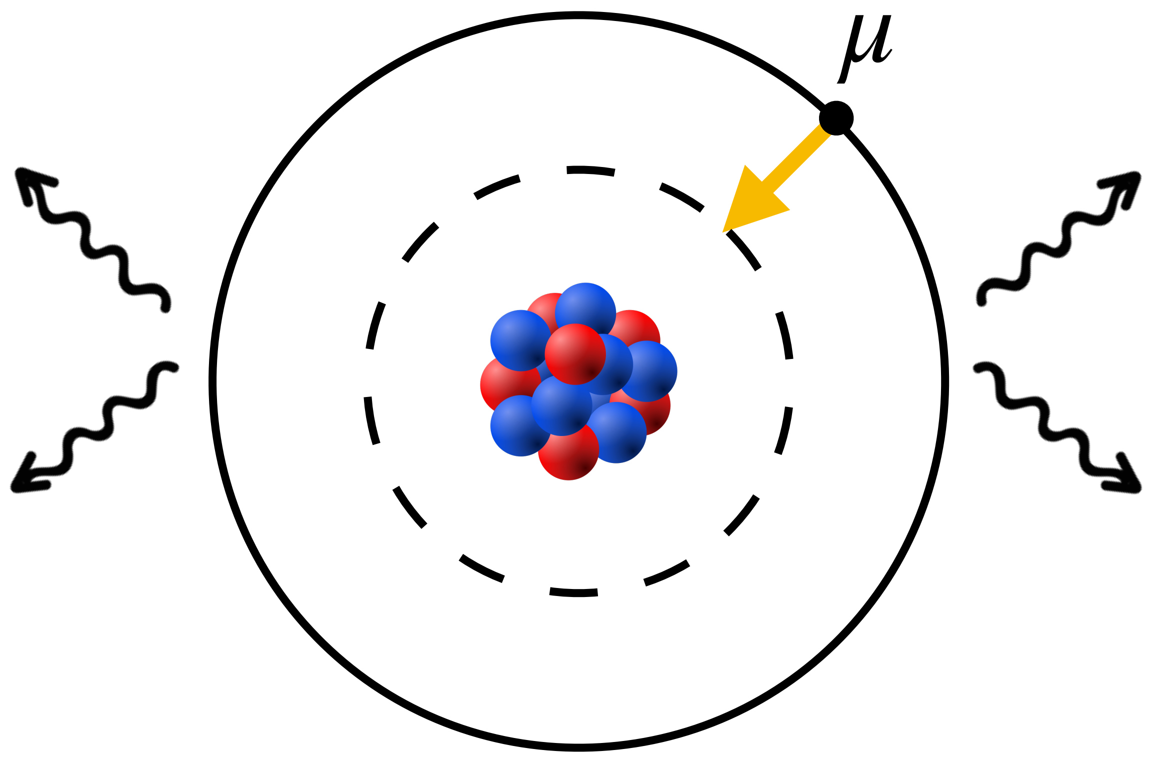
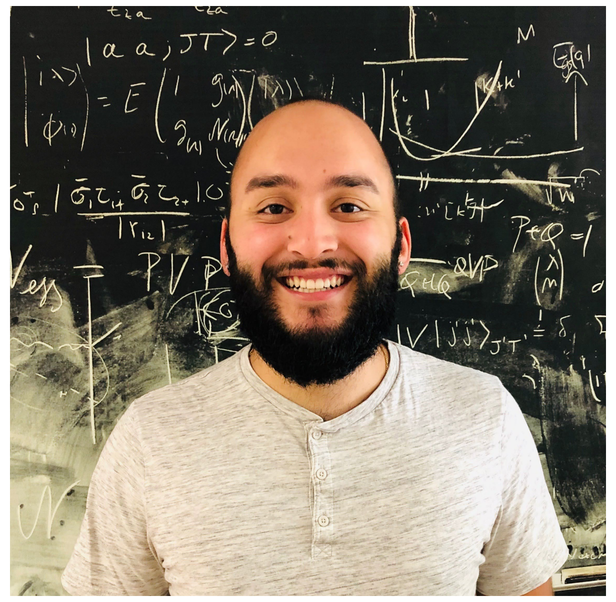
Ab initio nuclear corrections to light muonic atoms

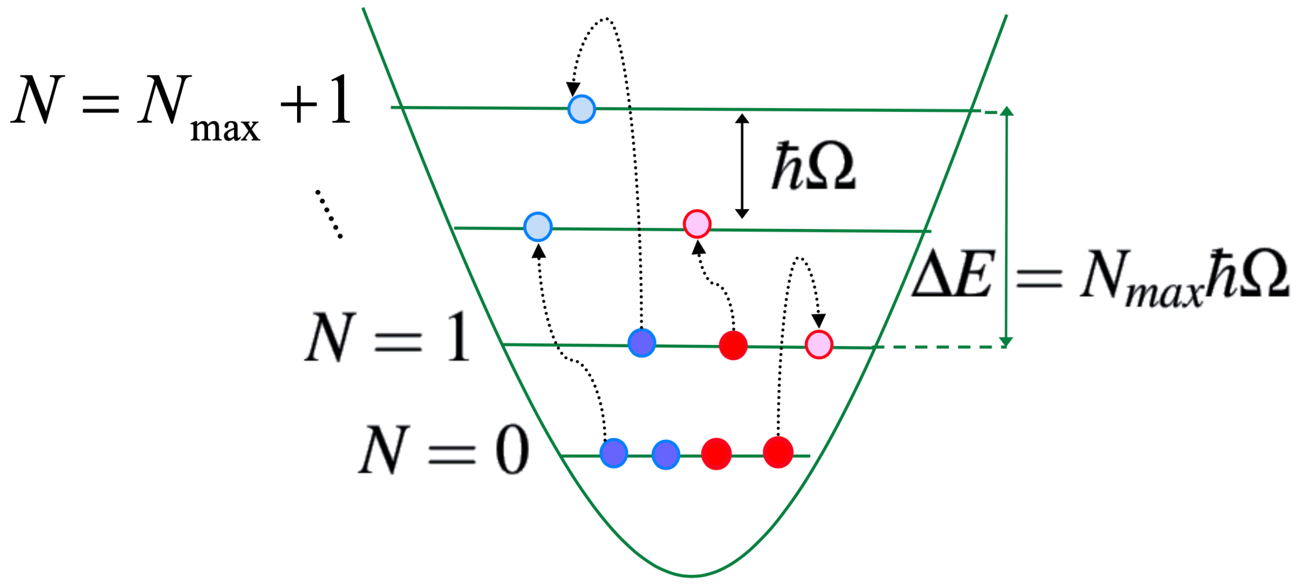
No-Core Shell Model calculations of polarizability for $^{6}\mathrm{Li}$ and $^{7}\mathrm{Li}$ atoms



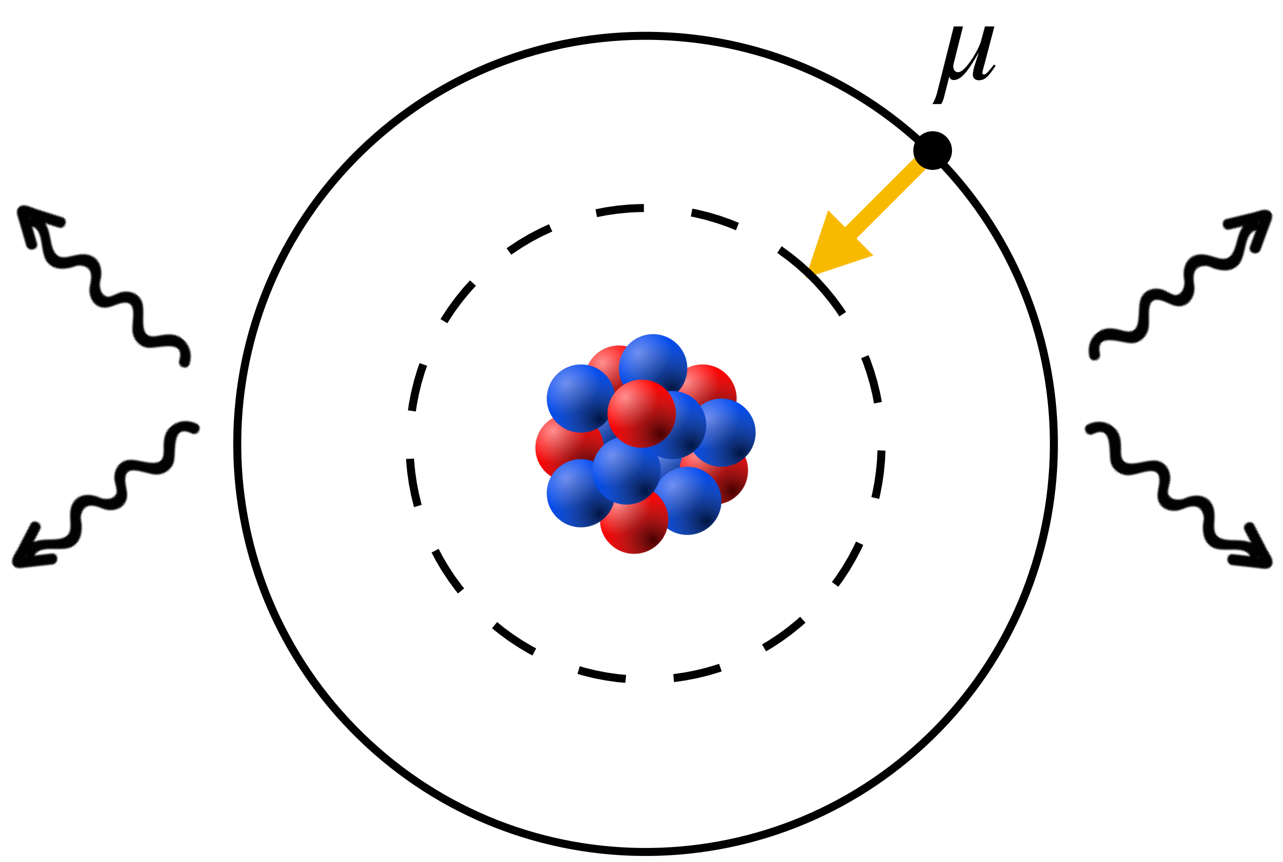

Michael Gennari


Diego Araujo Nájera


Petr Navratil



Muonic atoms as a precision probe



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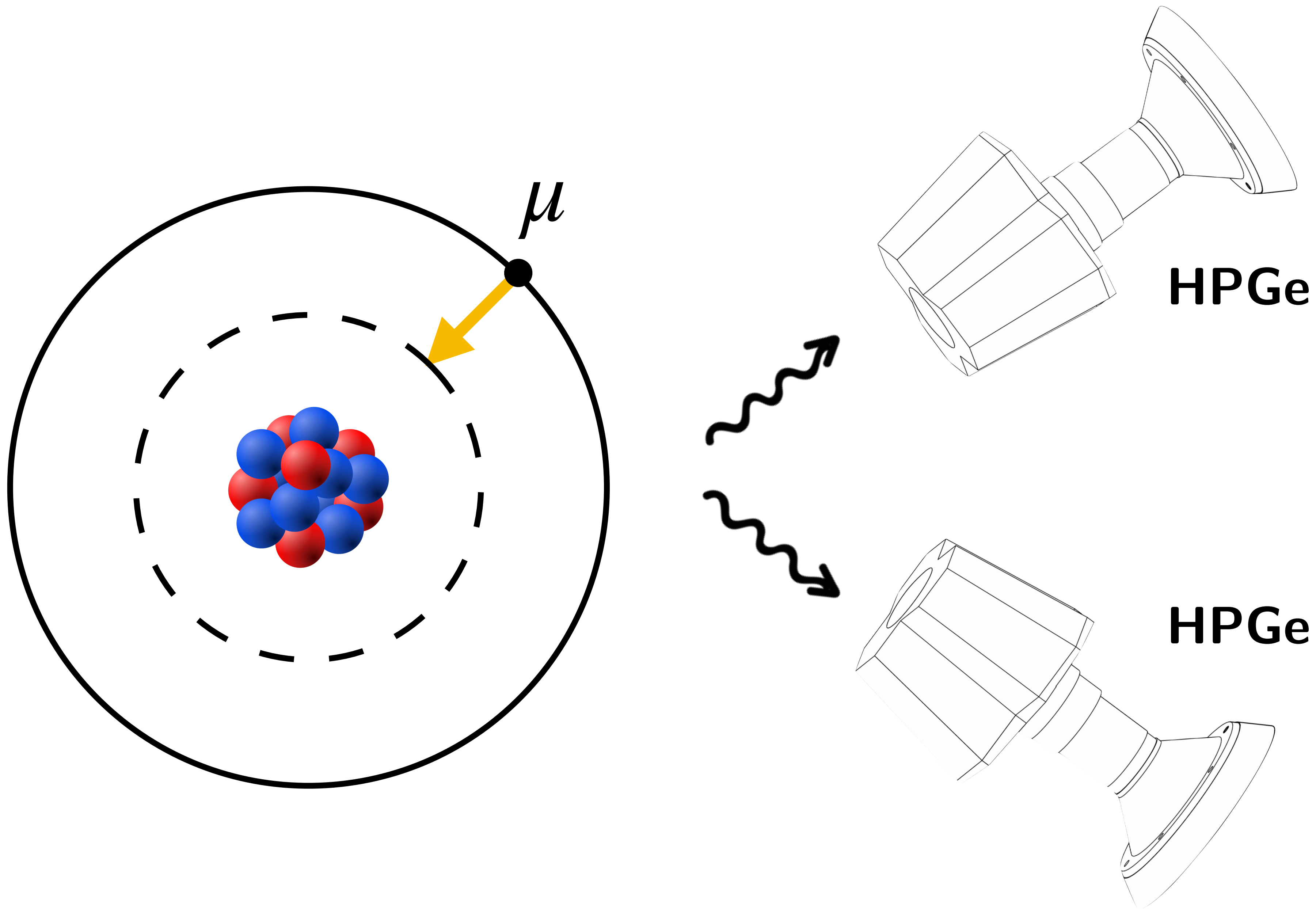
Observing muonic atoms



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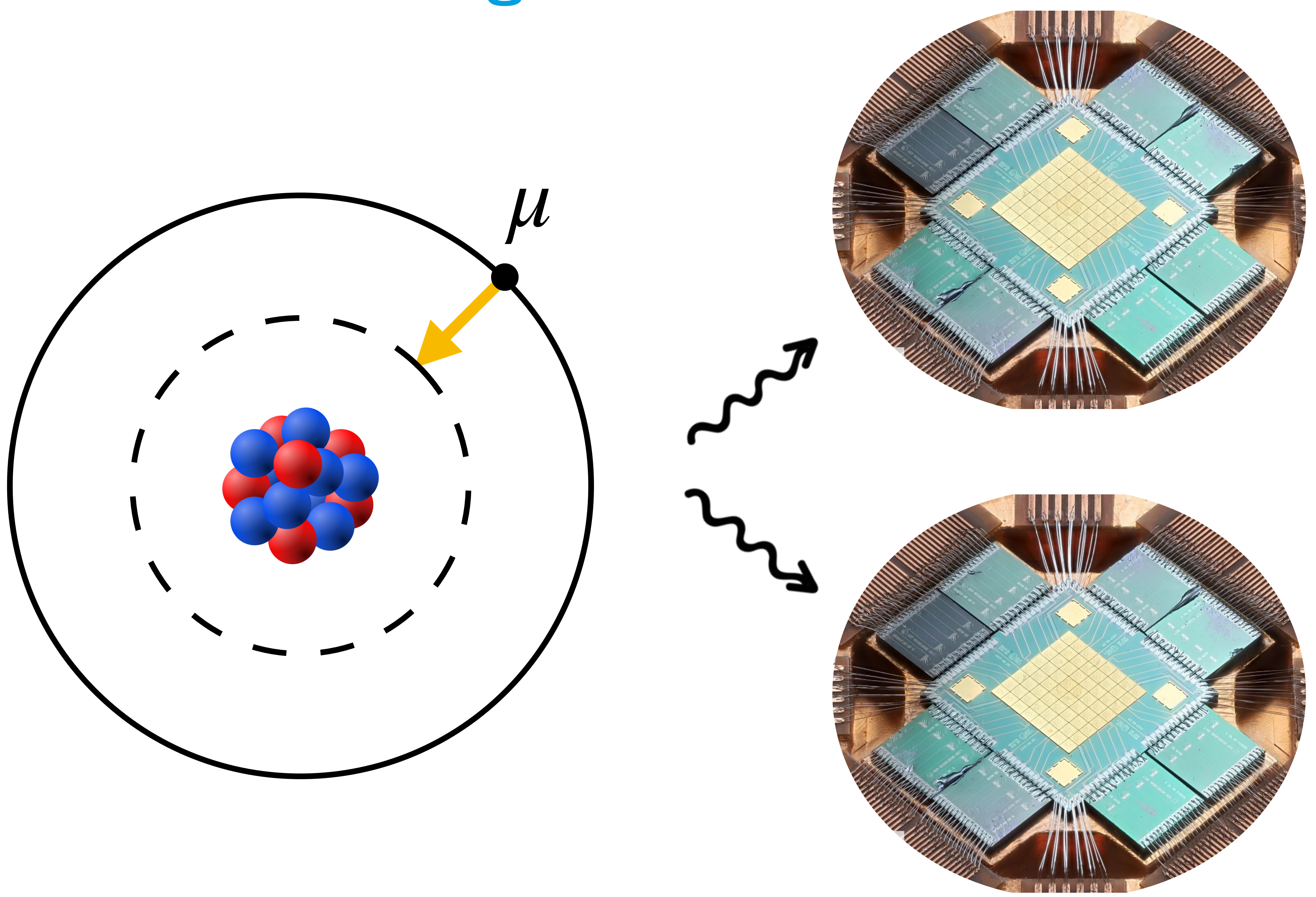

[Unger et al. J. Low Temp. Phys. (2024)]

QUARTET collaboration

- Improving energy resolution with MMC
  - Quantum sensor detector to reach low-Z nuclei
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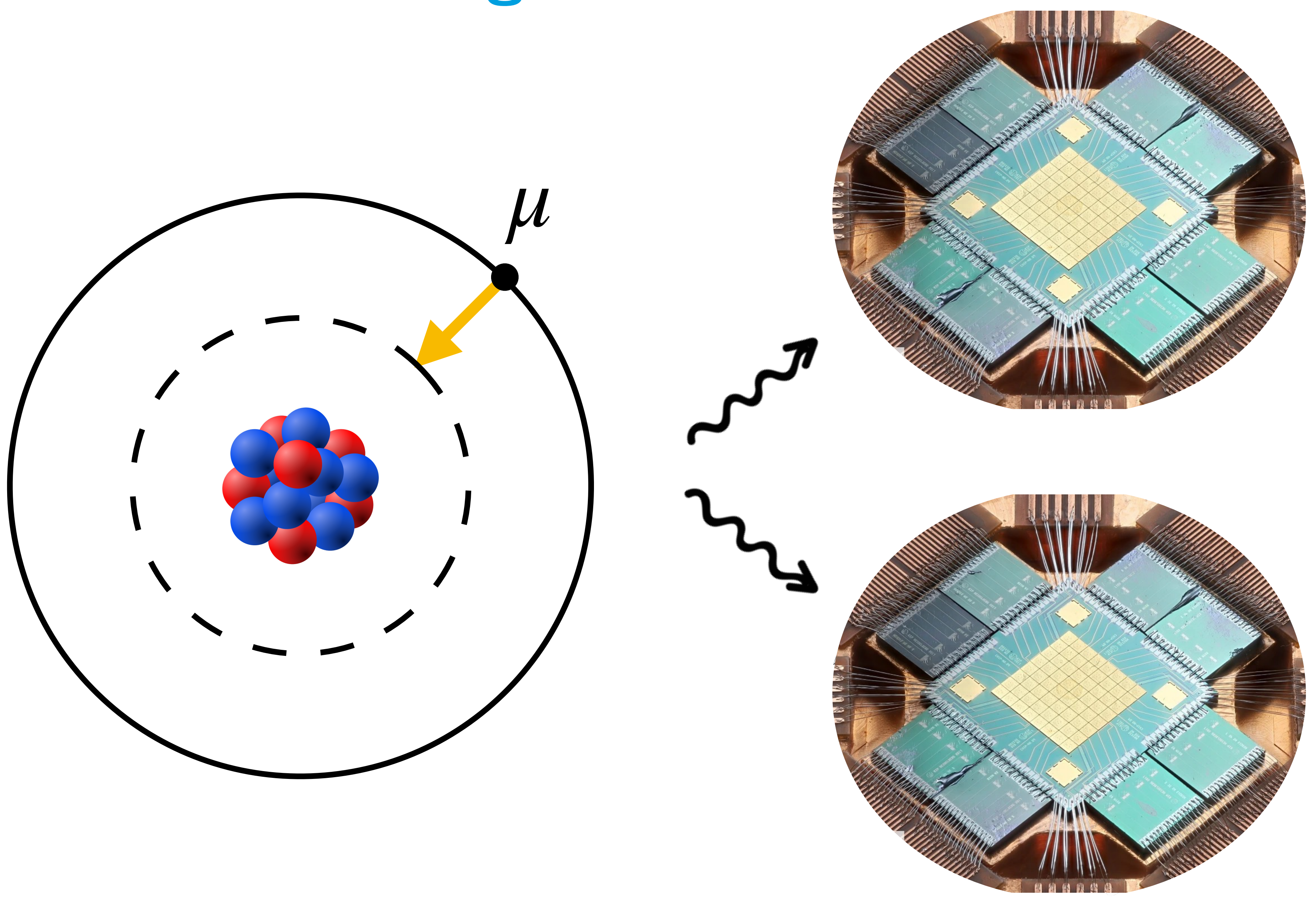

