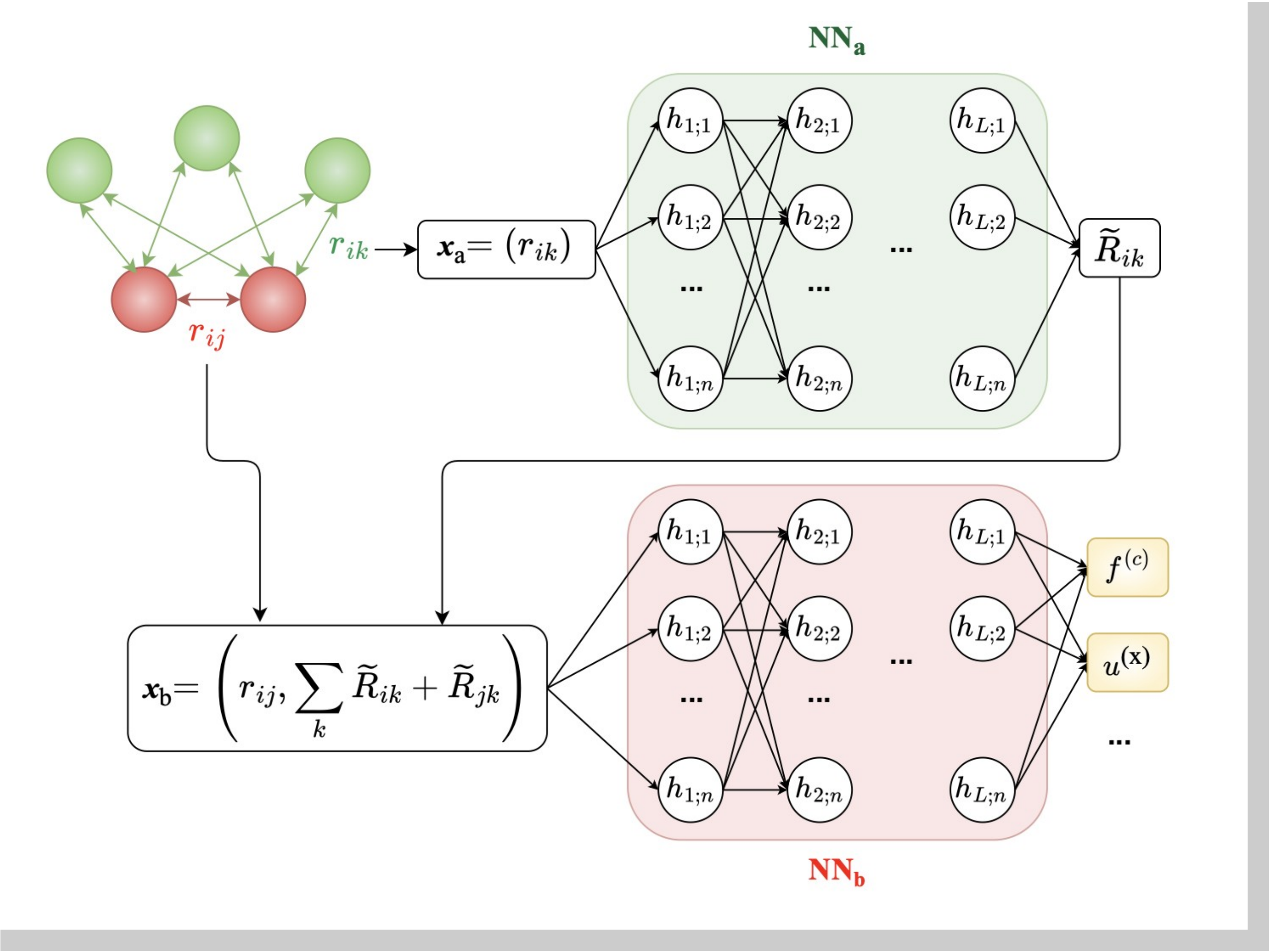

P. Weng, A. Gezerlis, and J. Holt, arXiv:2505.11442

Neural networks for light nuclei

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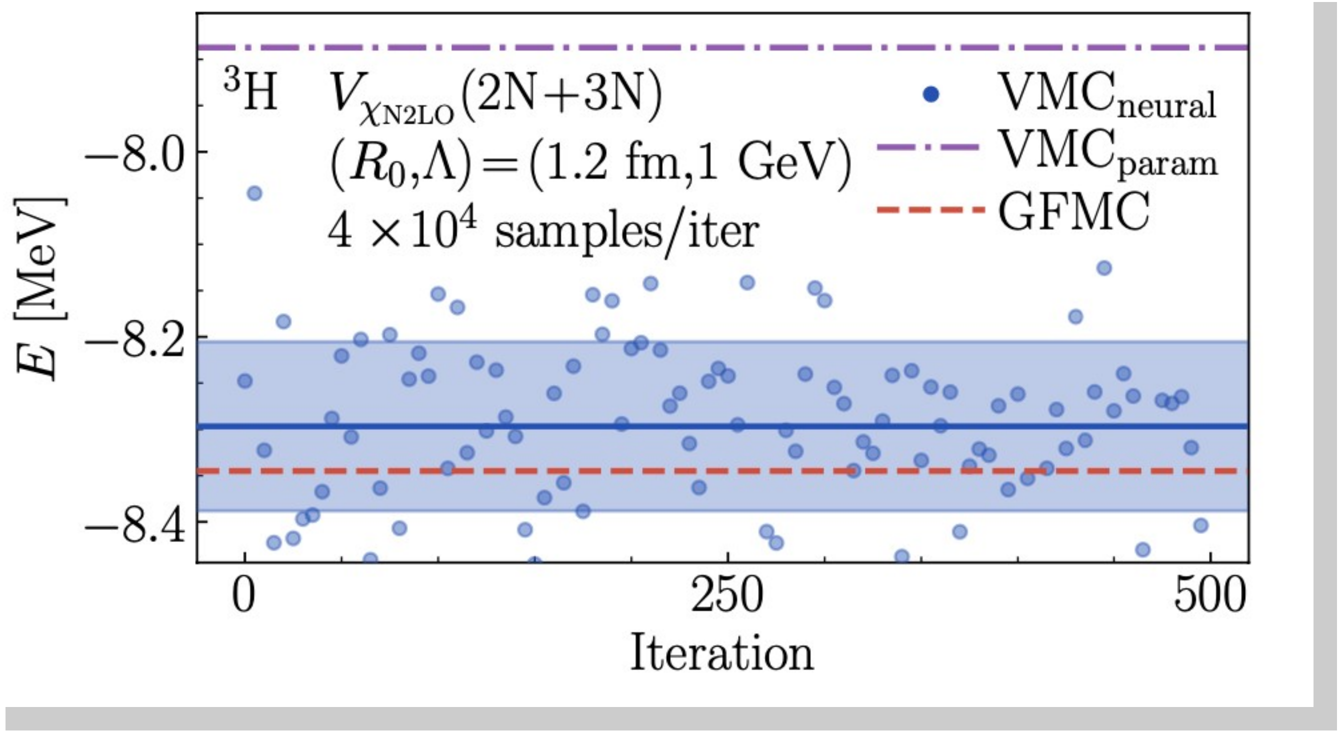
Nearly reproduces GFMC results
already at the VMC level

$E = E_k + V_{\chi N^2LO}(2N)$				
	R_0 [fm]	E_{neural} [MeV]	E_{GFMC} [MeV]	$\vert\Delta E\vert / \vert E_{GFMC}\vert$
^3H	1.0	-7.338 ± 0.008	-7.554 ± 0.007	2.9%
	1.1	-7.500 ± 0.006	-7.625 ± 0.005	1.6%
	1.2	-7.678 ± 0.005	-7.740 ± 0.005	0.8%
^2H	1.0	-2.217 ± 0.005	-2.21 ± 0.02	0.3%
	1.2	-2.212 ± 0.004	-2.20 ± 0.03	0.5%

P. Weng, A. Gezerlis, and J. Holt, arXiv:2505.11442

Neural networks for light nuclei

Dramatic improvement over standard/parametric VMC employed before, e.g., AFDMC



P. Weng, A. Gezerlis, and J. Holt, arXiv:2505.11442

Neural networks for light nuclei

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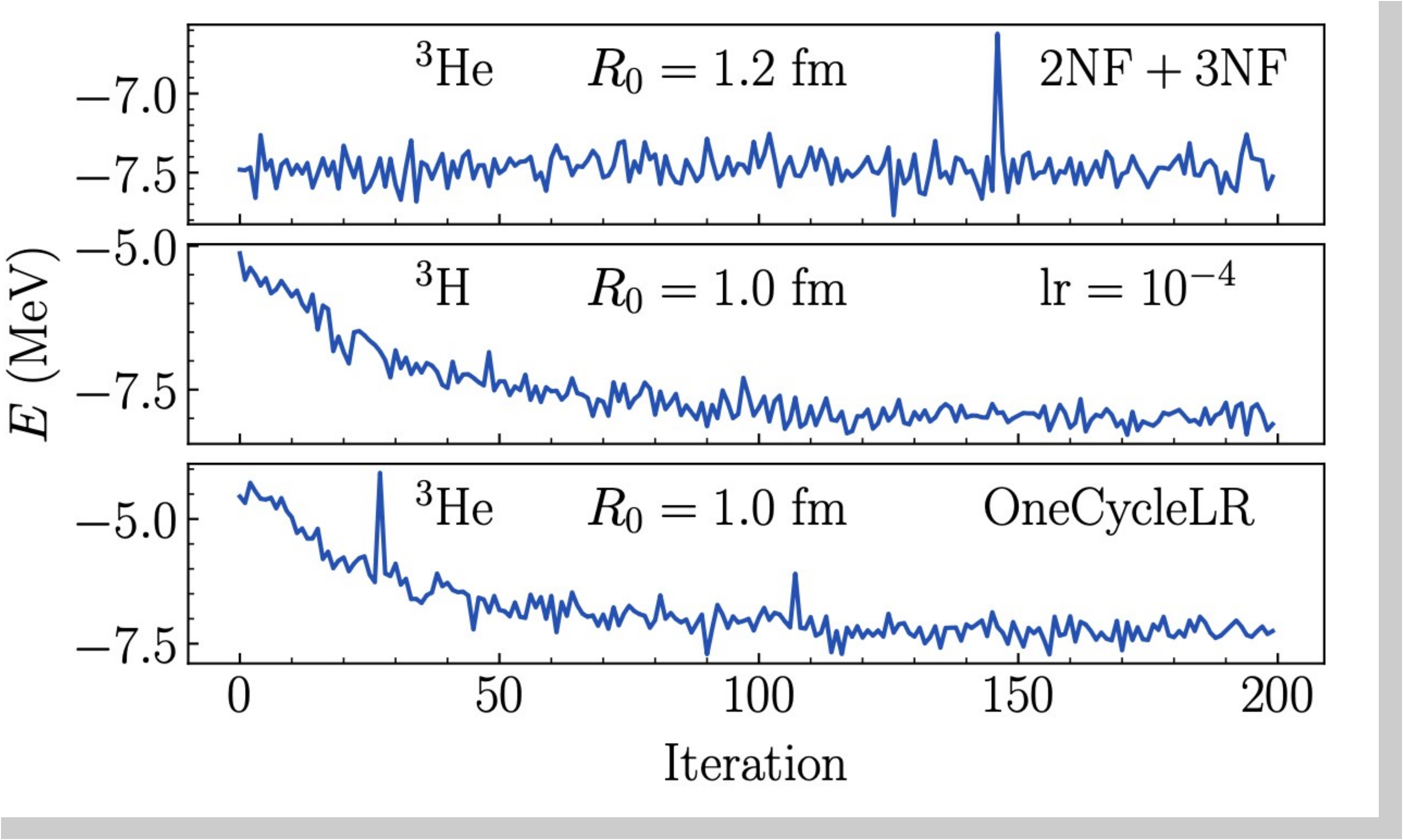
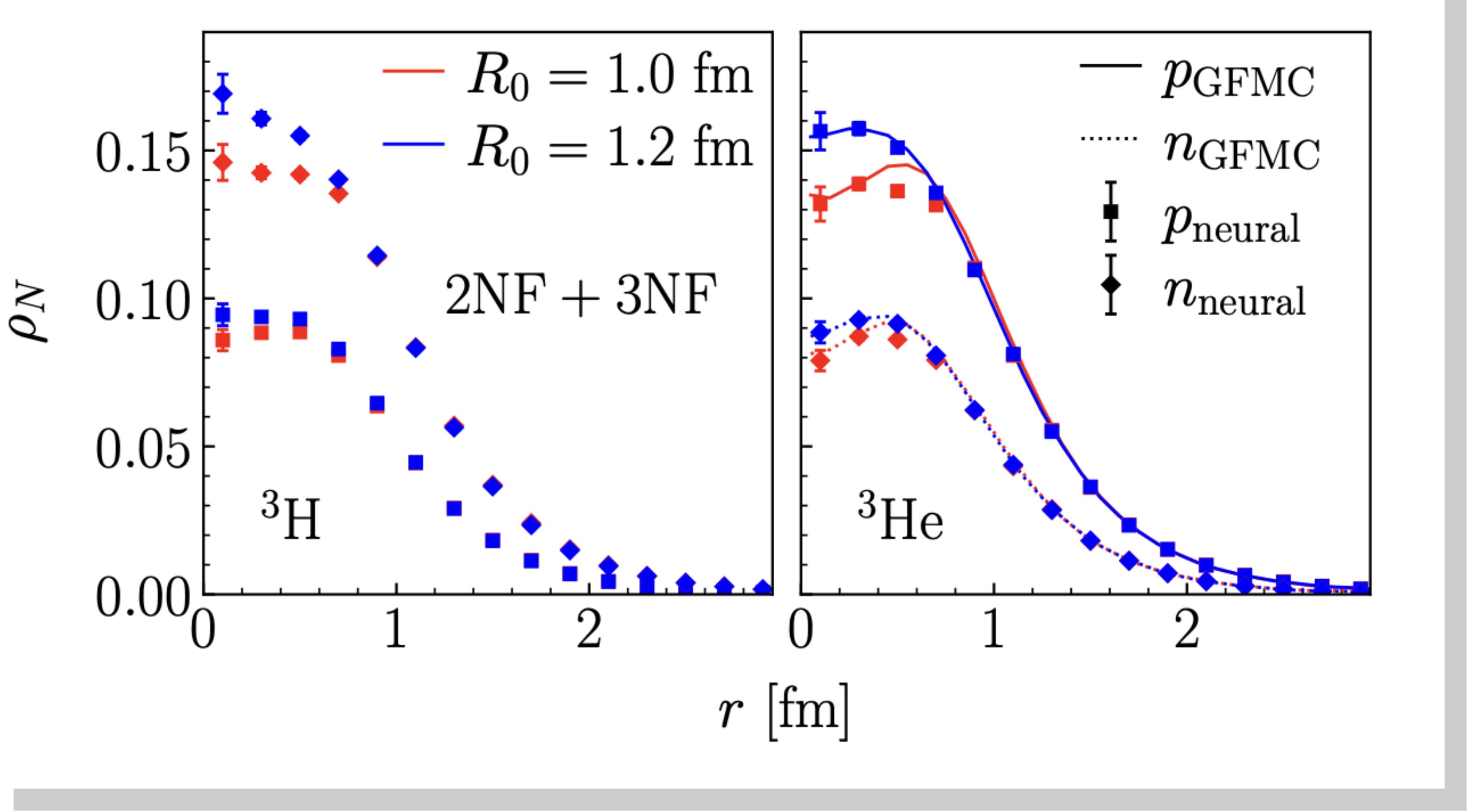
Transfer training: use one potential/nucleus as a starting point for other studies



P. Weng, A. Gezerlis, and J. Holt, arXiv:2505.11442

Neural networks for light nuclei

Point-particle distributions: $$\rho_N(r) = \frac{1}{4\pi r^2}\left\langle \psi \middle| \sum_{i=1}^{A} \frac{1+\zeta\tau_{z,i}}{2}\delta(r - |\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm}|) \middle| \psi \right\rangle$$