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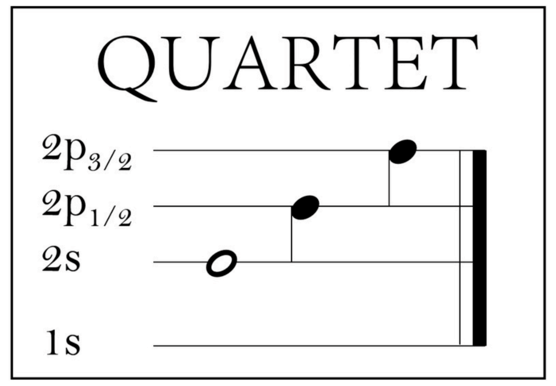

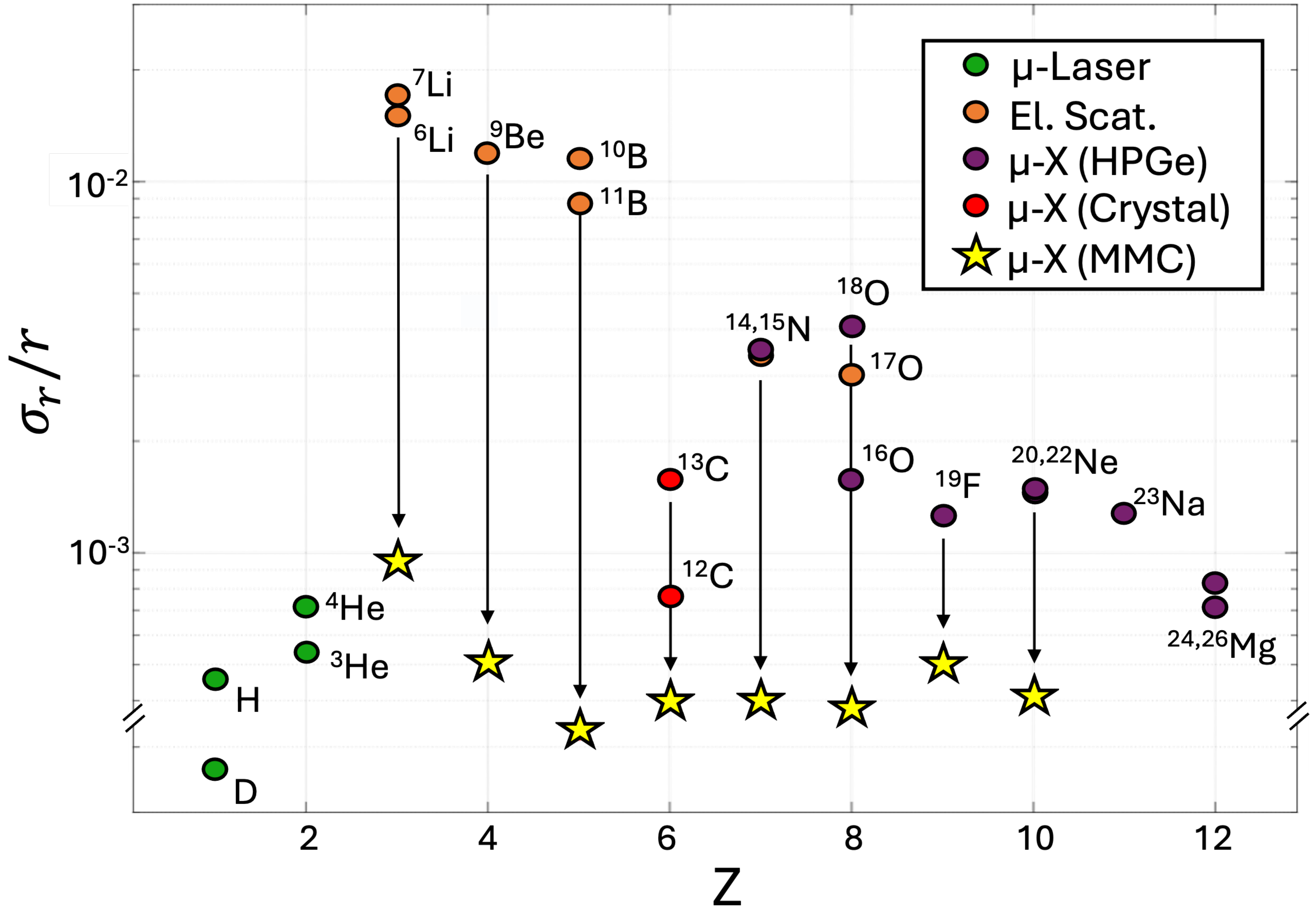

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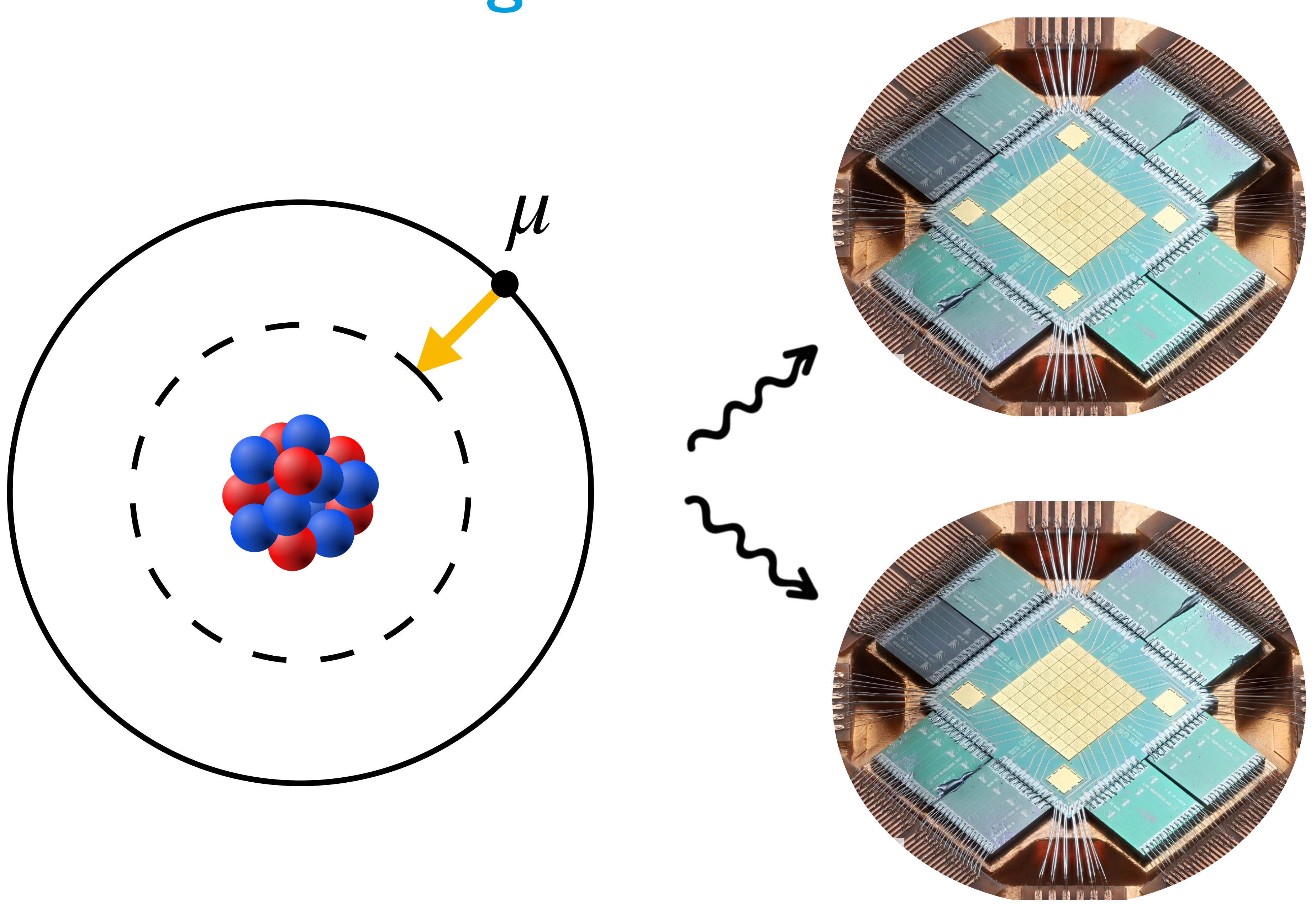

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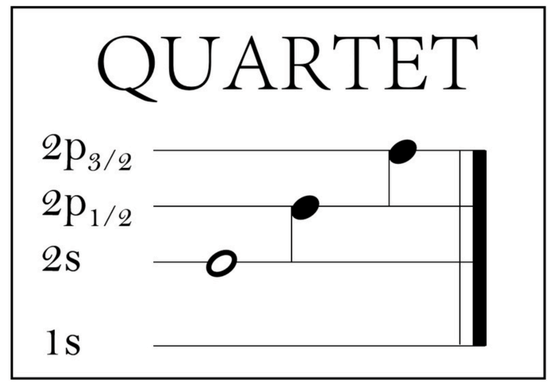

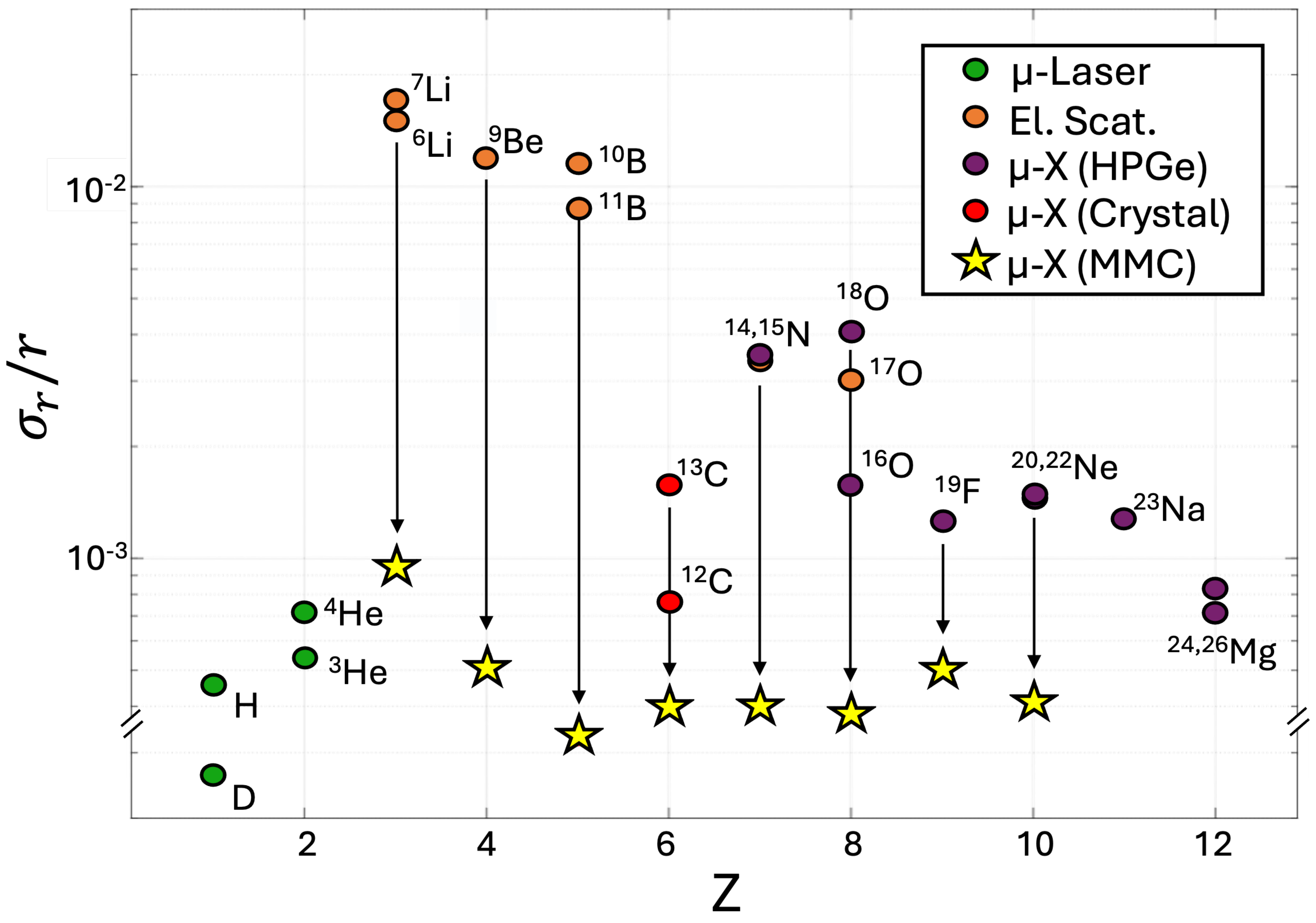

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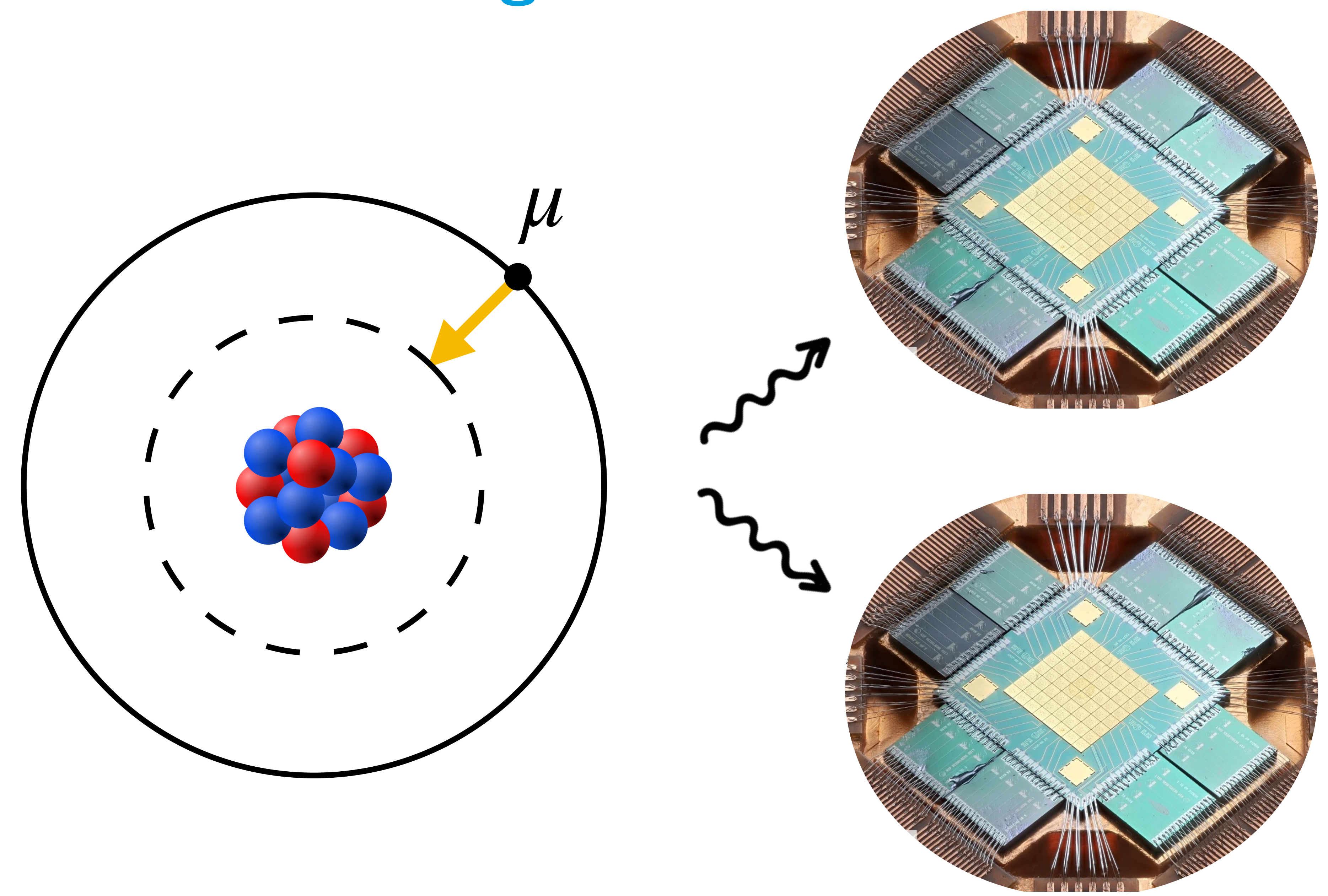

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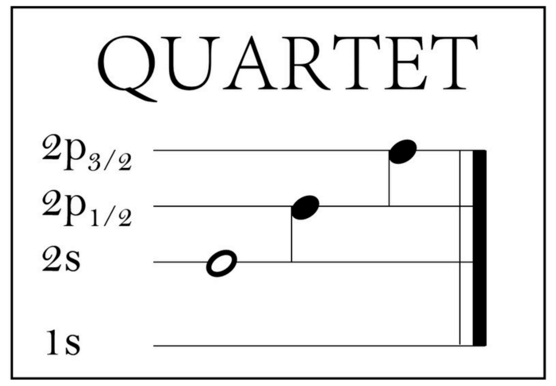