P. Weng, A. Gezerlis, and J. Holt, arXiv:2505.11442

Conformal prediction for nuclear physics

H. Yousefi Dezdarani, R. Curry, and A. Gezerlis, Phys. Rev. C, **113**, 014004 (2026)

H. Yousefi Dezdarani, R. Curry, C. L. Armstrong, and A. Gezerlis, *in preparation*

A hint of the philosophy of statistics

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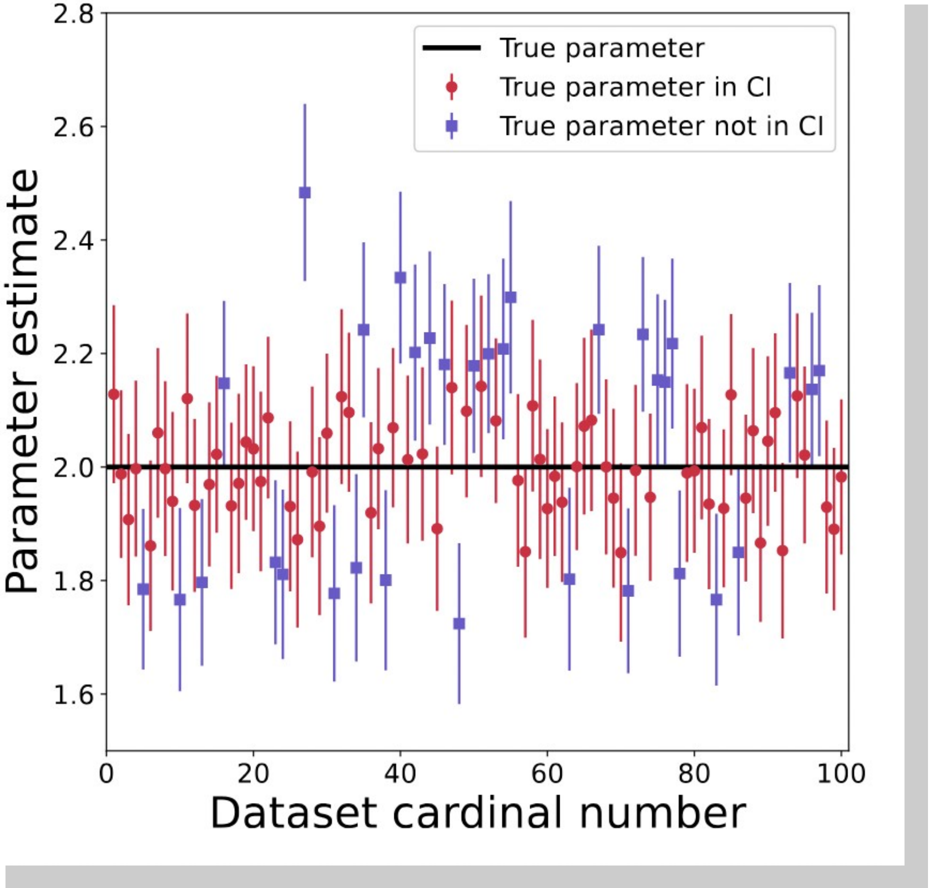
(Frequentist) confidence interval

E.g., maximize likelihood and take an $n\sigma$ error bar around that point.

Confidence interval *does not* imply degree of belief about our single dataset, but *does* provide guaranteed coverage across datasets.



A. Gezerlis and M. Williams, Eur. Phys. J Plus **138**, 19 (2023)

A hint of the philosophy of statistics

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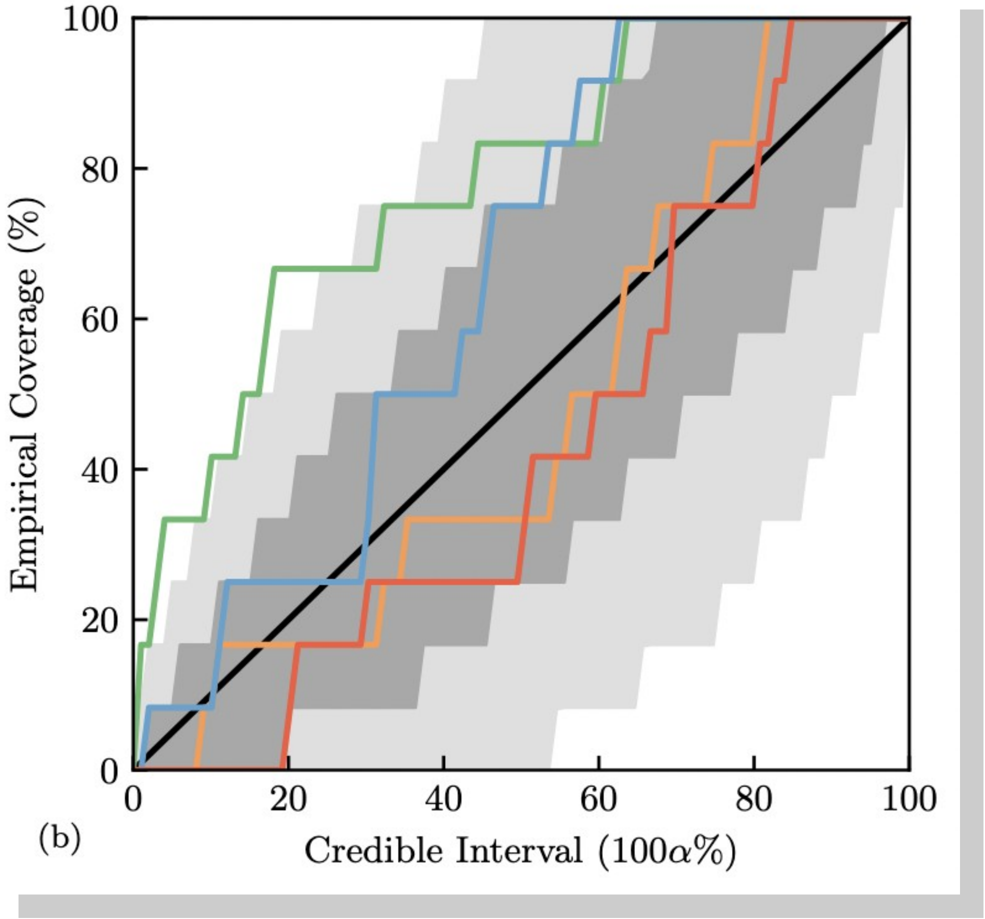
(Bayesian) credible interval

E.g., maximize posterior and take an $n\sigma$ error bar around that point.

Credible interval *does* imply degree of belief about our single dataset, but *does not* provide guaranteed coverage across datasets.

N.B. BUQEYE collaboration interprets coverage order-by-order



J. A. Melendez, R. J. Furnstahl, D. R. Phillips, M. T. Pratola, and S. Wesolowski, Phys. Rev. C **100**, 044001 (2019)

A hint of the philosophy of statistics

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Summarizing attractive features

- (Frequentist) confidence interval has guaranteed coverage across datasets

- (Bayesian) credible interval combines prior and likelihood to encapsulate degree of belief about our single dataset

Can you get the best of both worlds?

- Yes (*contra* Betteridge's law of headlines)

- Conformal prediction is a tool that post-processes any pre-trained model to produce guaranteed coverage

Conformal prediction

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- Distribution-free and model-agnostic uncertainty-quantification method

- Provides finite-sample prediction intervals with guaranteed coverage

- It accomplishes this by employing the quantile function (inverse of the cumulative distribution function) in an ingenious way:

$$C(X_{n+1}) = [Q_Y\left(\tfrac{\alpha}{2} \mid X_{n+1}\right) - q,\ Q_Y\left(1 - \tfrac{\alpha}{2} \mid X_{n+1}\right) + q]$$

where $q = Q_S(1-\alpha)$

H. Yousefi Dezdarani, R. Curry, and A. Gezerlis, Phys. Rev. C, **113**, 014004 (2026)

Conformal prediction: scattering

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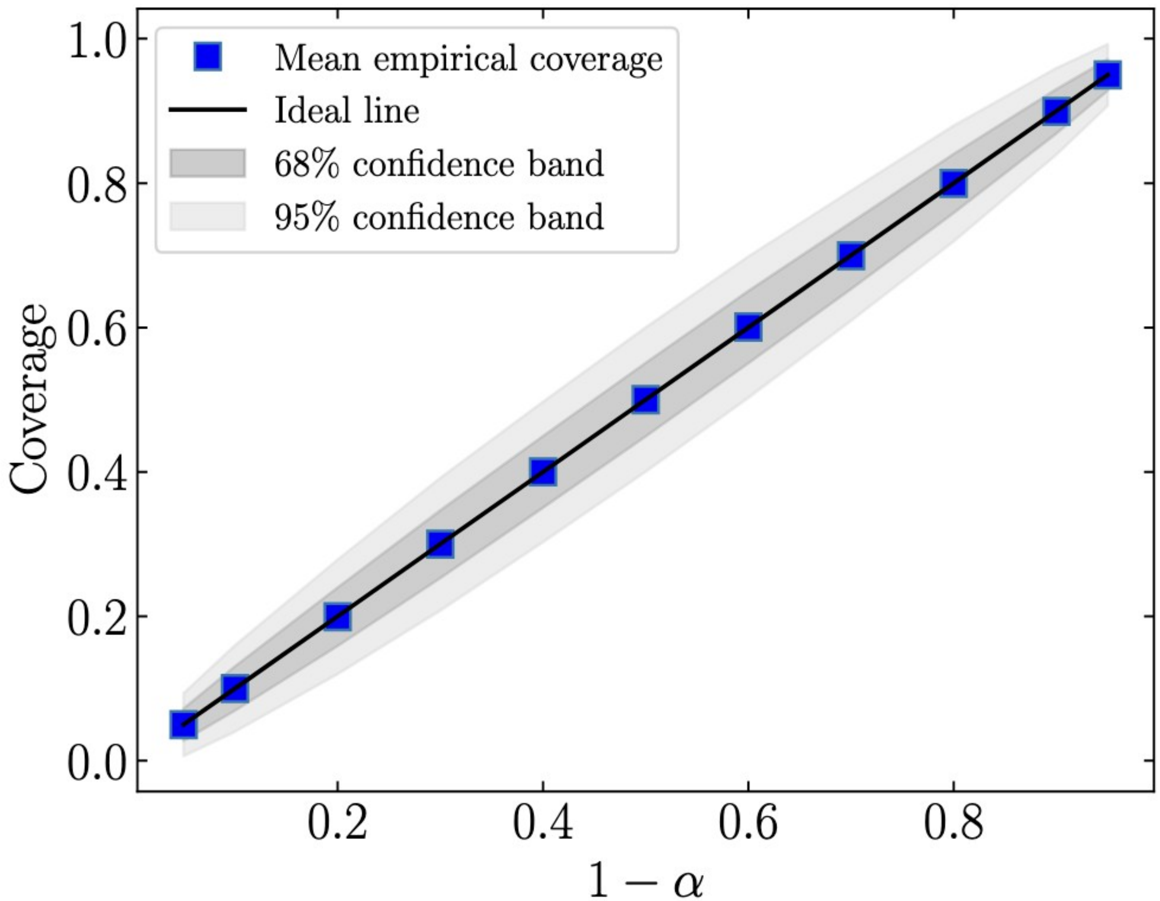
Application: BUQEYE pointwise model

- Two-nucleon total cross section at E = 50 MeV
- Empirical coverage over 4000 independent trials
- There is near-perfect alignment between empirical coverage and ideal line