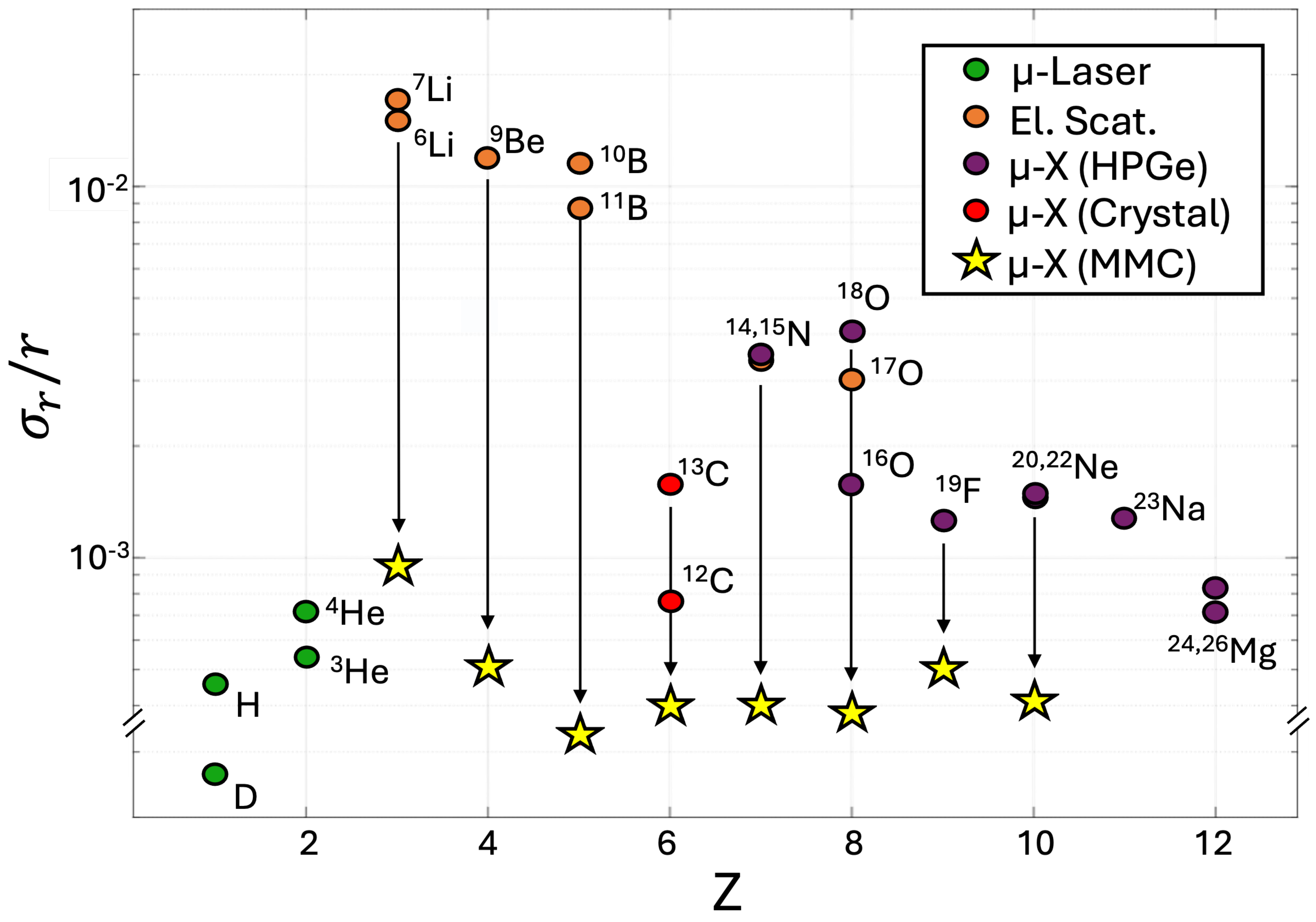

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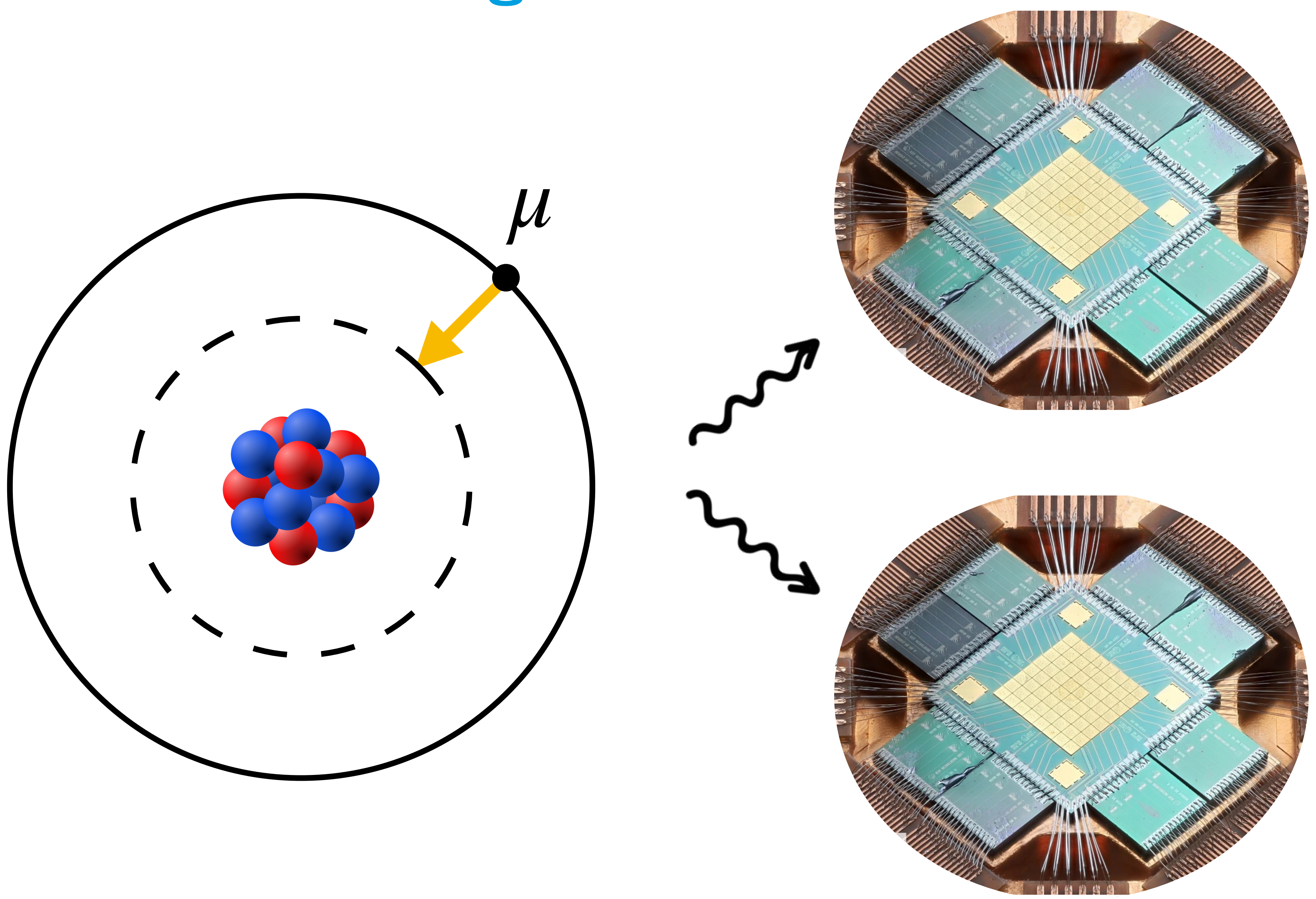

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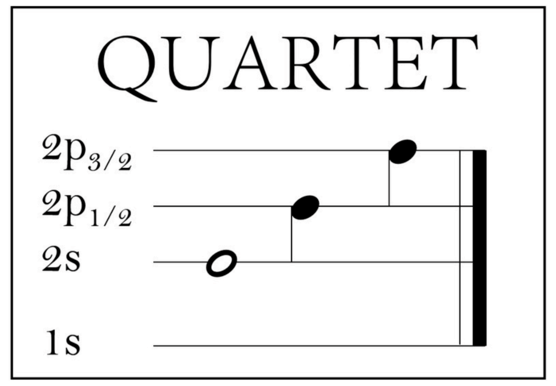

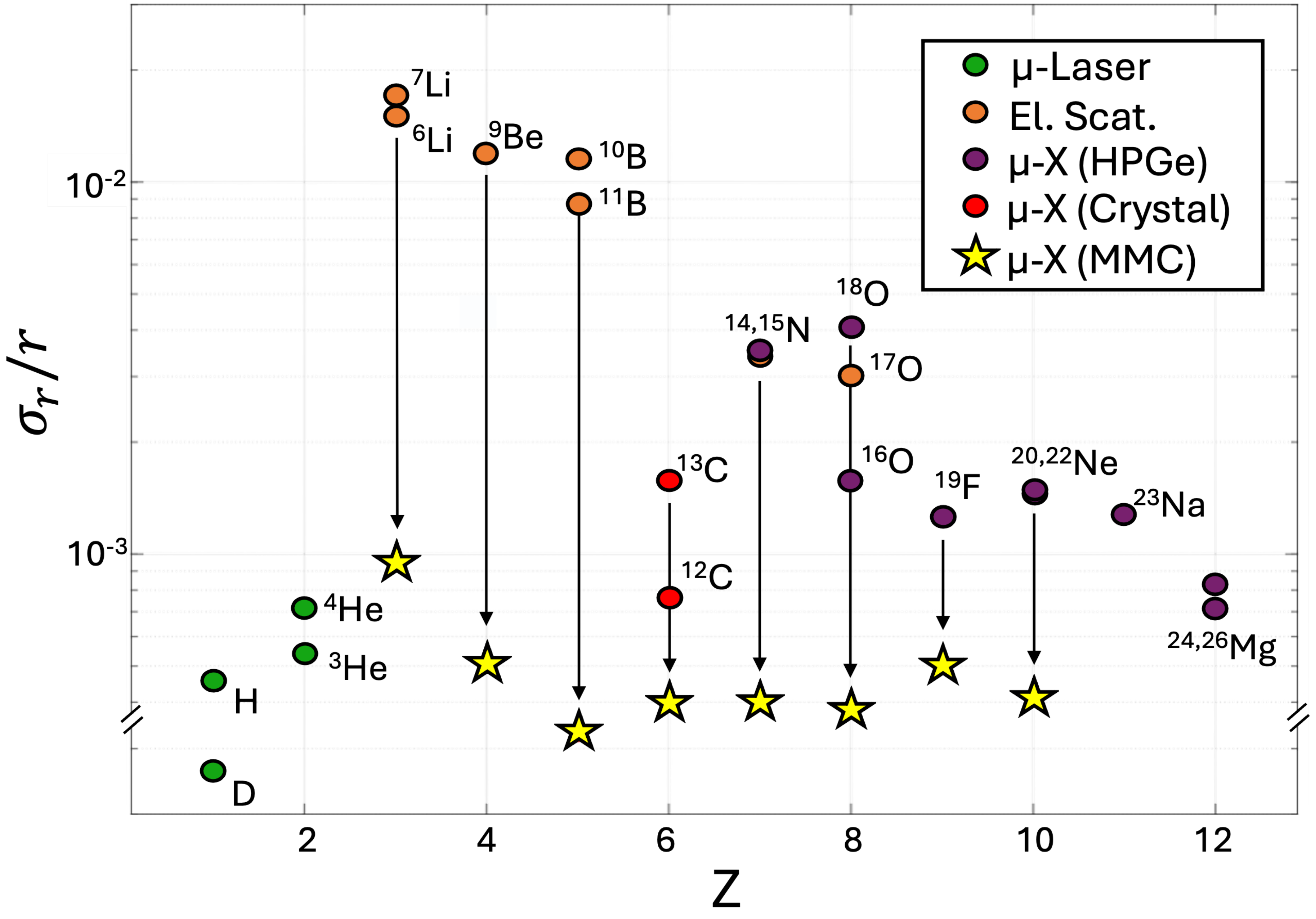

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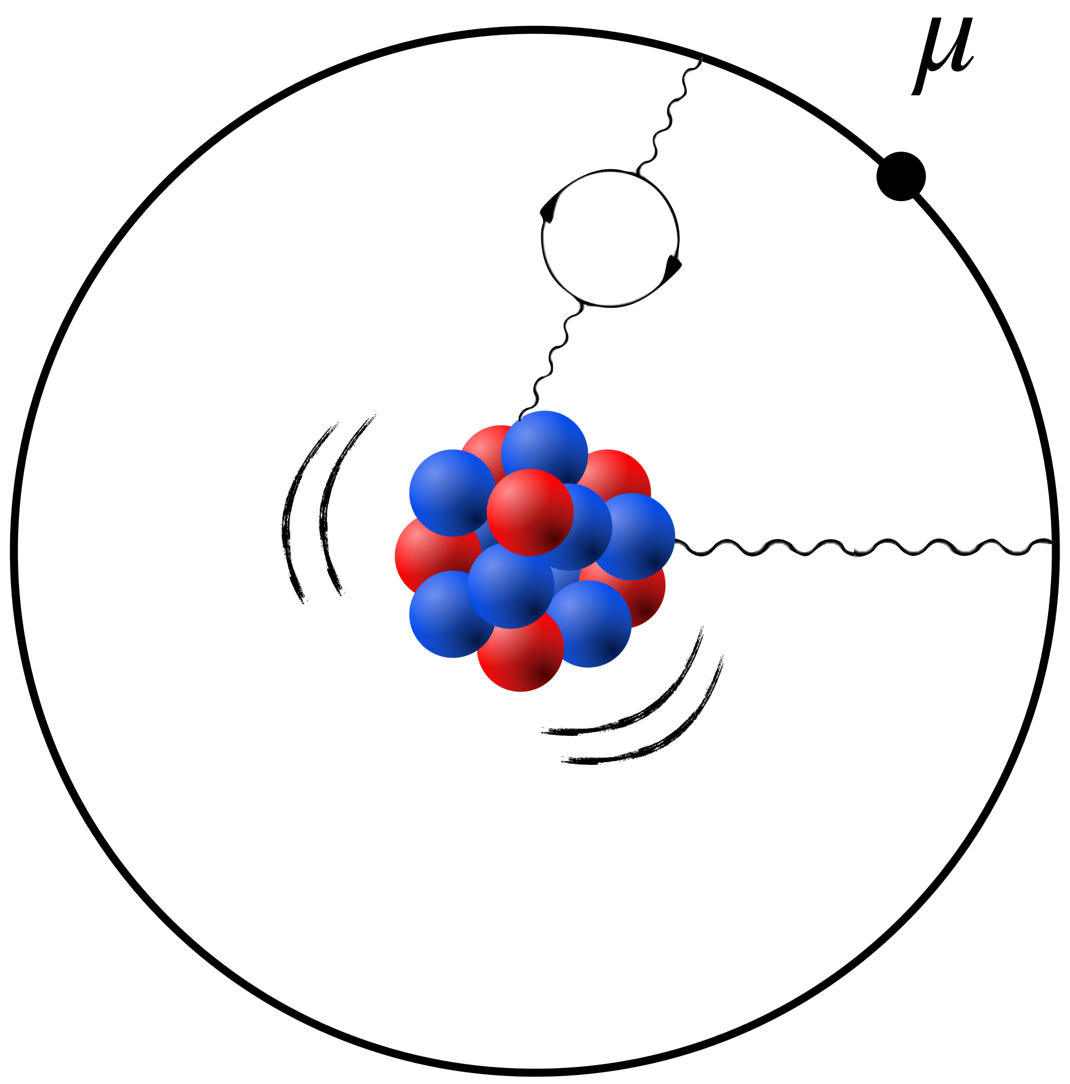
Beyond Nuclear: δ_C estimation for $|V_{ud}|$ extraction

[V. Katyal et al. PRA (2025)] [C.-Y. Seng, M. Gorchtein, PLB (2023)]
[B. Ohayon, Atomic Data and Nuclear Data Tables (2025)]

Generating effective potentials



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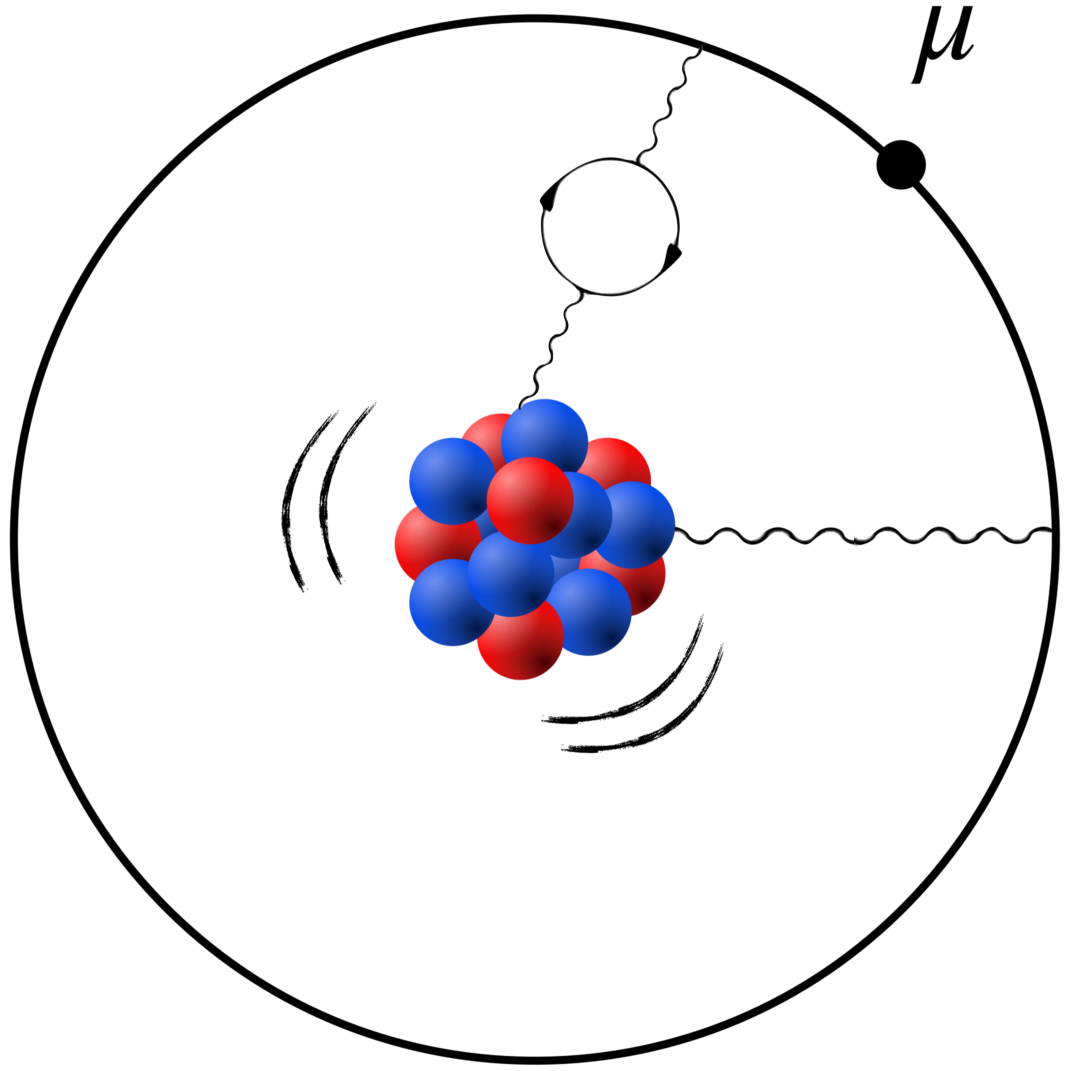

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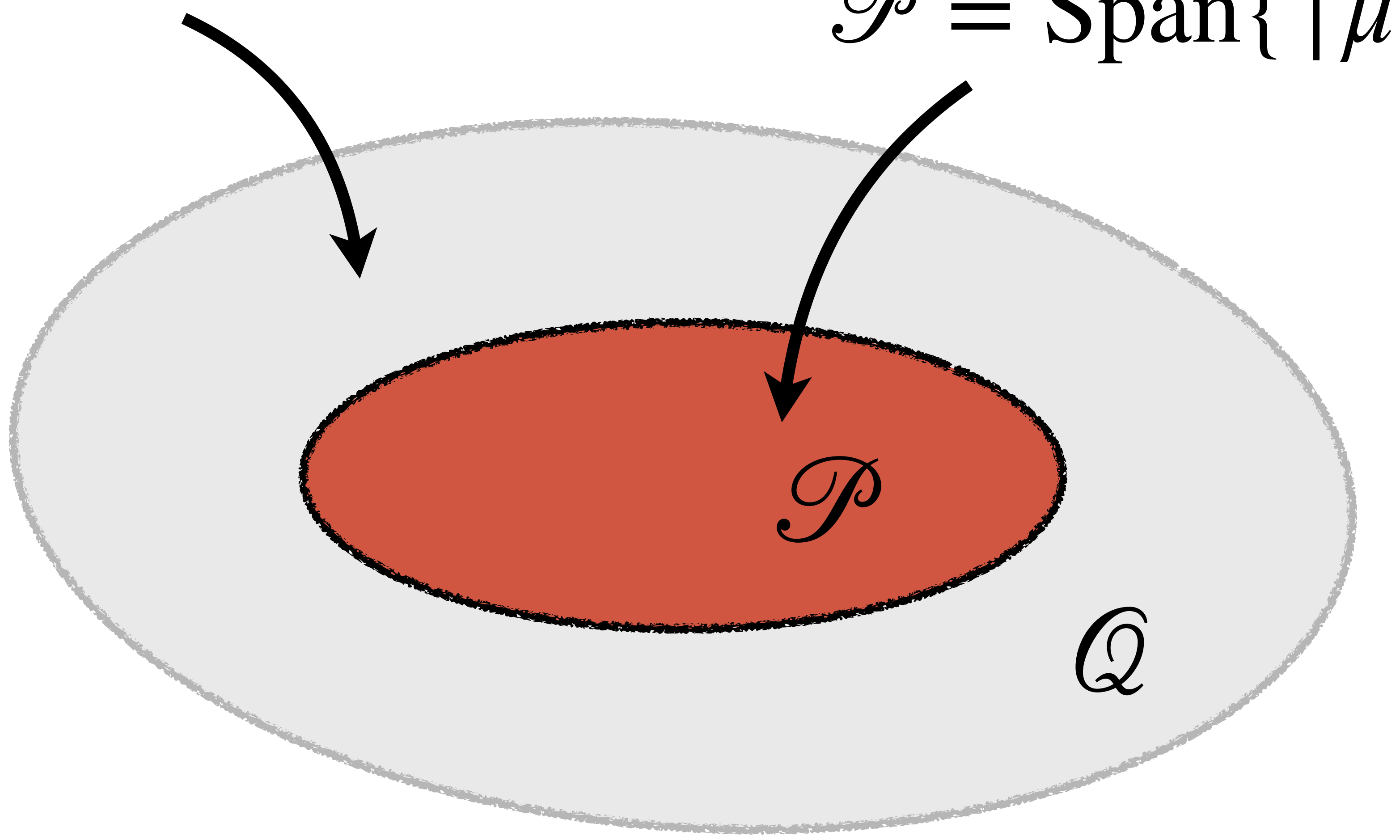
$$\mathscr{P} = \mathrm{Span}\{\,|\mu\rangle\} \otimes |\Psi_0\rangle$$