H. Yousefi Dezdarani, R. Curry, and A. Gezerlis, Phys. Rev. C, **113**, 014004 (2026)

Conformal prediction: scattering

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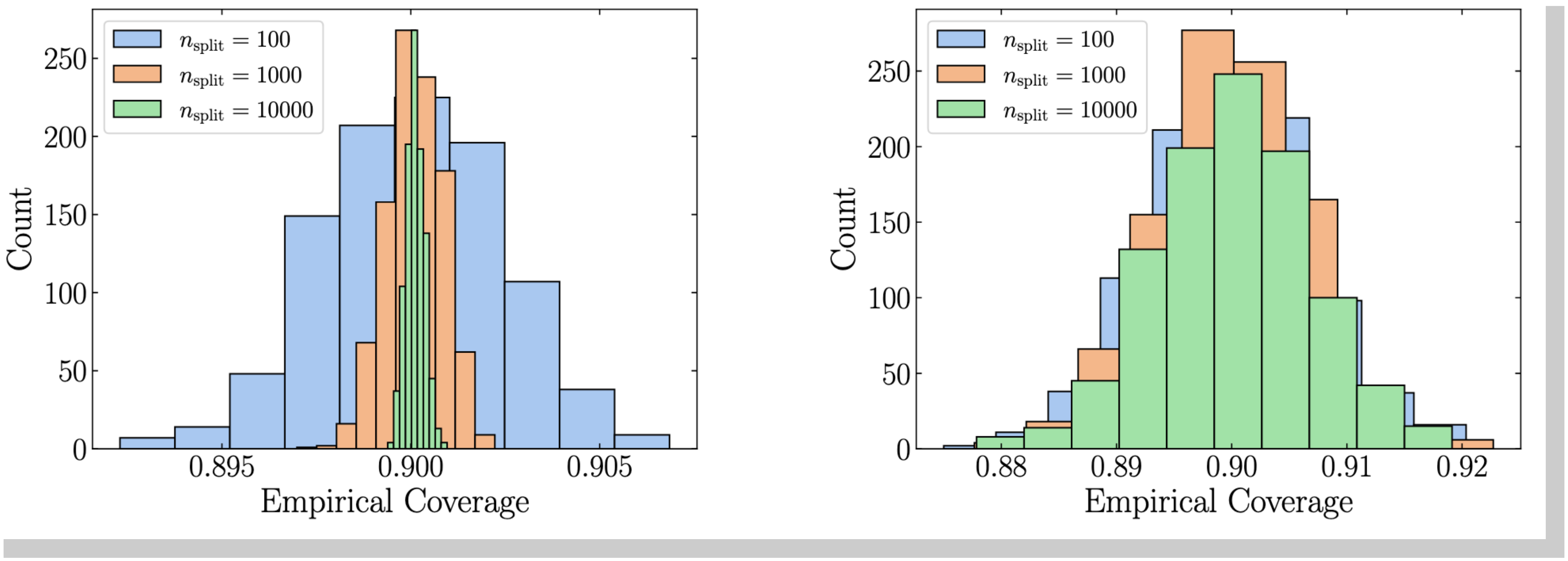
Application: BUQEYE Gaussian-process model

We drew posterior samples using the BUQEYE open-source code



conformal | Bayesian

H. Yousefi Dezdarani, R. Curry, and A. Gezerlis, Phys. Rev. C, **113**, 014004 (2026)

Conformal prediction: neutron stars

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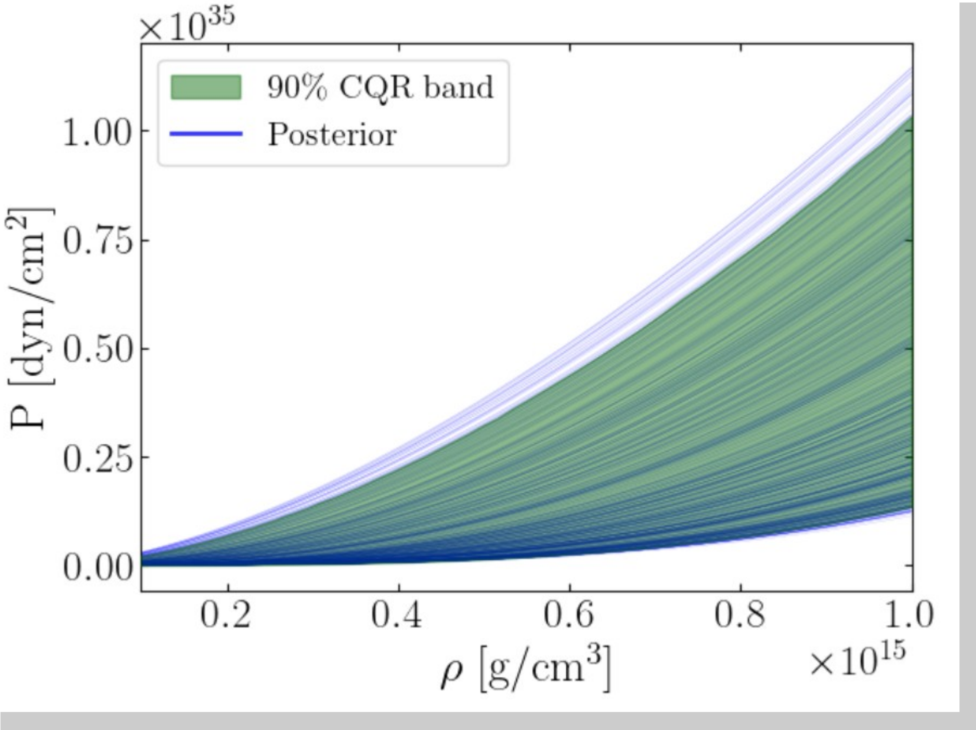
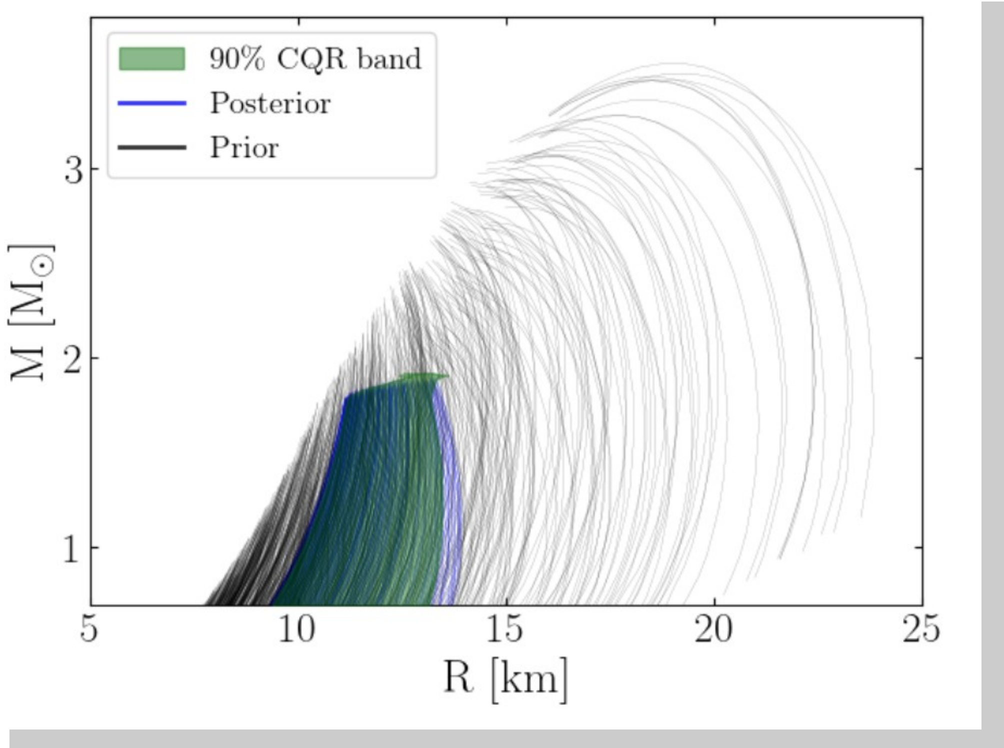
Application: TOV equations for polytrope