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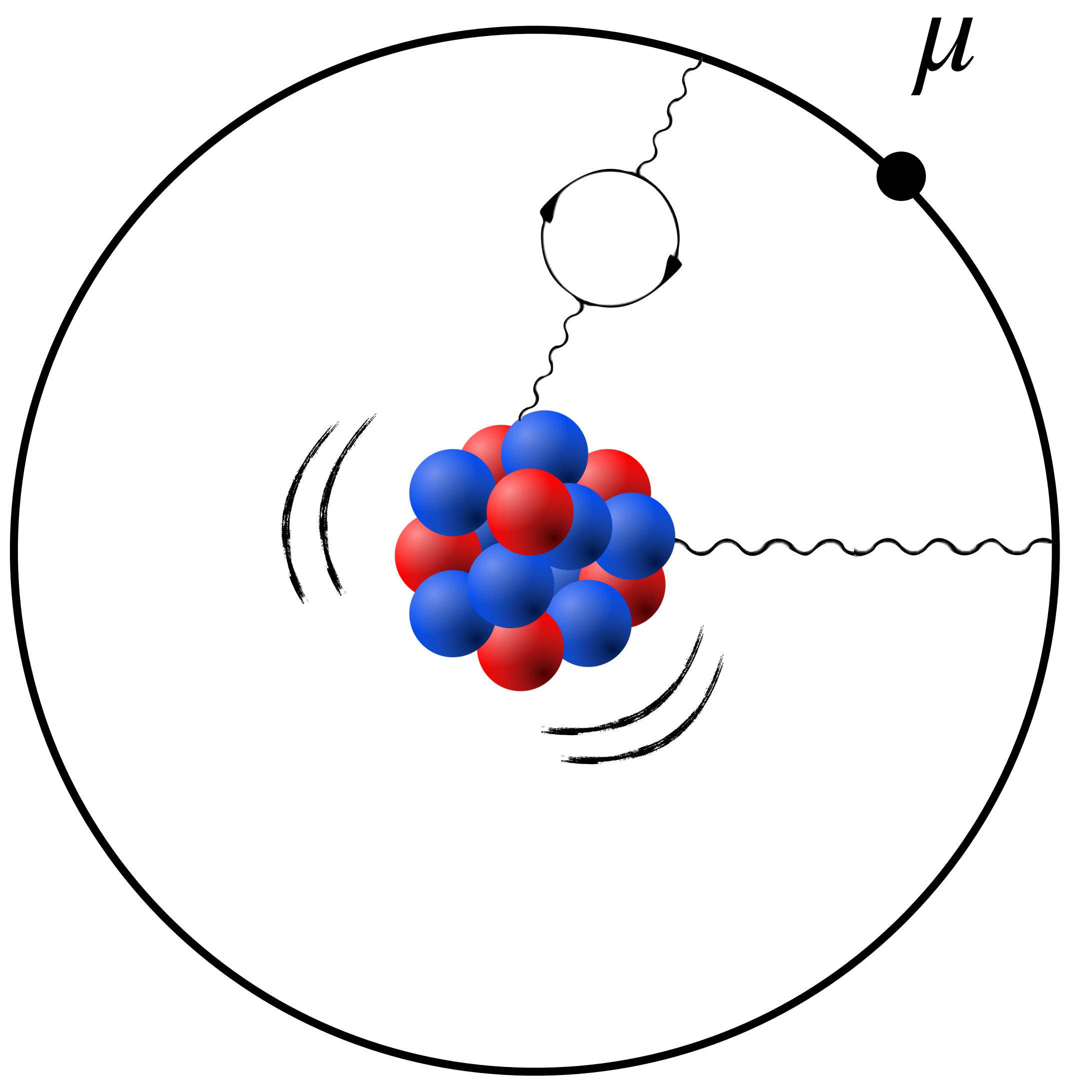
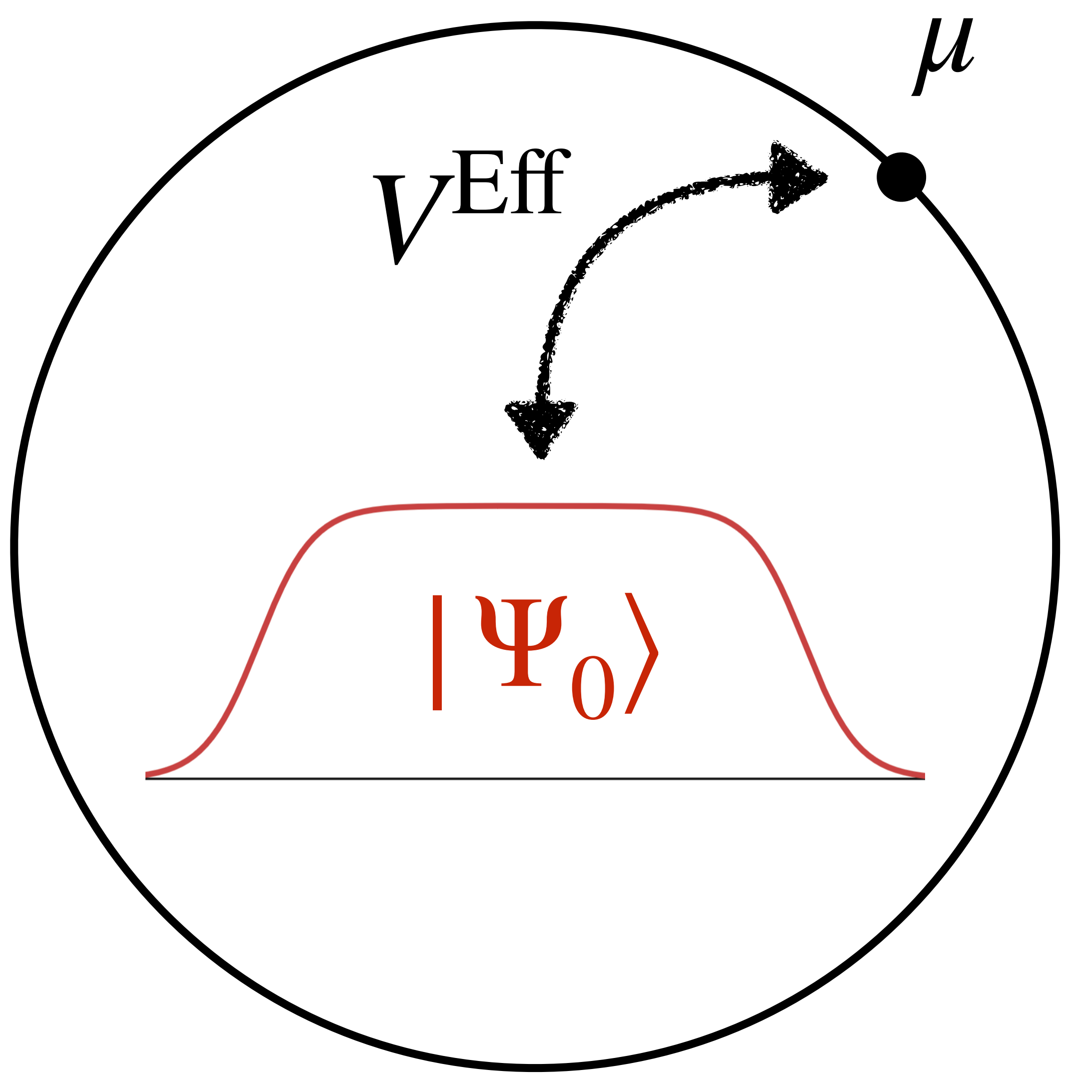


$$V_{\mu}^{\mathrm{Eff}} = V_{\mu N}^{\mathscr{P}} + \Delta V_{\mu N}$$

[Borie and Rinker, RMP (1982)]



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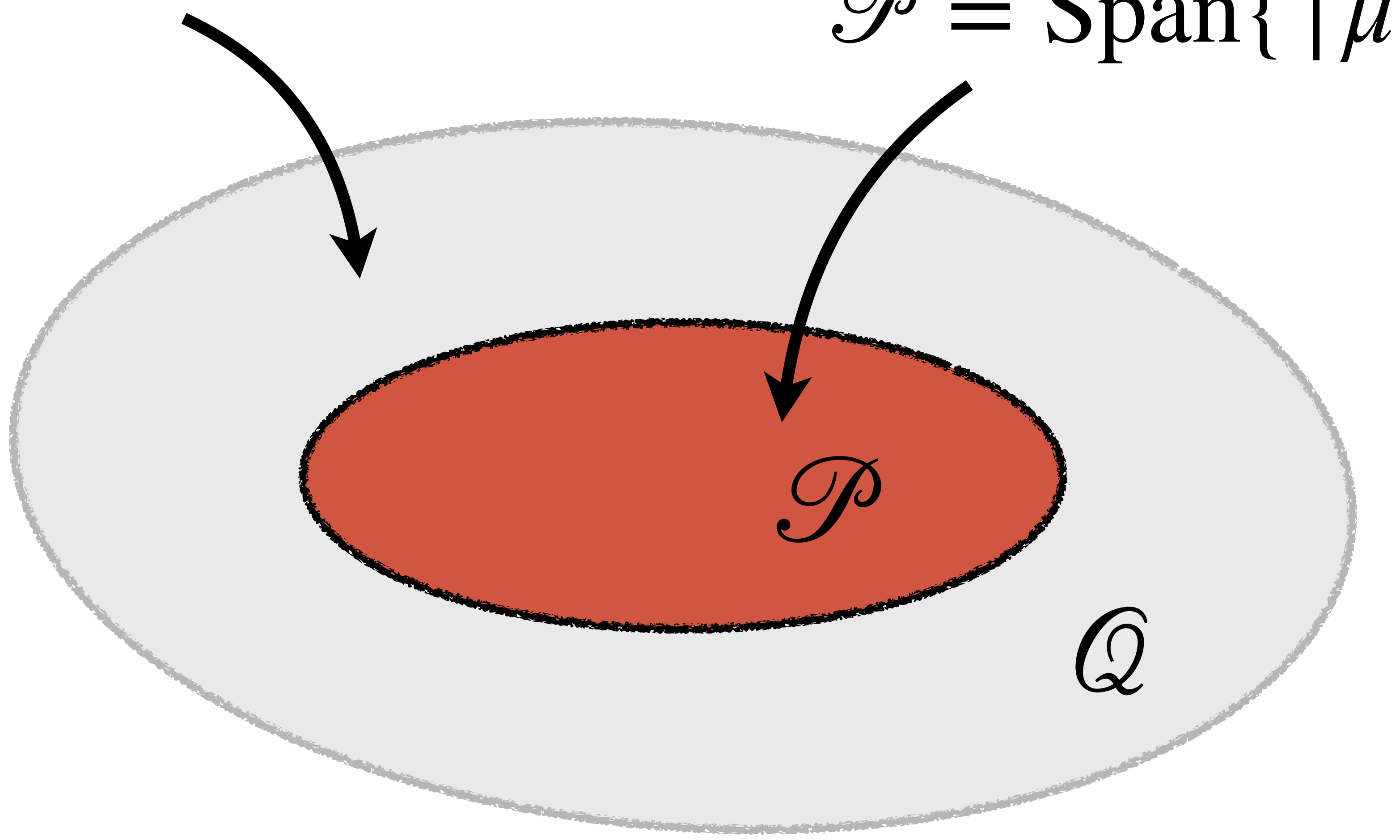

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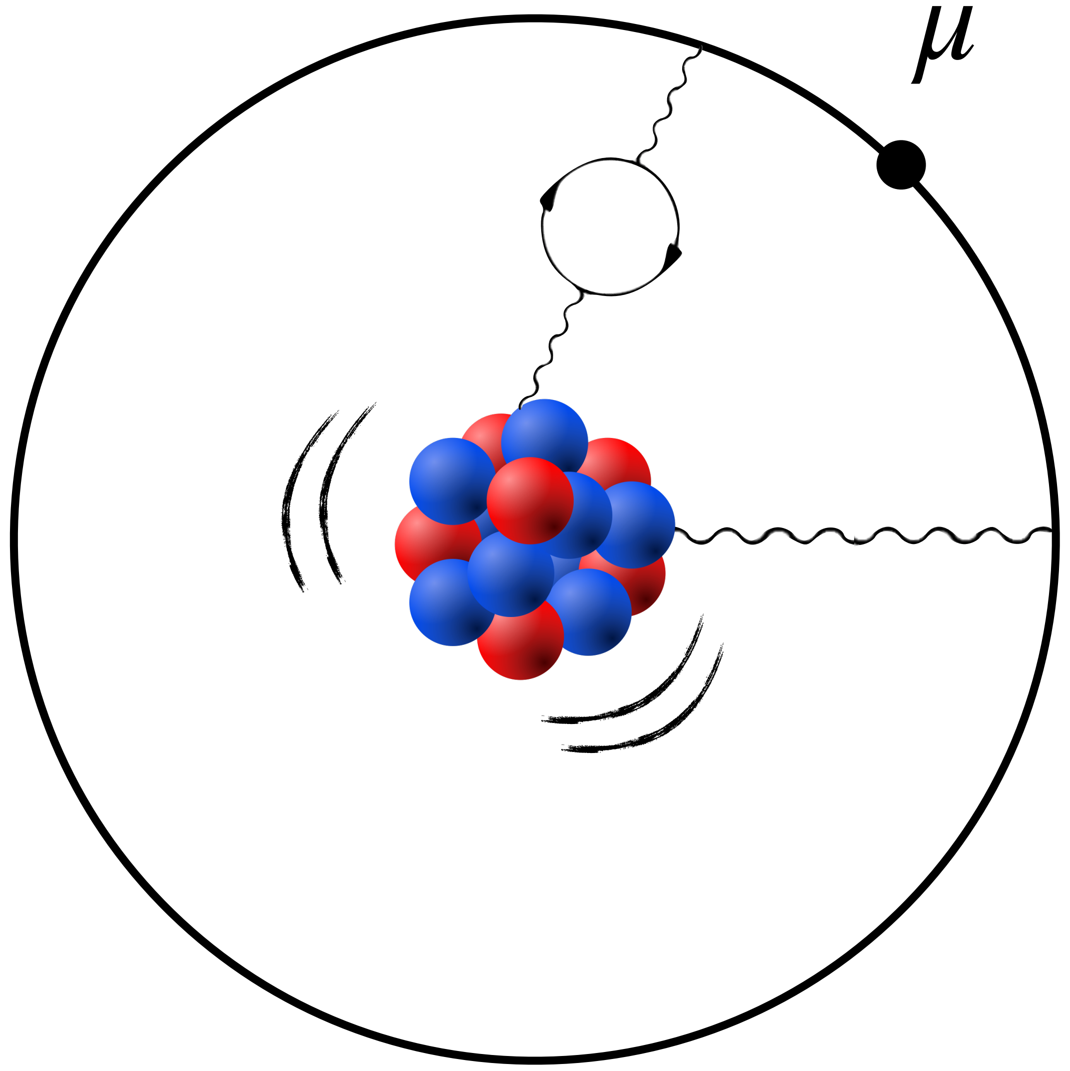

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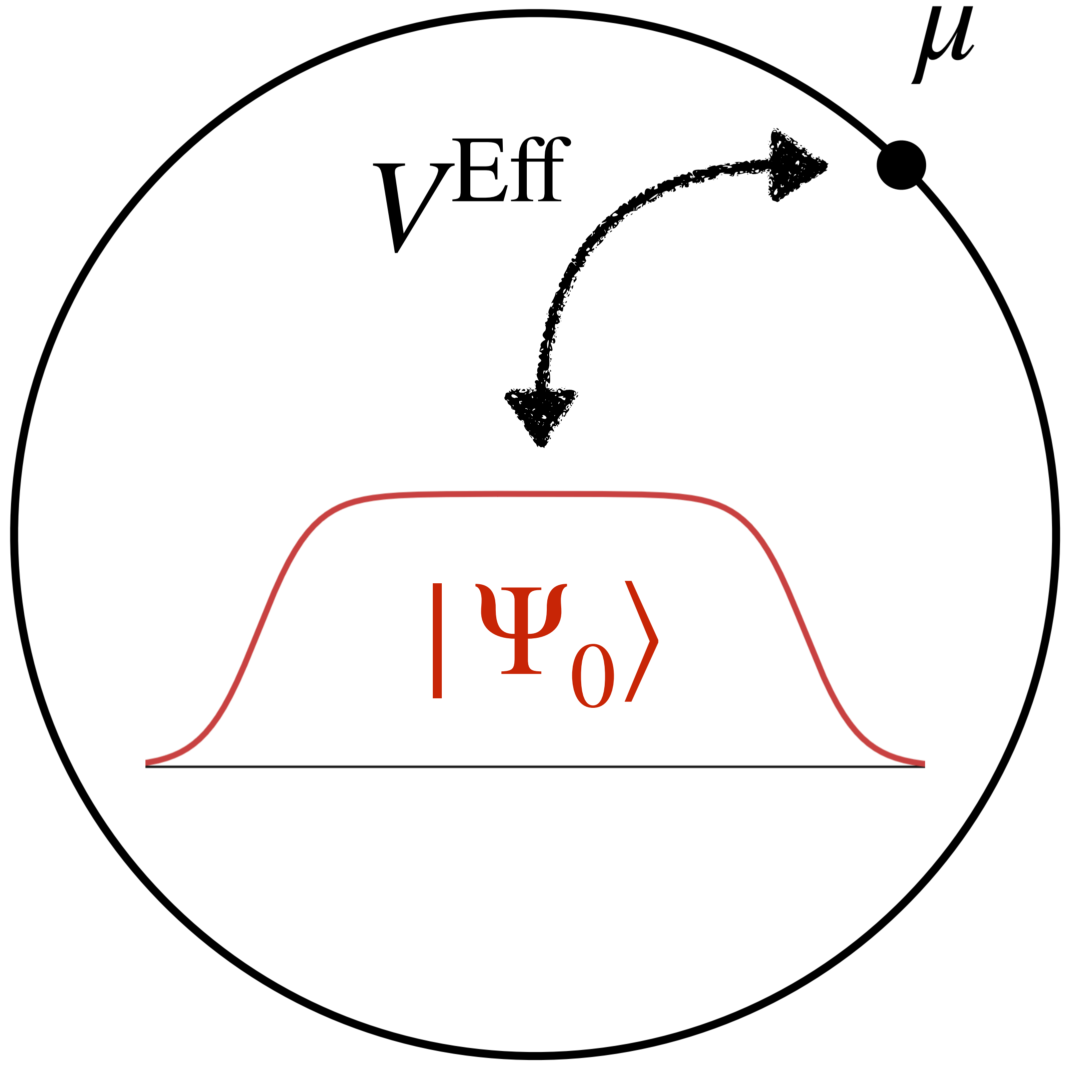