$$\frac{dP(r)}{dr} = -\frac{G\,m(r)\,\rho(r)}{r^2}\left[1+\frac{P(r)}{\rho(r)c^2}\right]\left[1+\frac{4\pi r^3 P(r)}{m(r)c^2}\right]\left[1-\frac{2Gm(r)}{c^2 r}\right]^{-1}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$





H. Yousefi Dezdarani, R. Curry, C. L. Armstrong, and A. Gezerlis, *in preparation*

Conformal prediction: neutron stars

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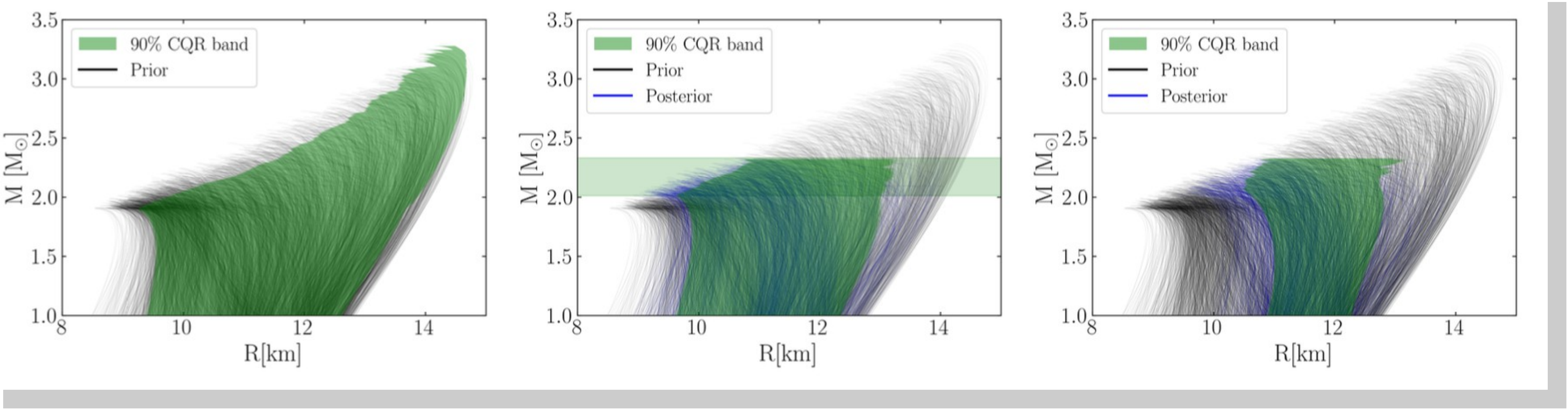
Application: multimessenger analysis (NMMA collaboration)



Post-processing carried out using individual EOS samples

T. Dietrich *et al*, Science **370**, 1450 (2020)

H. Yousefi Dezdarani, R. Curry, C. L. Armstrong, and A. Gezerlis, *in preparation*

Conformal prediction: neutron stars

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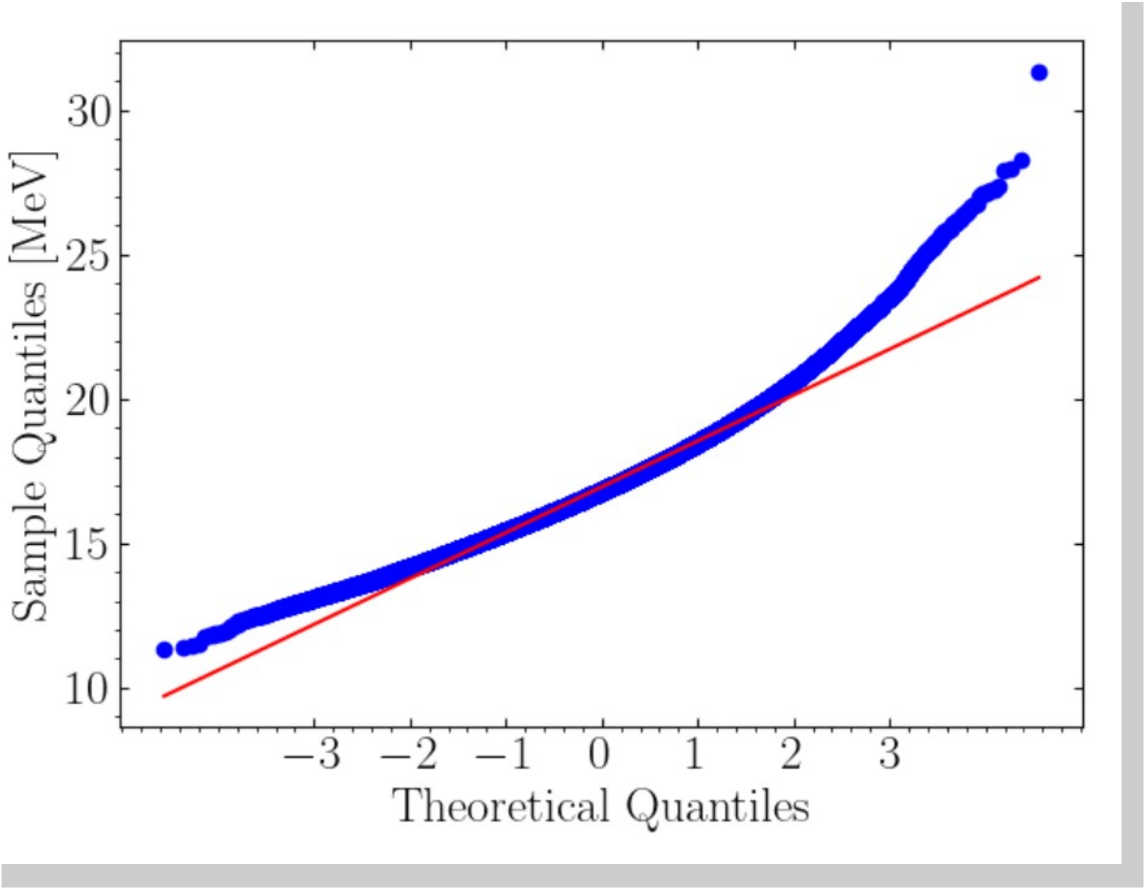
Application: QMC neutron matter equation of state (1)