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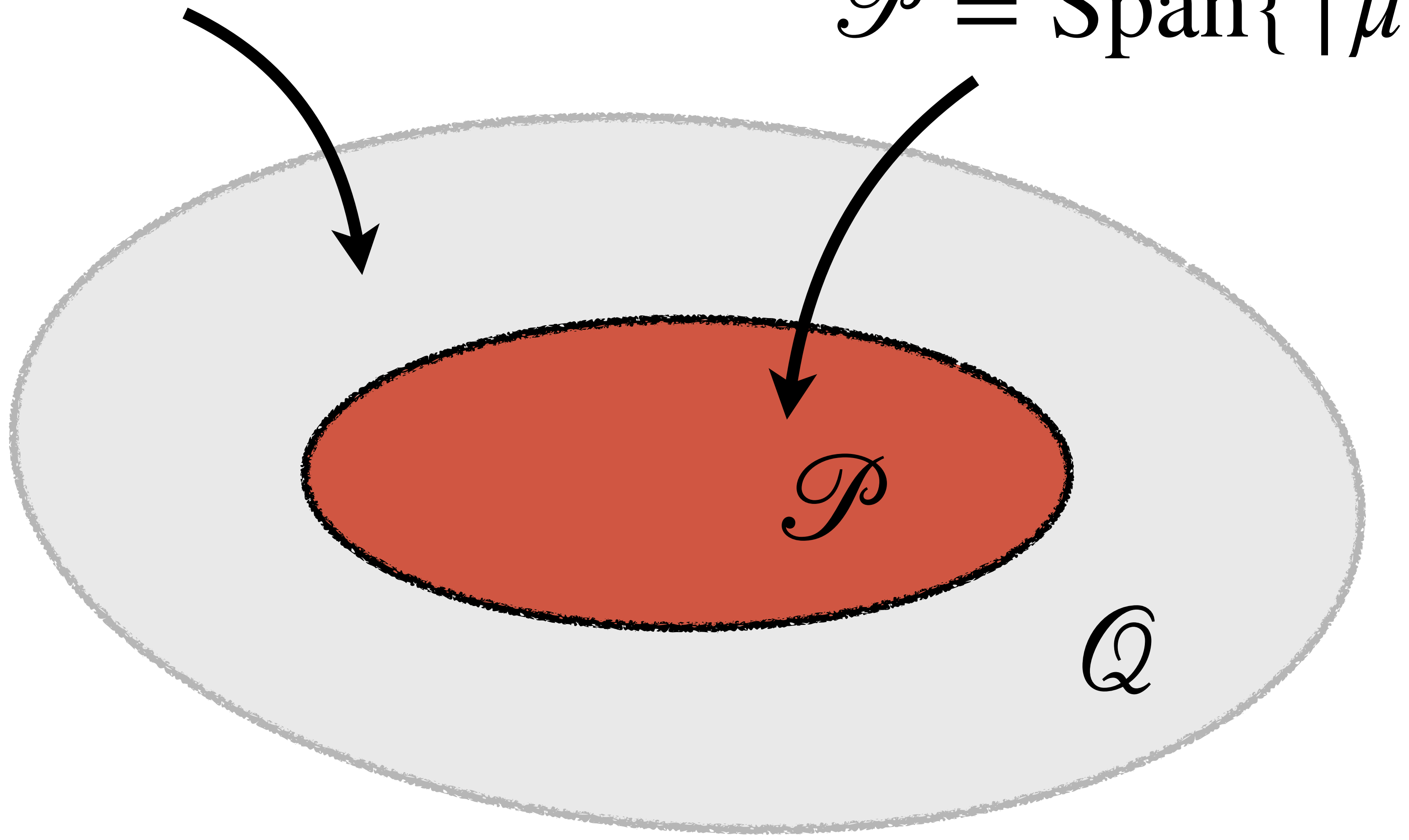

Effective muonic system

- Perturbation theory at 2nd order:

$$\Delta V_{\mu N} = \sum_q \frac{V_{\mu N}^{\mathscr{P}}|q\rangle\langle q|V_{\mu N}^{\mathscr{P}}}{E_0 - E_q}$$

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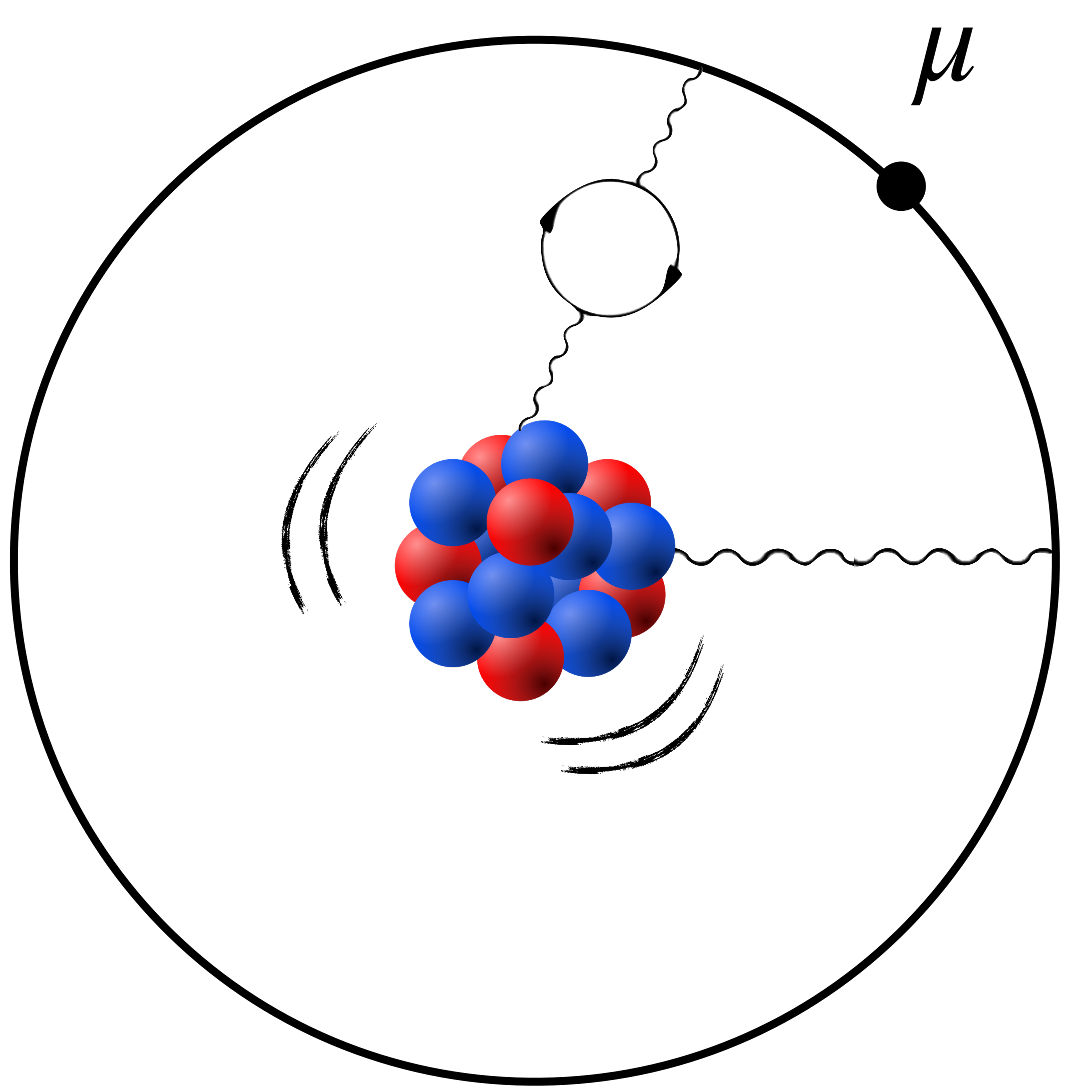

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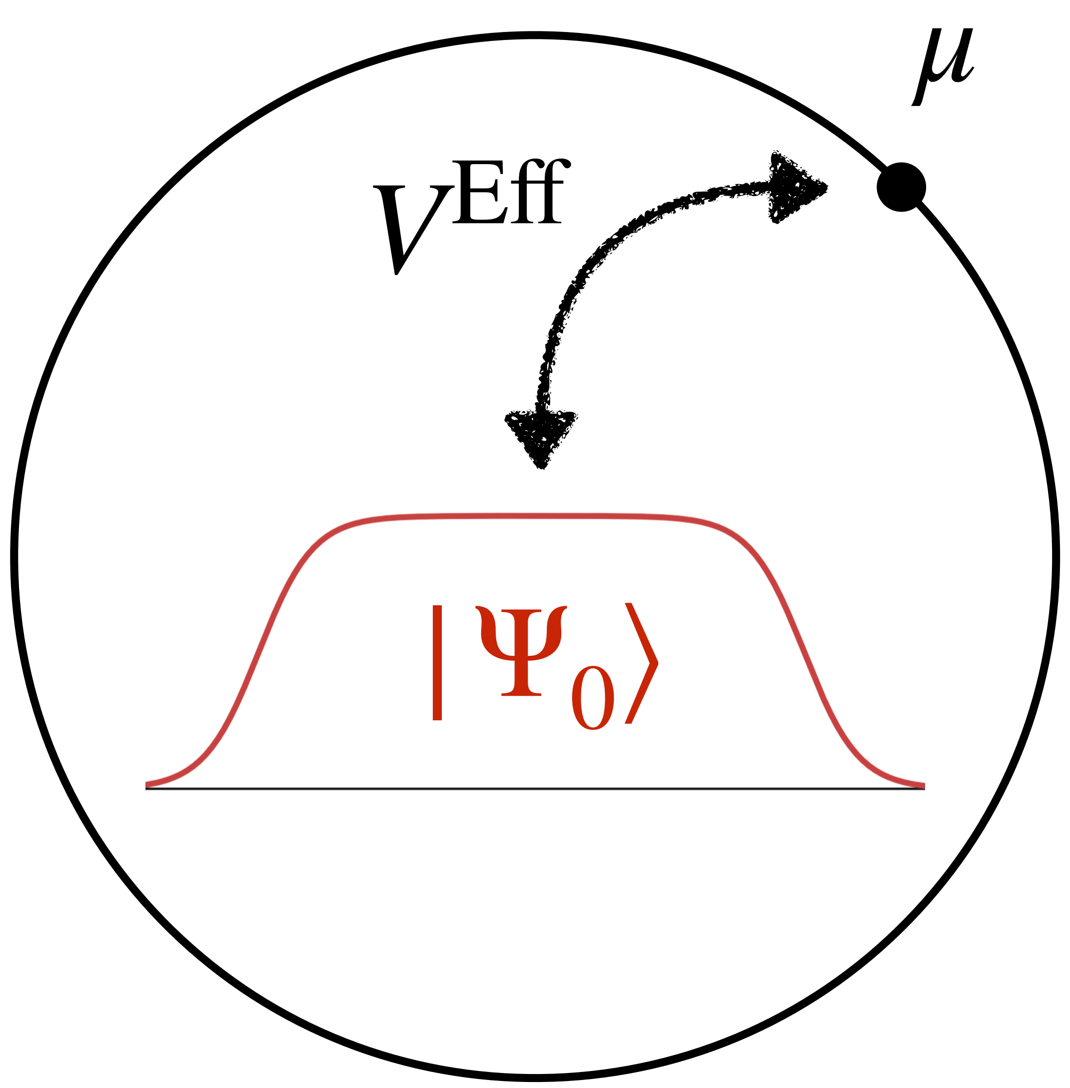

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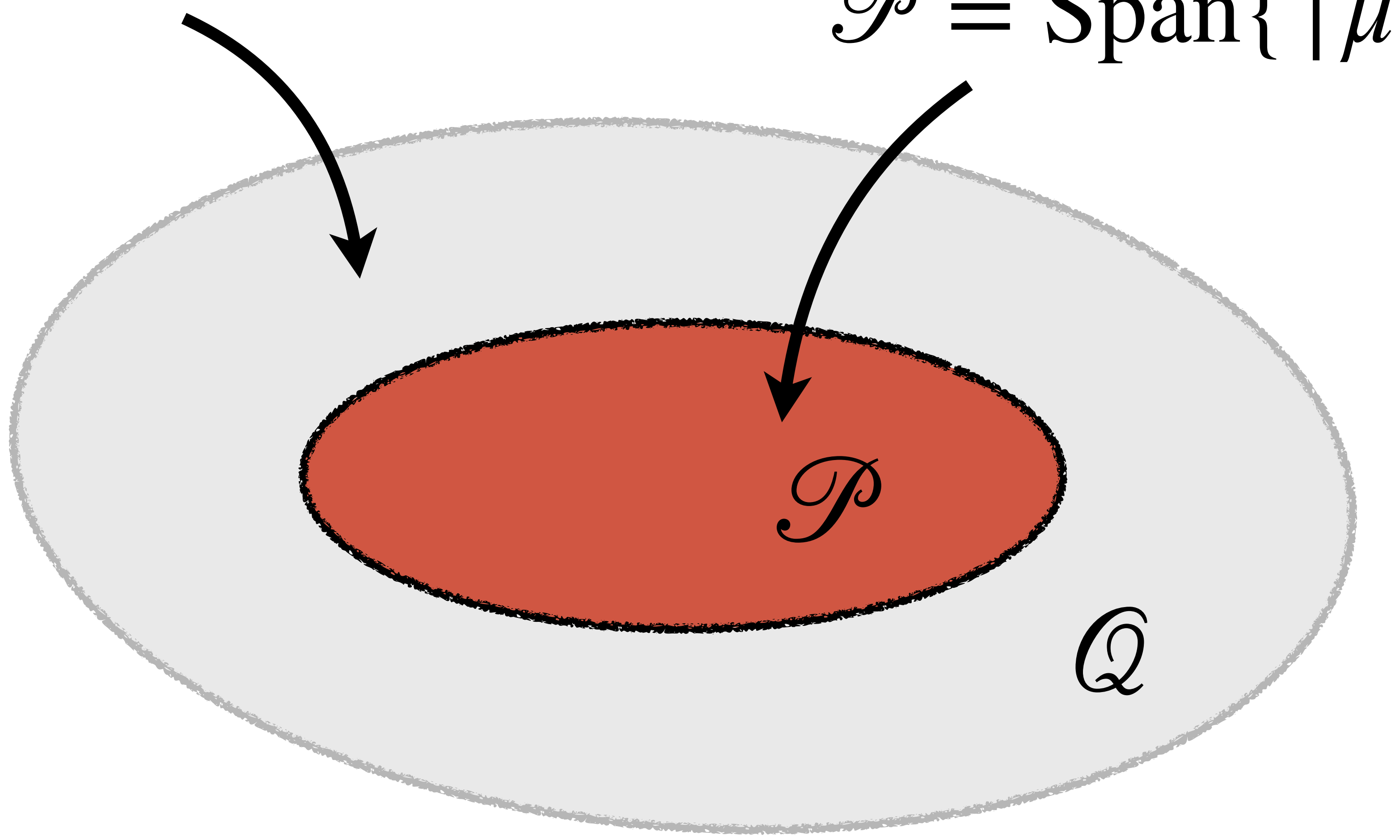

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  (ii) "Plane-wave" approx for internal/external muons

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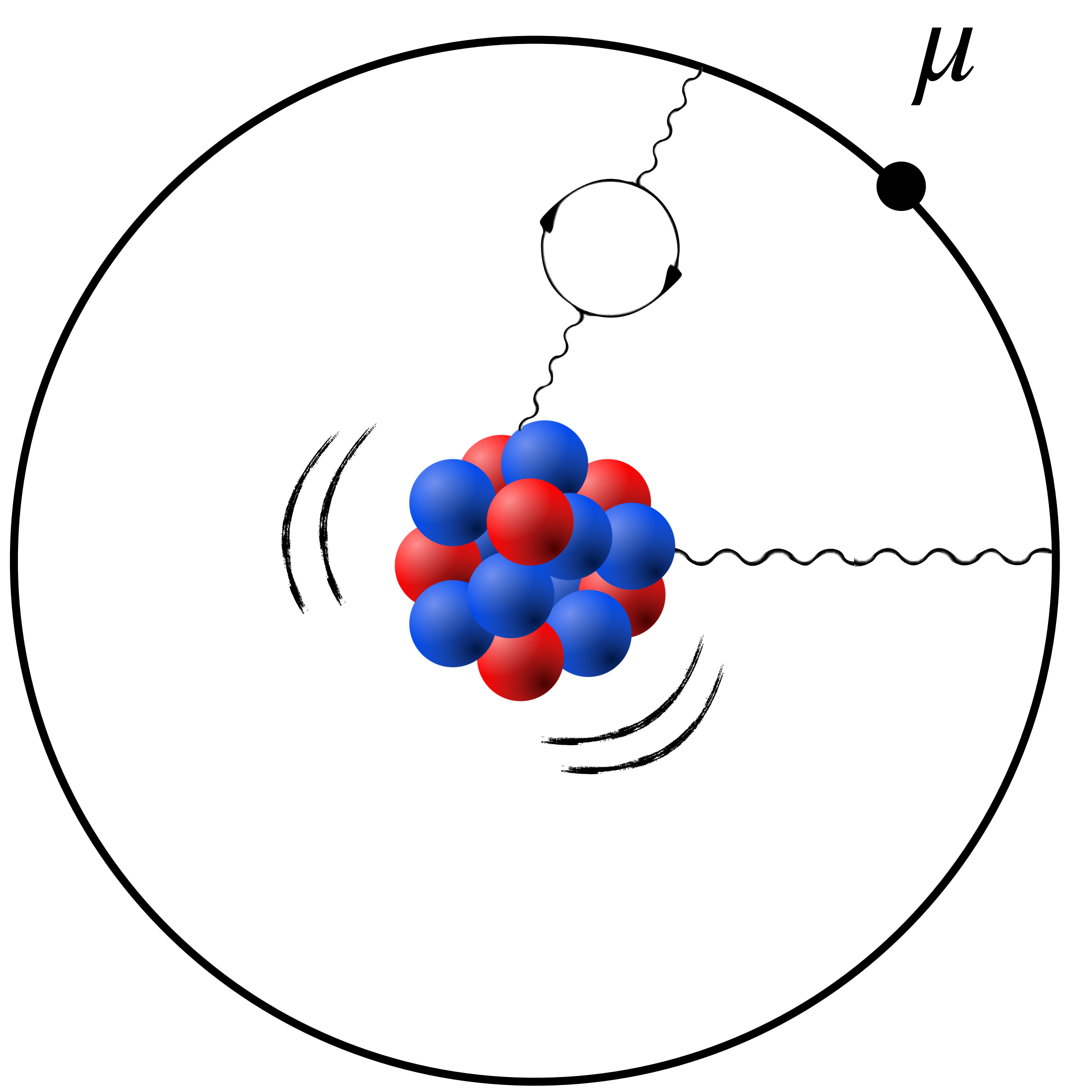

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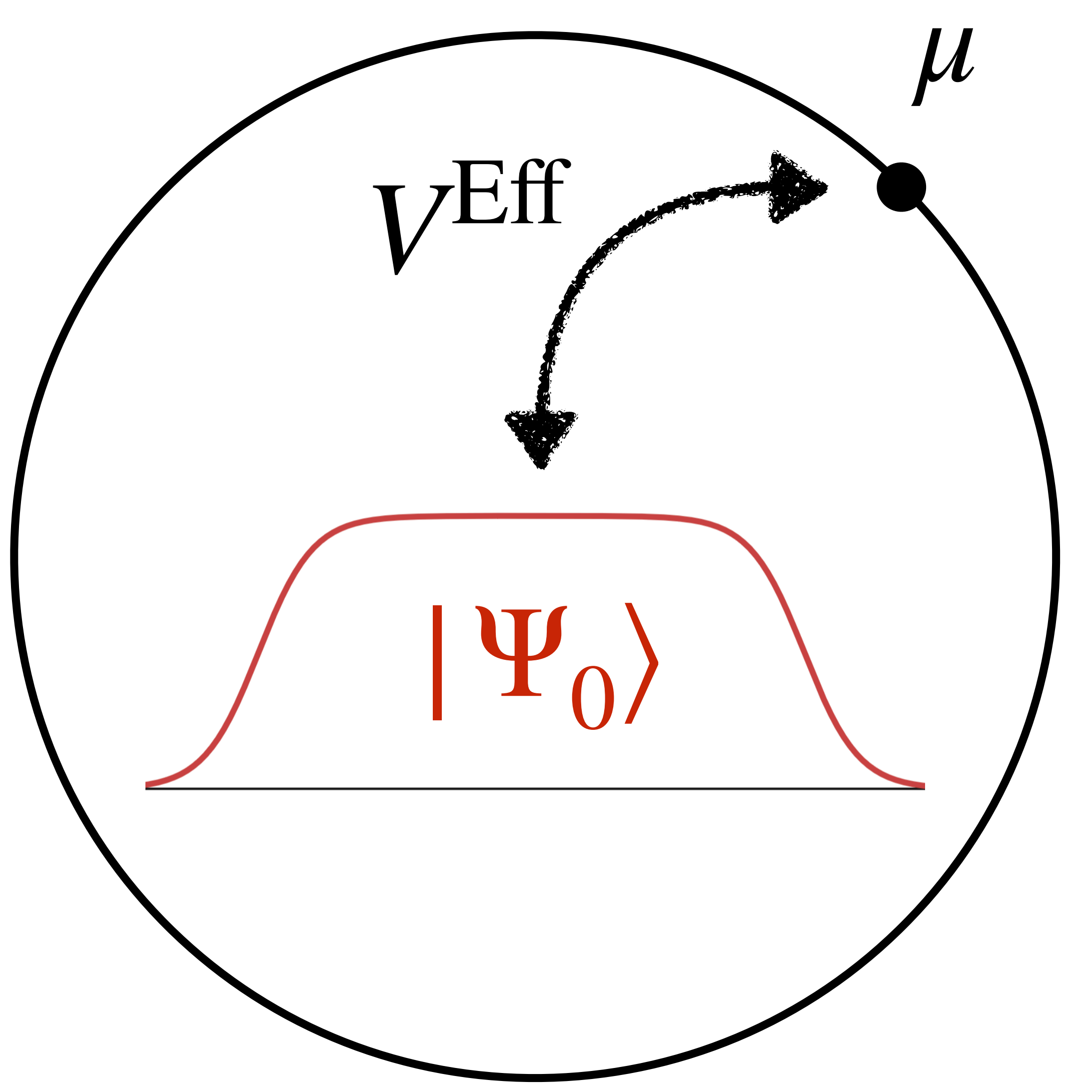

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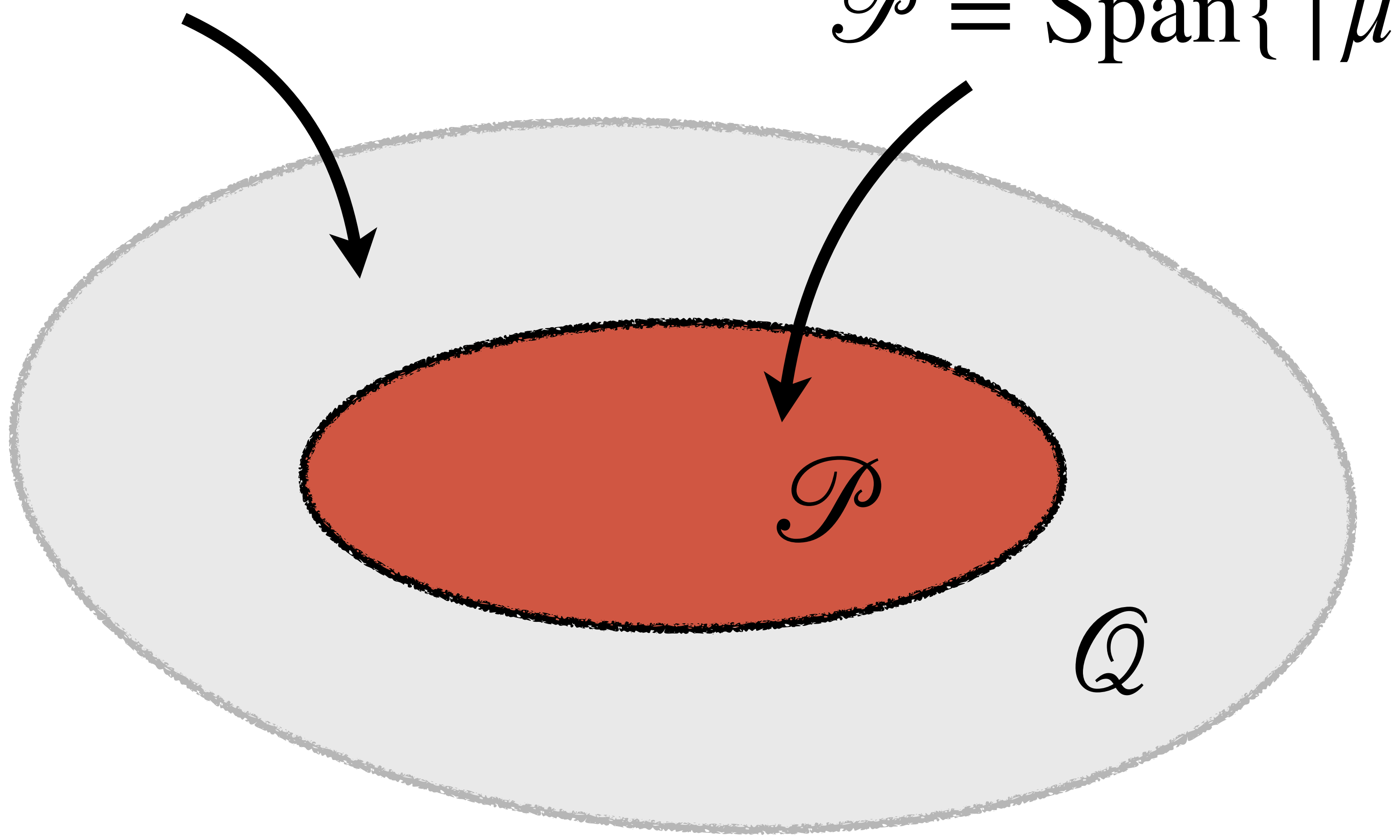

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- Future: control approx with **potential NRQED** EFT
  [Peset et al., EPJA (2015)]

The two-photon exchange nuclear correction

Radius extraction master formula

$$\delta_{\mathrm{E}} = \delta_{\mathrm{QED}} + \mathscr{C}\ r_c^2 + \delta_{\mathrm{NS}}$$

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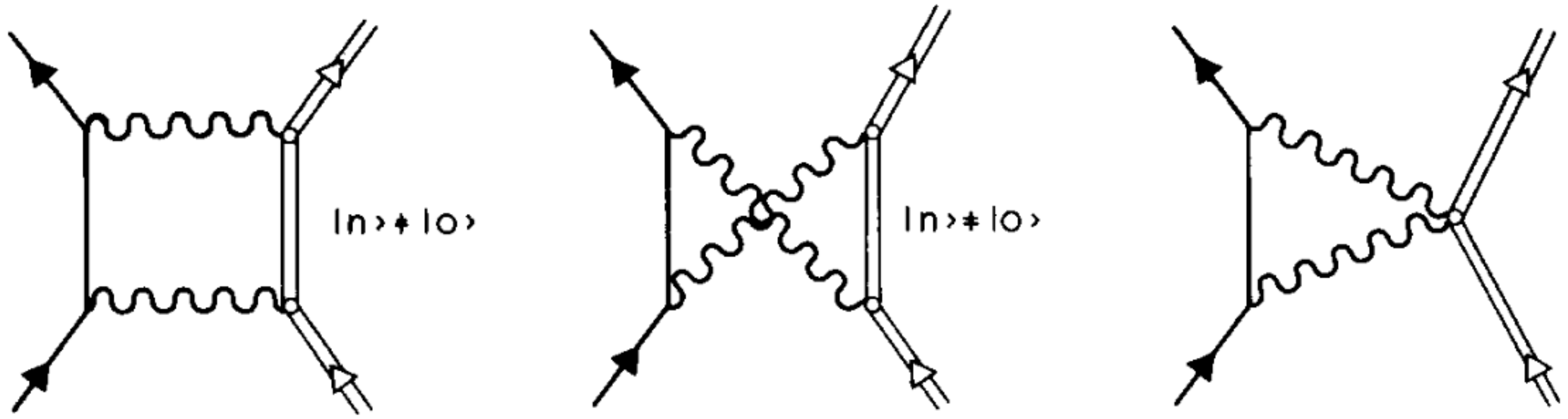
Two-photon polarizability contributions



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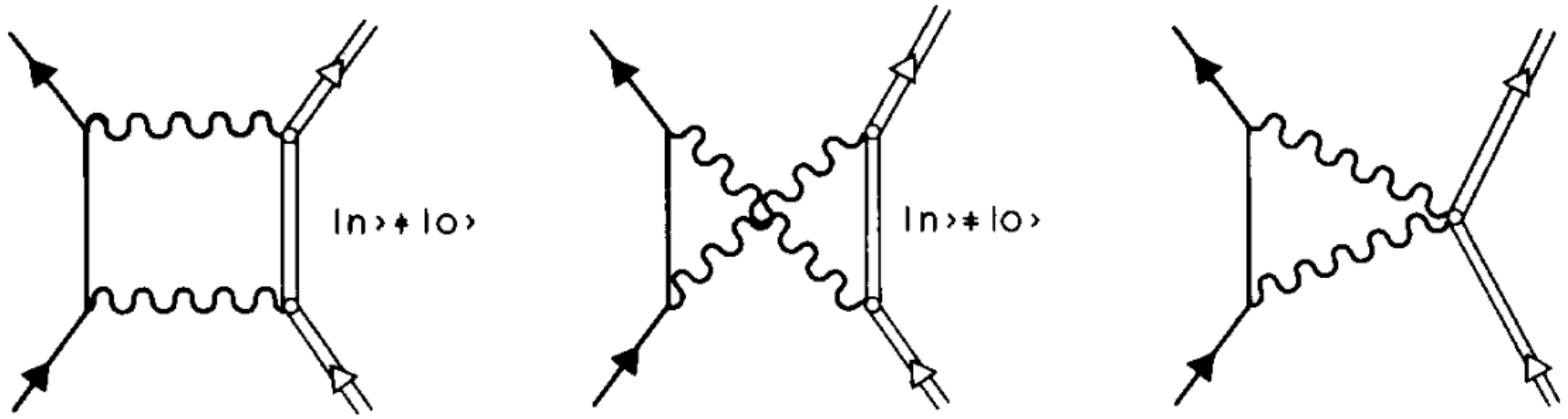

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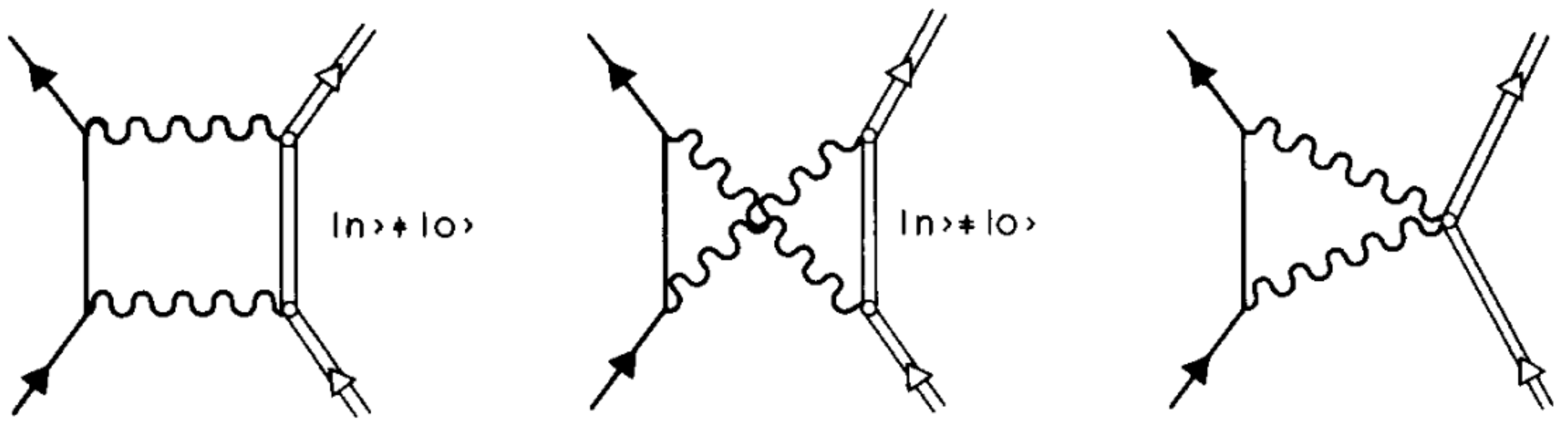

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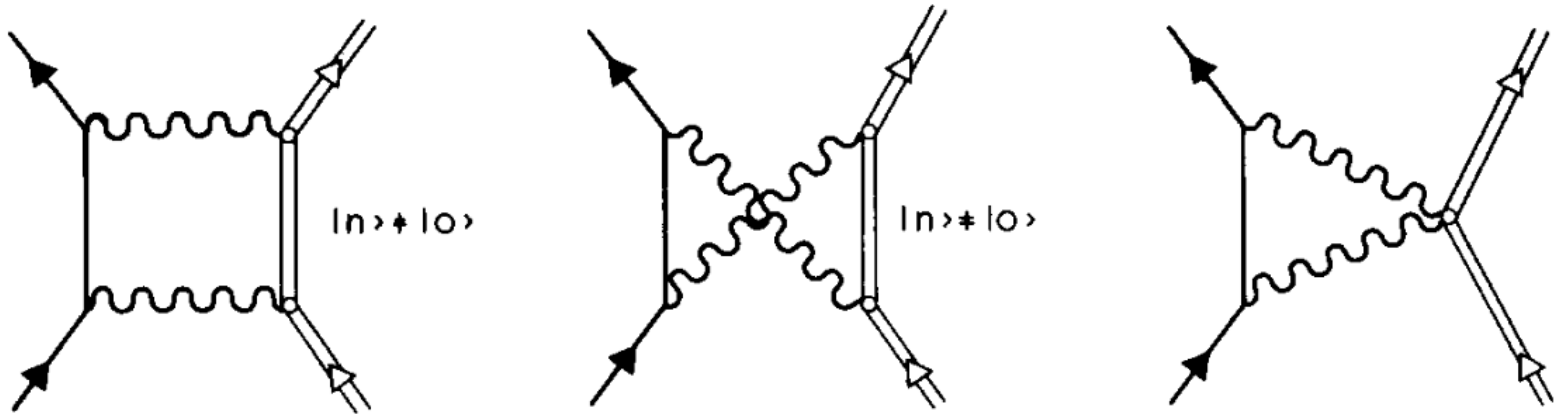

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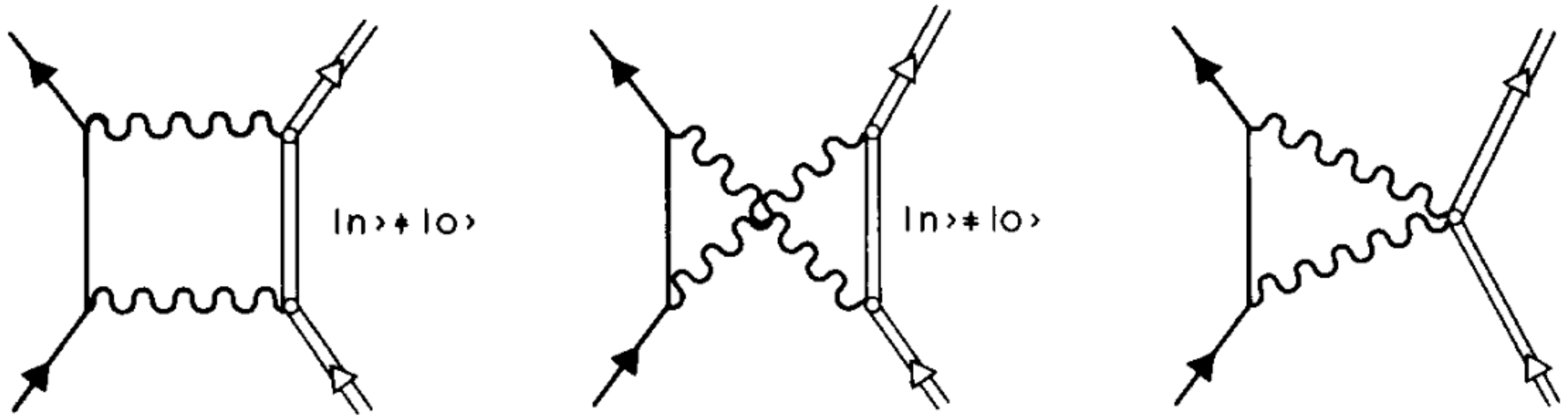

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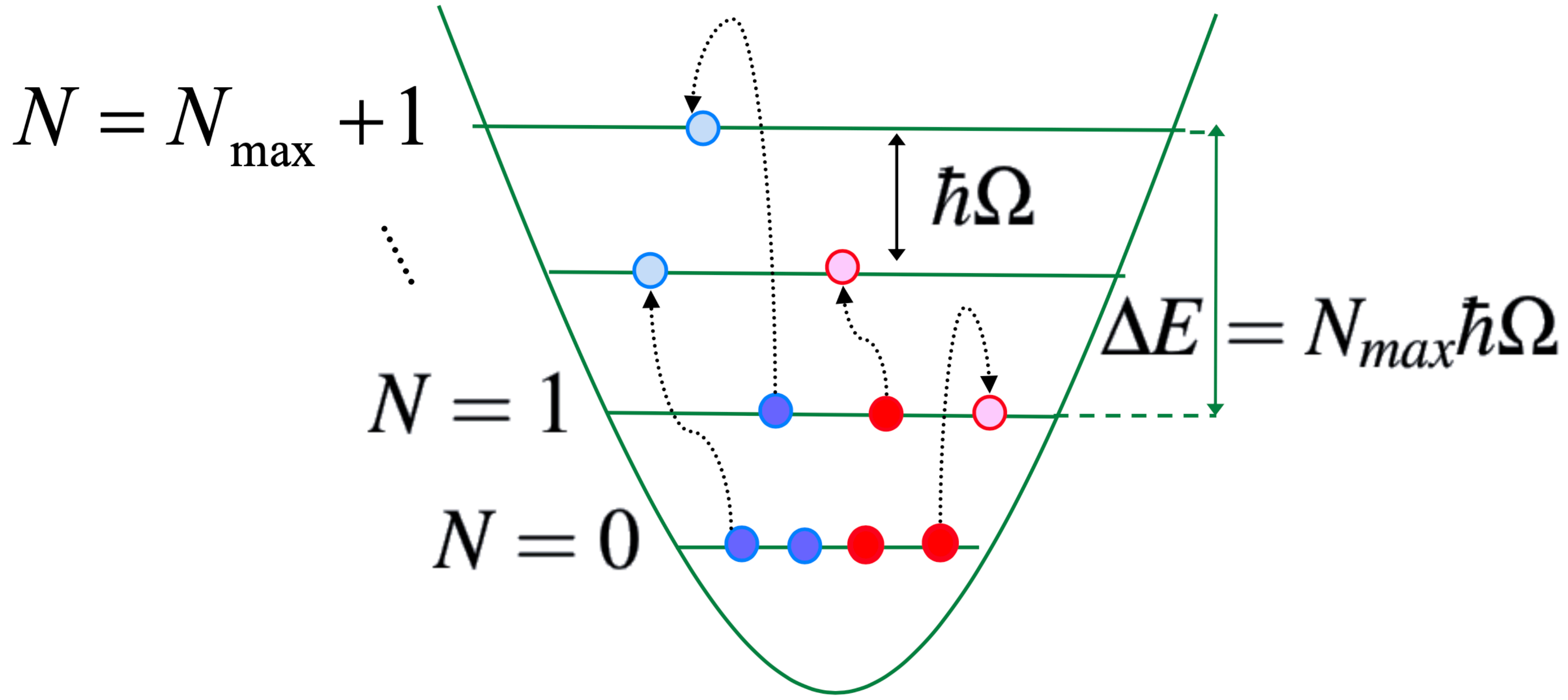
- Response functions:
  - ➡ $S_X(\omega,q) \equiv \sum_{J\geq 0}\sum_{N\neq 0} |\langle N\,|\,O_{X,J}(q)\,|\,\Psi\rangle|^2\,\delta(E_N - E_0 - \omega)$