- Deviations from the diagonal reflect a non-normal distribution
- Non-gaussianity motivates use of distribution-free method



C. L. Armstrong *et al*, Phys. Rev. Lett. **135**, 142501 (2025)

H. Yousefi Dezdarani, R. Curry, C. L. Armstrong, and A. Gezerlis, *in preparation*

Conformal prediction: neutron stars

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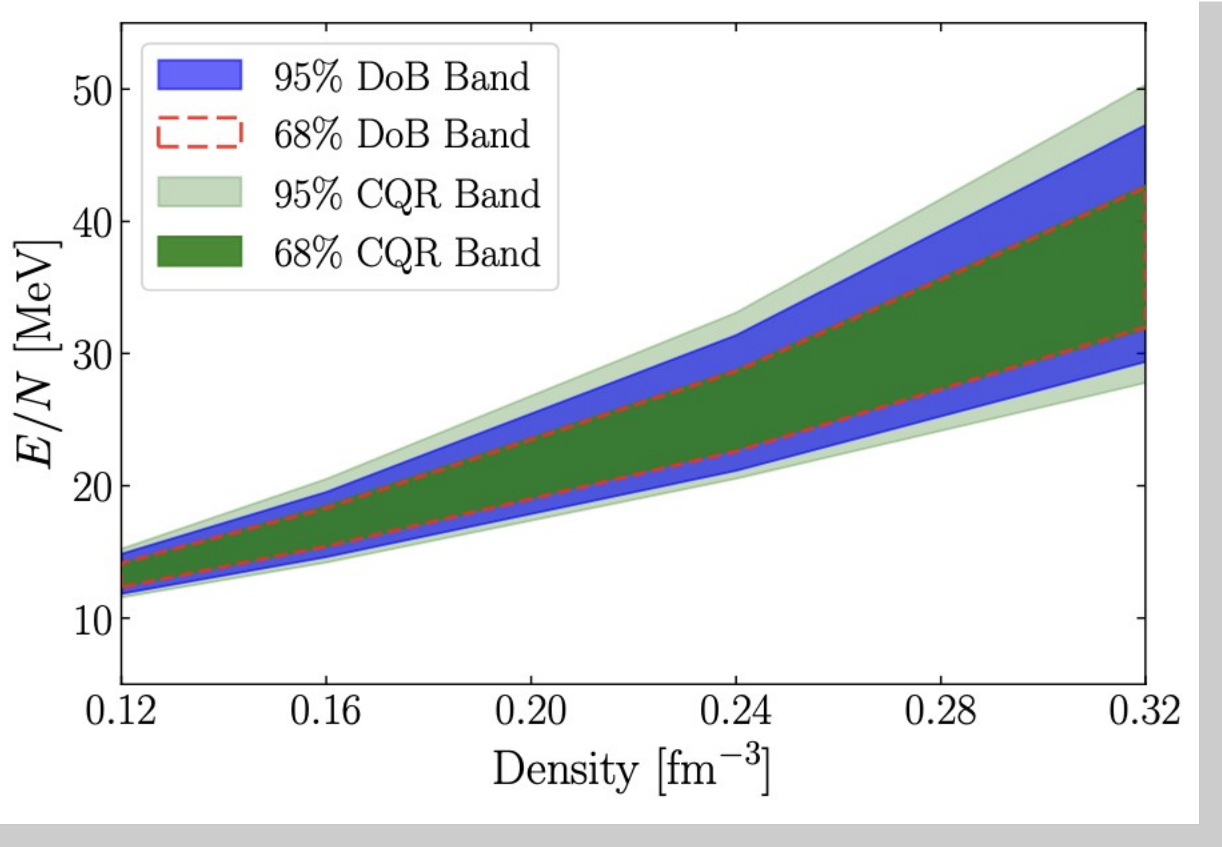
Application: QMC neutron matter equation of state (2)



DoB = Bayesian
CQR = conformal

Conformal prediction interval wider than Bayesian one

C. L. Armstrong *et al*, Phys. Rev. Lett. **135**, 142501 (2025)

H. Yousefi Dezdarani, R. Curry, C. L. Armstrong, and A. Gezerlis, *in preparation*

Conclusions

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- We used PMM and EC emulators together with QMC studies of light nuclei
- We used chiral Effective Field Theory interactions in neural-network studies of light nuclei
- We applied conformal prediction to nuclear scattering and neutron star EOSs

Acknowledgments

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Funding

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CANADA FOUNDATION FOR INNOVATION | FONDATION CANADIENNE POUR L'INNOVATION

Ontario
MINISTRY OF RESEARCH AND INNOVATION
MINISTÈRE DE LA RECHERCHE ET DE L'INNOVATION

Collaborators

Guelph

- Ryan Curry
- Habib Yousefi Dezdarani

Los Alamos

- Rahul Somasundaram
- Ingo Tews

Michigan State

- Cassandra Armstrong

Texas A&M

- Jeremy Holt
- Pengsheng Weng

TU Darmstadt

- Kai Hebeler
- Achim Schwenk