Ab initio nuclear corrections



Nuclear physics modelling

Model used for nuclear currents

- Electromagnetic current modelling
  - General one-body current for point-like particles
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  - Harmonic oscillator Slater determinant
  - Vary many-body basis: (Ω, N_{max})
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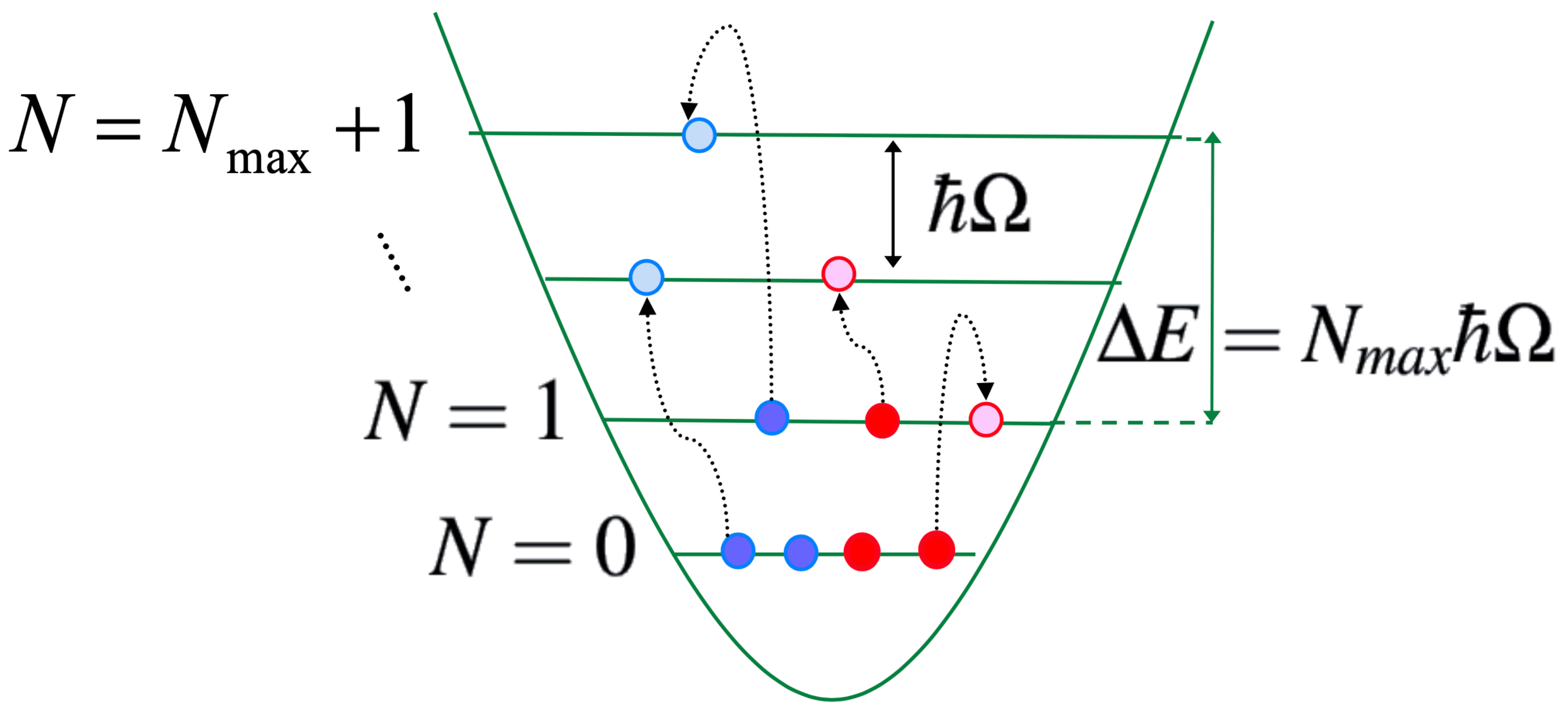
- Many-body approximation
  - No-Core Shell Model
  - More details in next section

  ➡ **Negligible many-body approximation uncertainty**

Ab initio No-Core Shell Model

Anti-symmetrized products of many-body HO states



Ab initio No-Core Shell Model

Lanczos tridiagonalization algorithm [Lanczos (1950)]

- Initialization: normalized pivot $|\phi_1\rangle$
- Recursion: α_i, β_i and $|\phi_i\rangle$
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- Output:
  - Lanczos basis and coefficients $\{|\phi_i\rangle, \alpha_i, \beta_i\}$ → H in Lanczos basis

$$\begin{pmatrix} \alpha_1 & \beta_2 & & & & \\ \beta_2 & \alpha_2 & \beta_3 & & & \\ & \beta_3 & \alpha_3 & \ddots & & \\ & & \ddots & \ddots & \beta_{k-1} & \\ & & & \beta_{k-1} & \alpha_{k-1} & \beta_k \\ & & & & \beta_k & \alpha_k \end{pmatrix}$$

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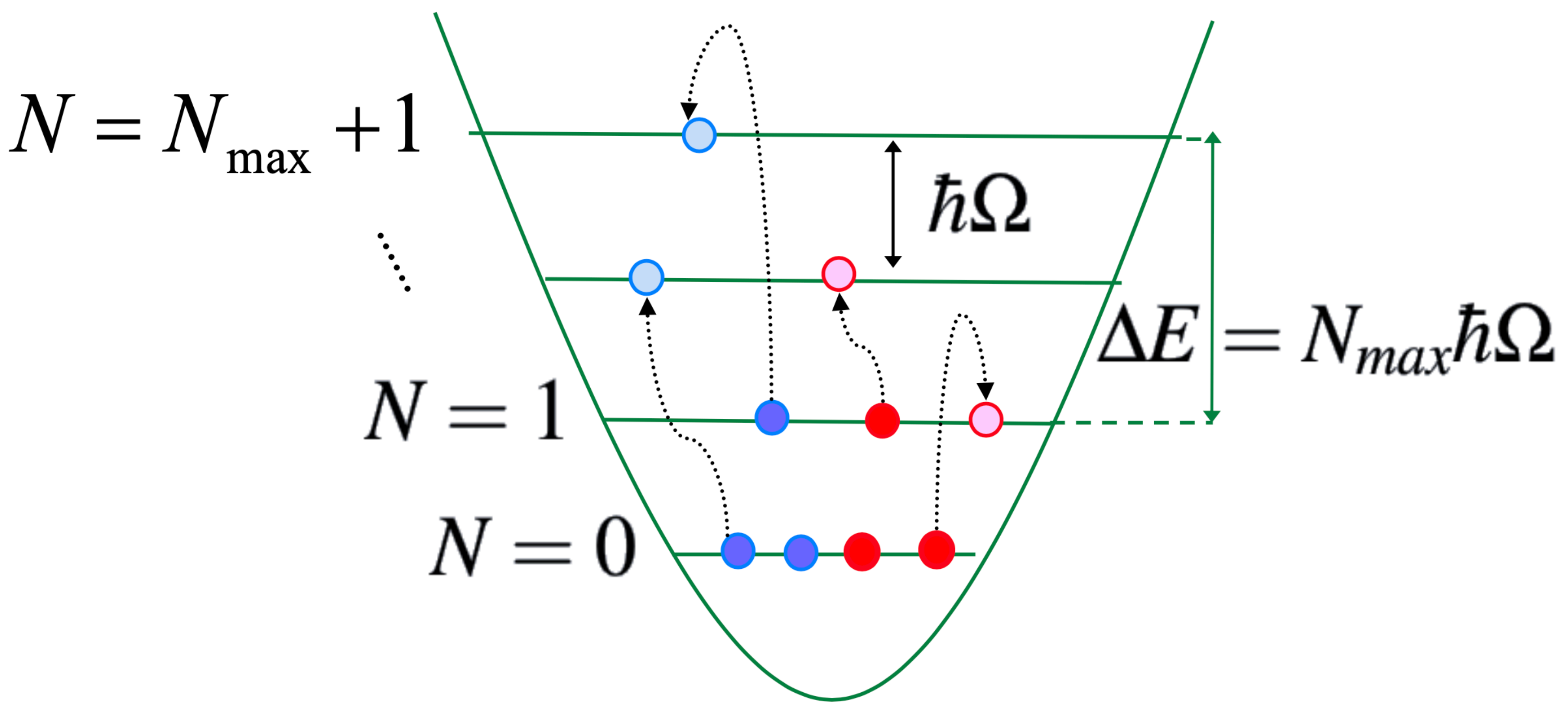

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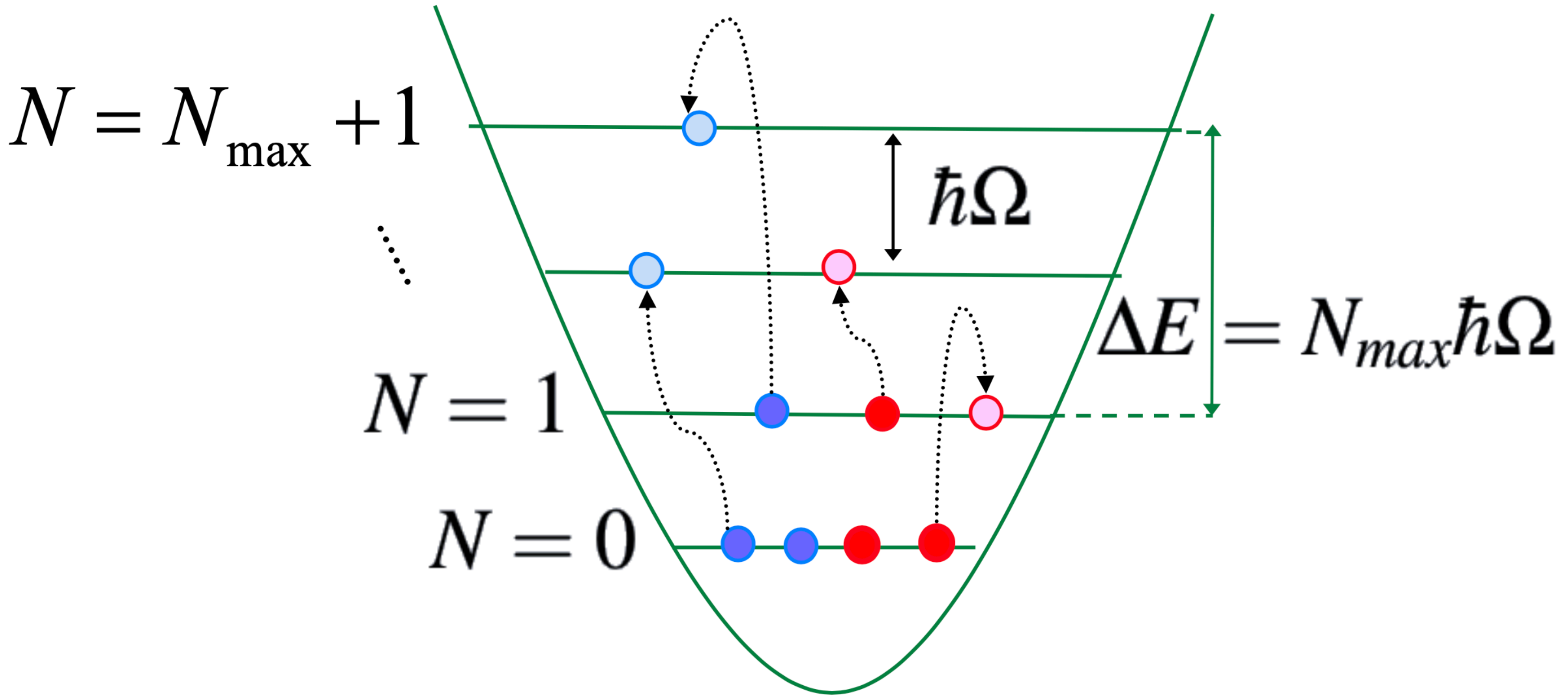

Application to nuclear structure

- Efficient calculation of low-lying spectra
  - Selection rules ⇒ **Fast matrix-vector multiplication**
  - In practice: $N_L \sim 100 - 200$ is sufficient
- Application to $^{6-7}\mathrm{Li}$
  - $N_L = 200$ for $N_{\max} = 1$ to 9
  - Ground-state of $^{6-7}\mathrm{Li}$ $|\Psi\rangle$ ⇒ **Starting point for $\delta^A_{\rm pol}$**

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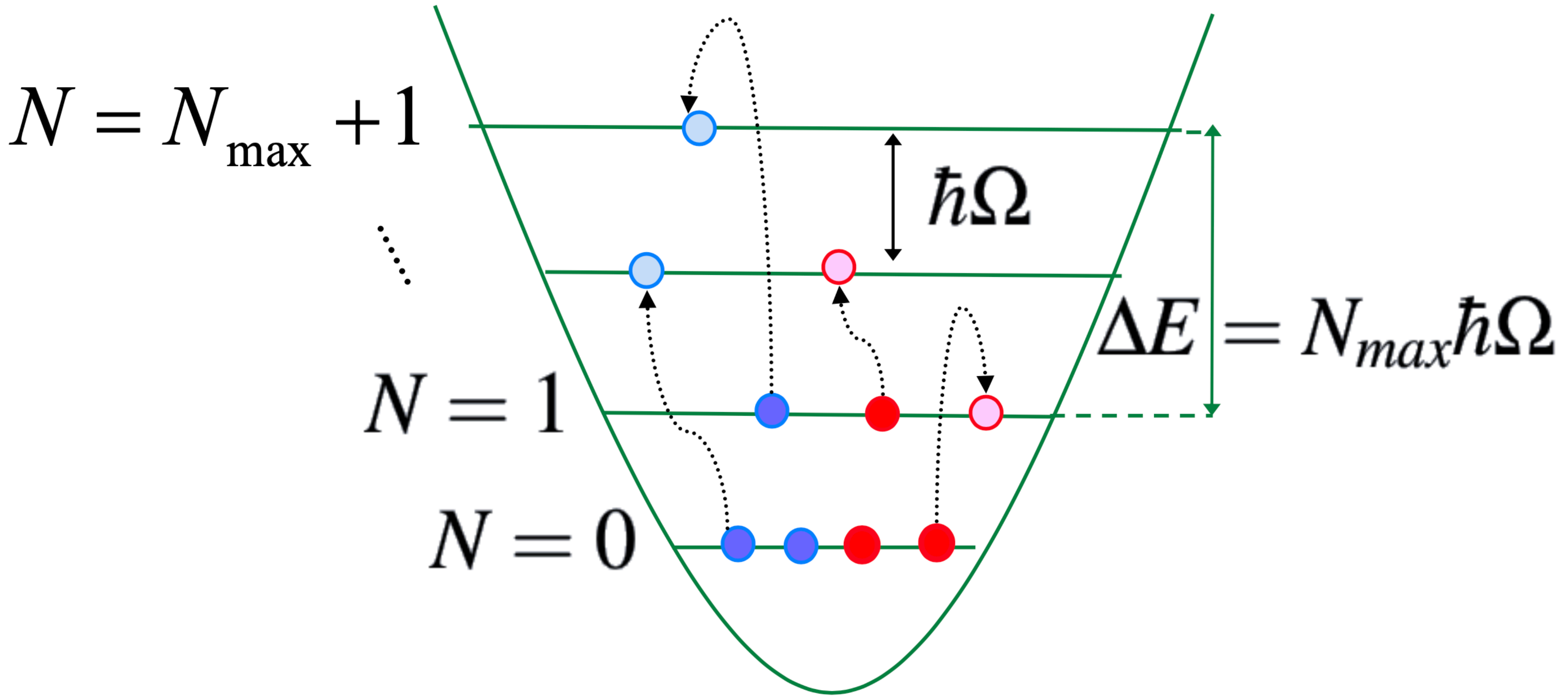

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Lanczos-strength algorithm for $\delta^A_{\rm pol}$

- Convergence problem
  - Often the **strength of $S_O(\omega)$ is fragmented**
  - And only low-lying states converged
- Lanczos strength algorithm
  - Pivot based on 1st Lanczos output: $|\phi_1'\rangle = O|\Psi\rangle$
  - Recovers **exactly** $\int d\omega\, \omega^n\, S_O(\omega)$ for $n \leq 2N_L - 1$

➡ **One additional NCSM run per operator**

Numerical results for $^{6-7}\mathrm{Li}$ isotopes

A first test case for N4LO-E7 and $N_{\max} = 7$

Numerical calculations

- $q_{\max} = 700$ MeV and $\Delta q = 10$ MeV
- 10 different operators for $J_{\max} = 3$

➡ **700 NCSM calculations at $N_{\max} = 7$**

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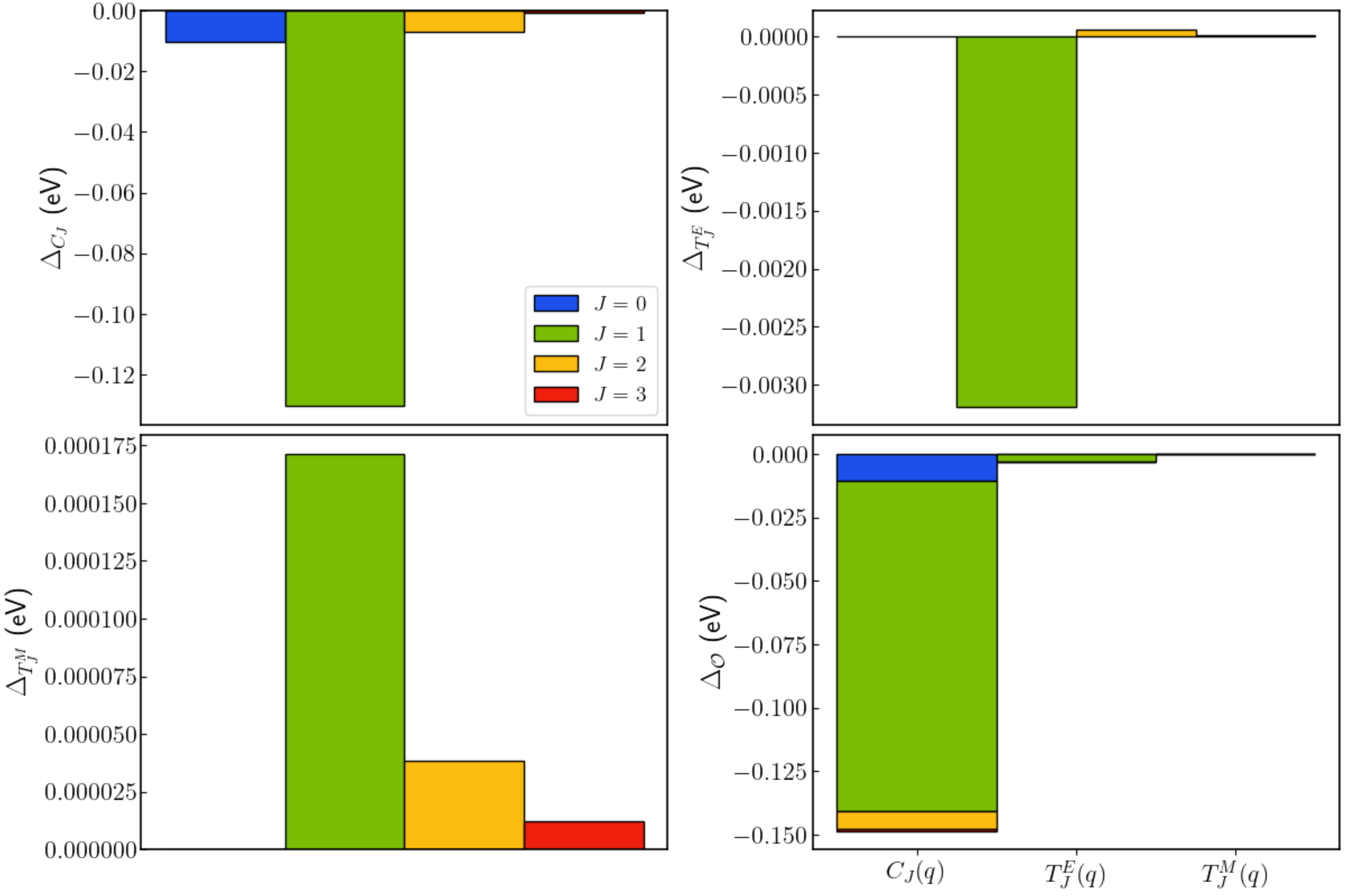

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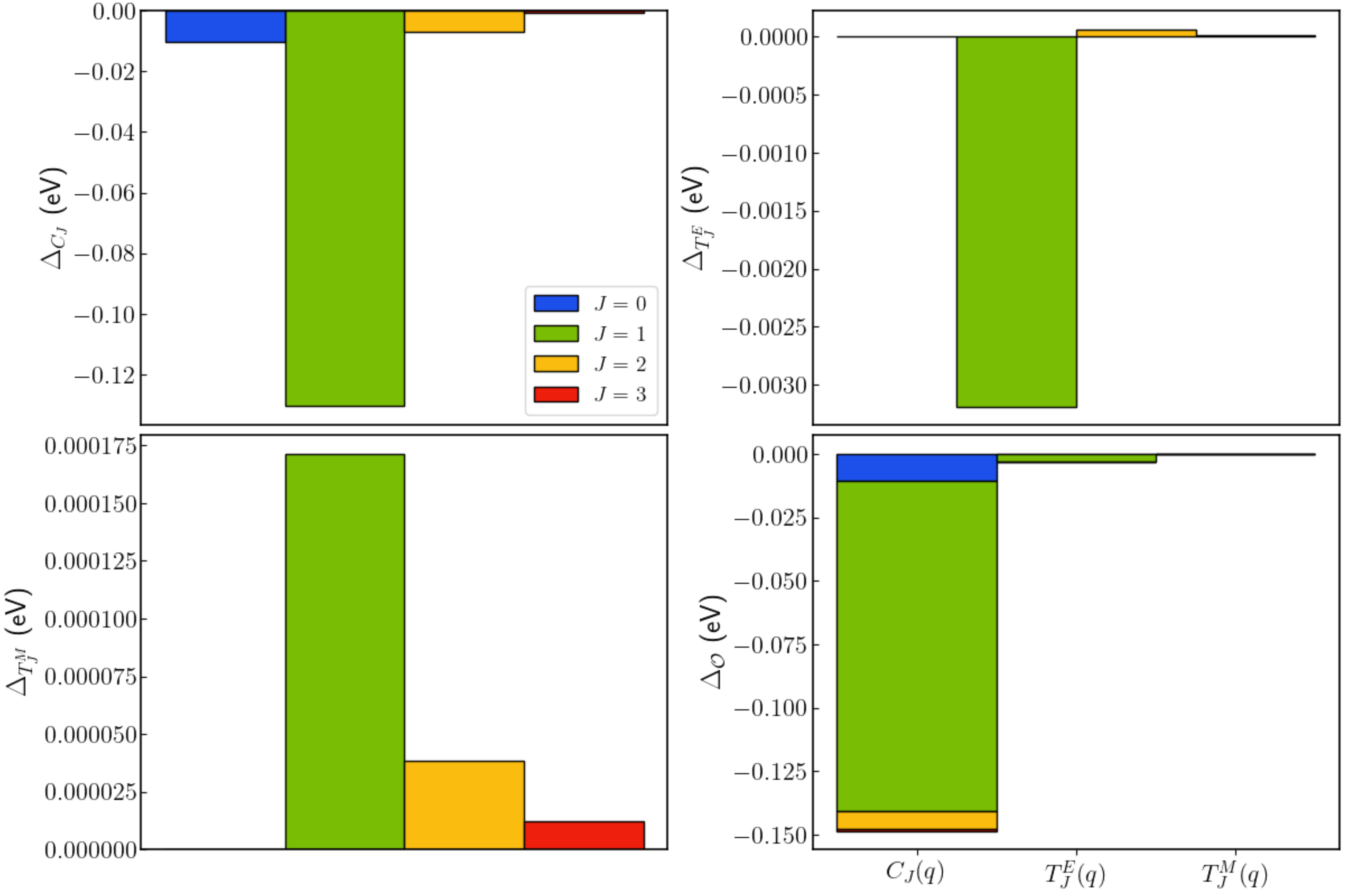

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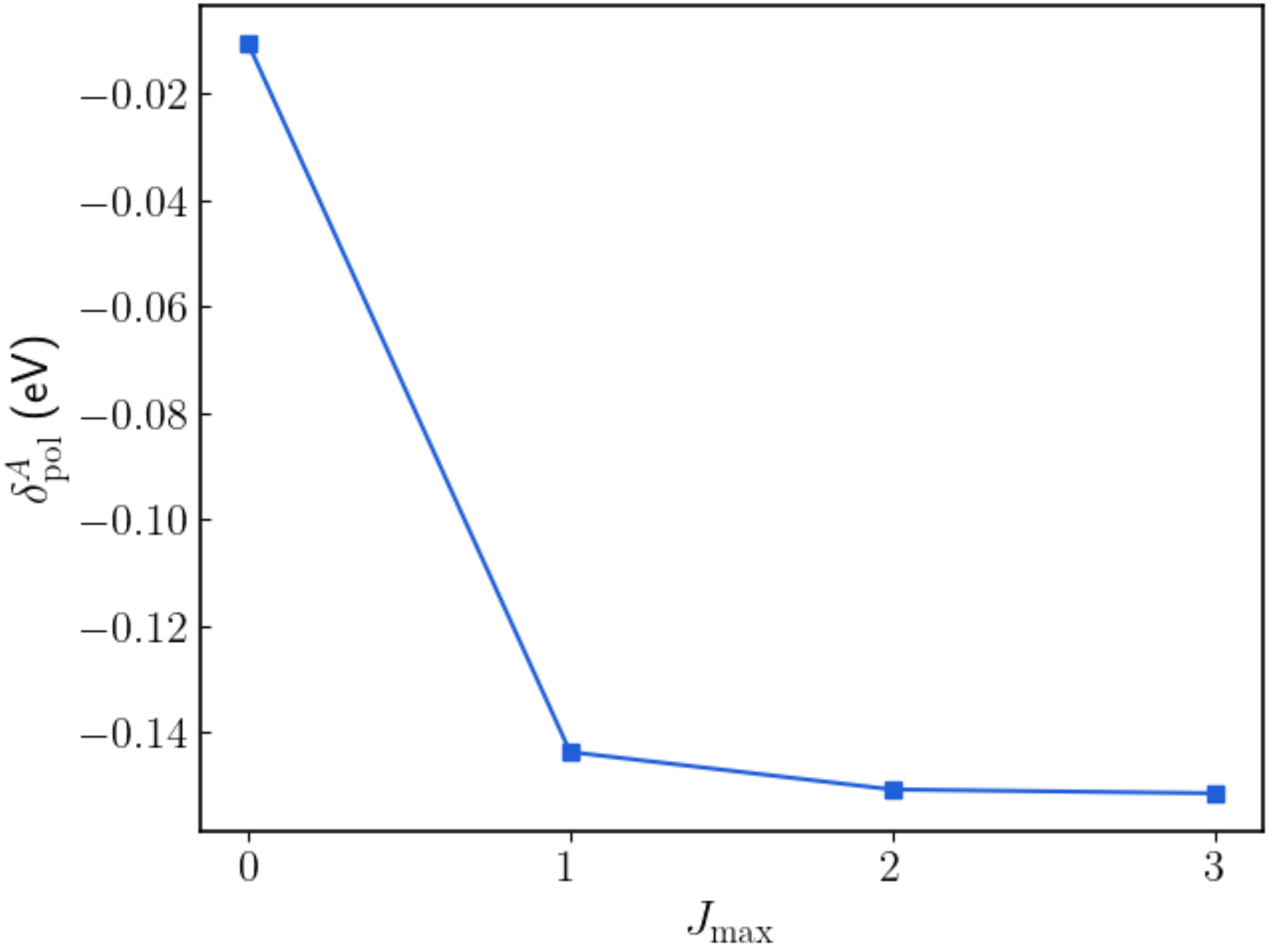
Observations

- Contribution repartitions
  - Well-known **dipole** dominance
  - **Charge** contributions are dominant
- Negligible contributions
  - TM is negligible for any J
  - TE is relevant only for $J = 1$

➡ **Only half the operators are relevant**

Checking convergence in $J_{\max}$



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- Here shown for $N_{\max} = 7$ and N4LO-E7
- All other cases are similar

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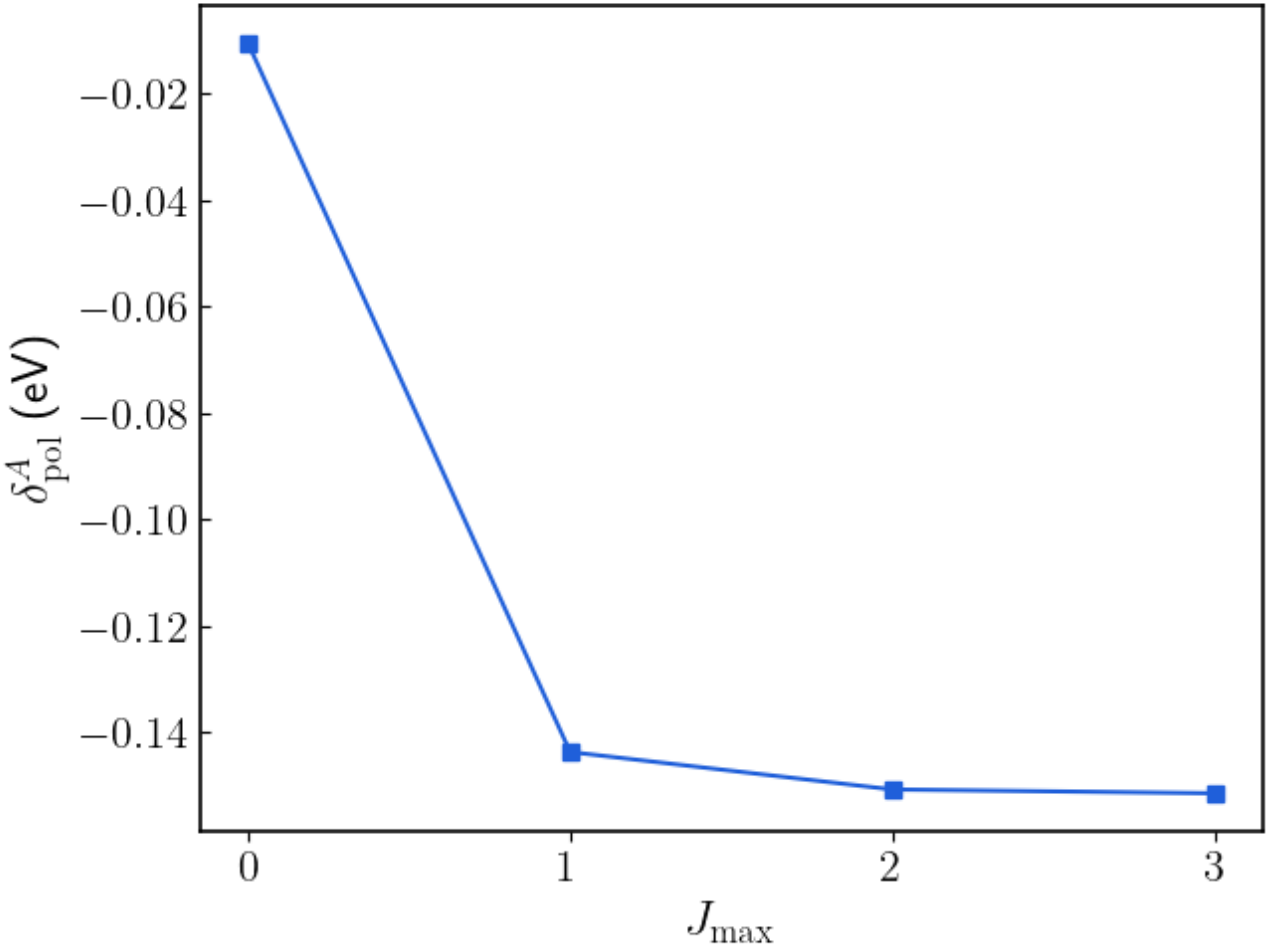

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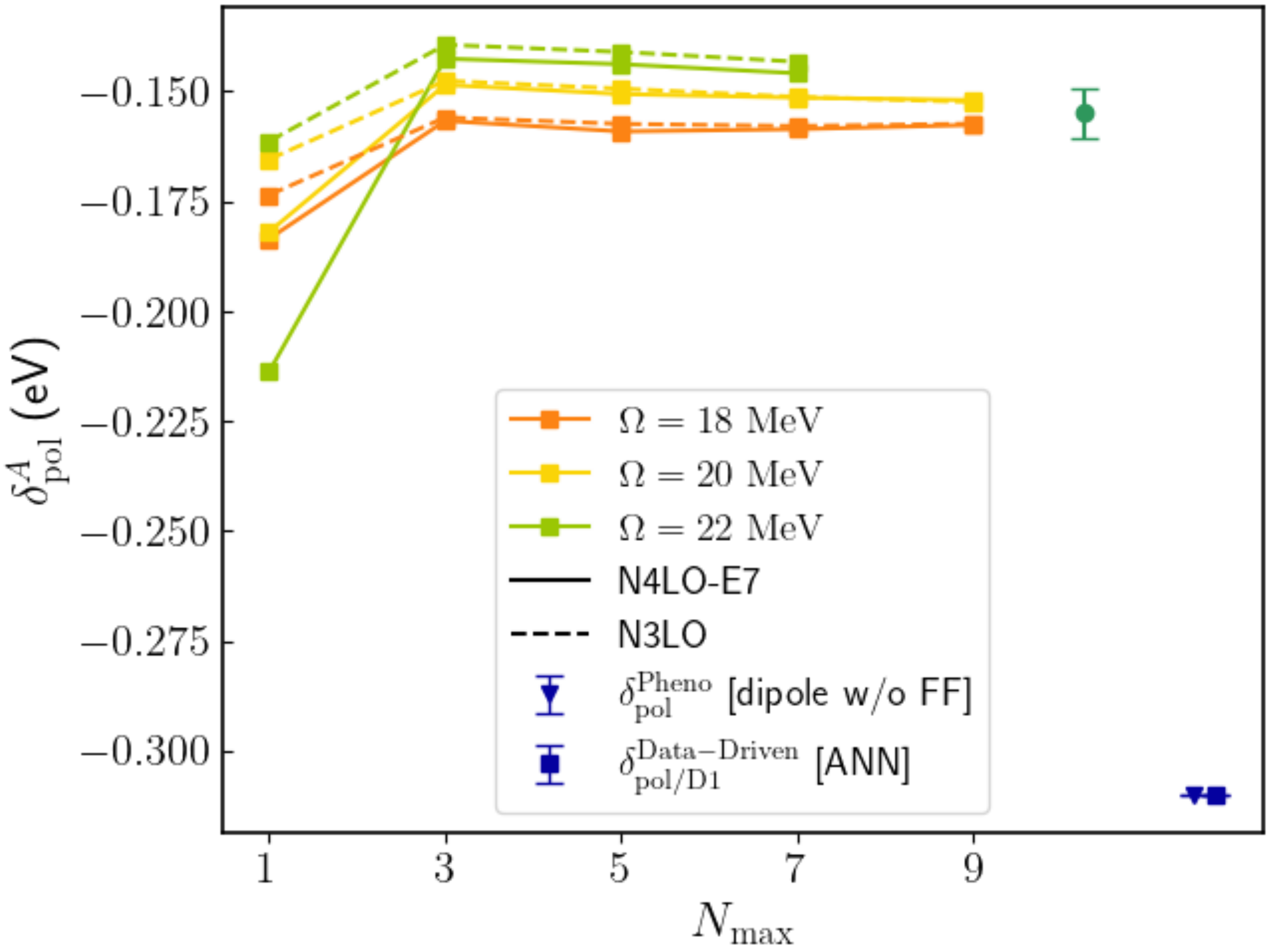
$$\epsilon_{J_{\max}} \lesssim 1 - 10 \text{ meV}$$

Multipole truncation ⇒ Negligible uncertainty

Dependence on $(\Omega, N_{\max})$ and the interaction



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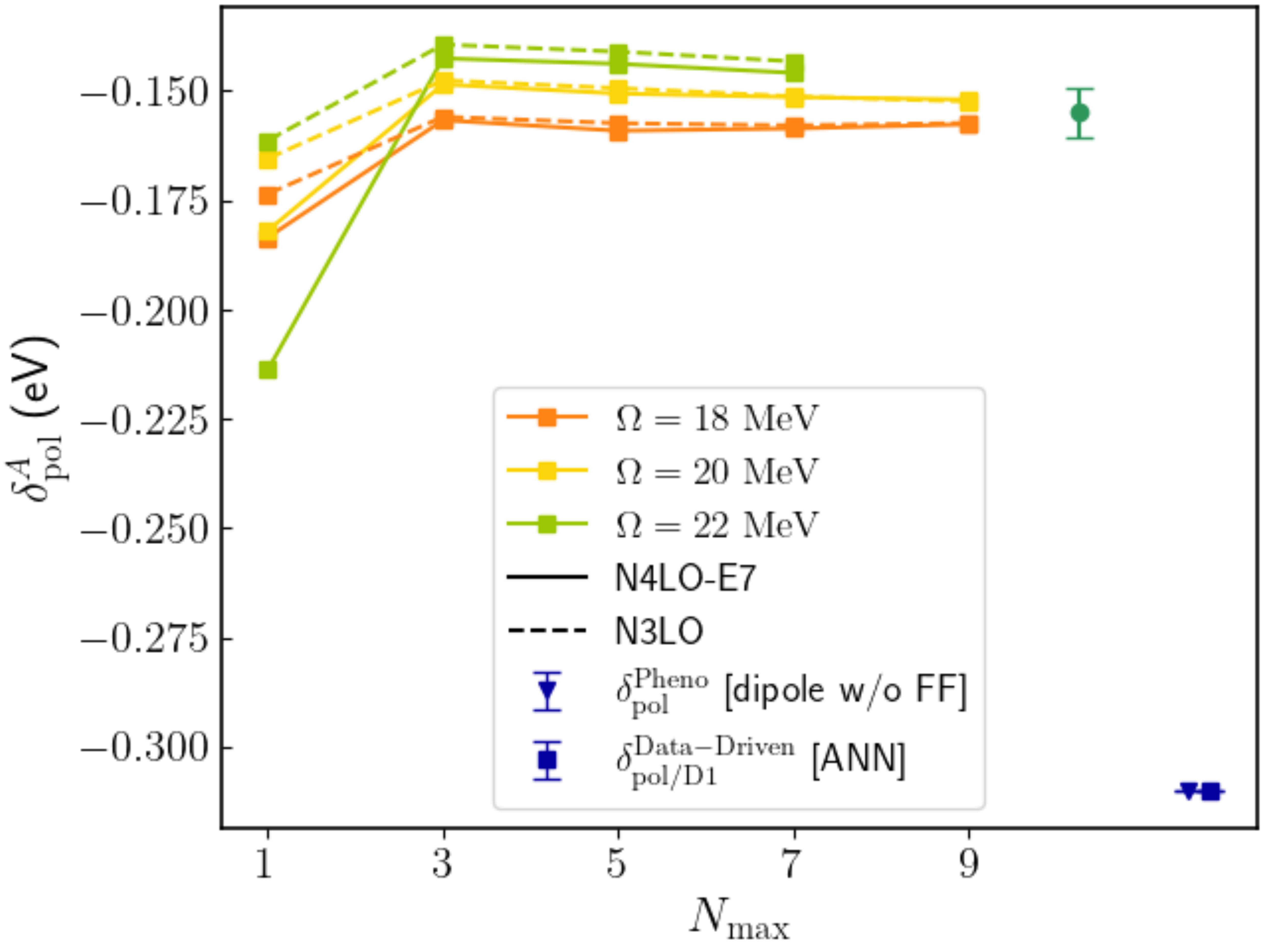

Preliminary numerical results

- Model-space dependence
  - Optimal frequency around 20 MeV
  - Run calculations for $\Omega = 18,\ 20,\ 22$ MeV
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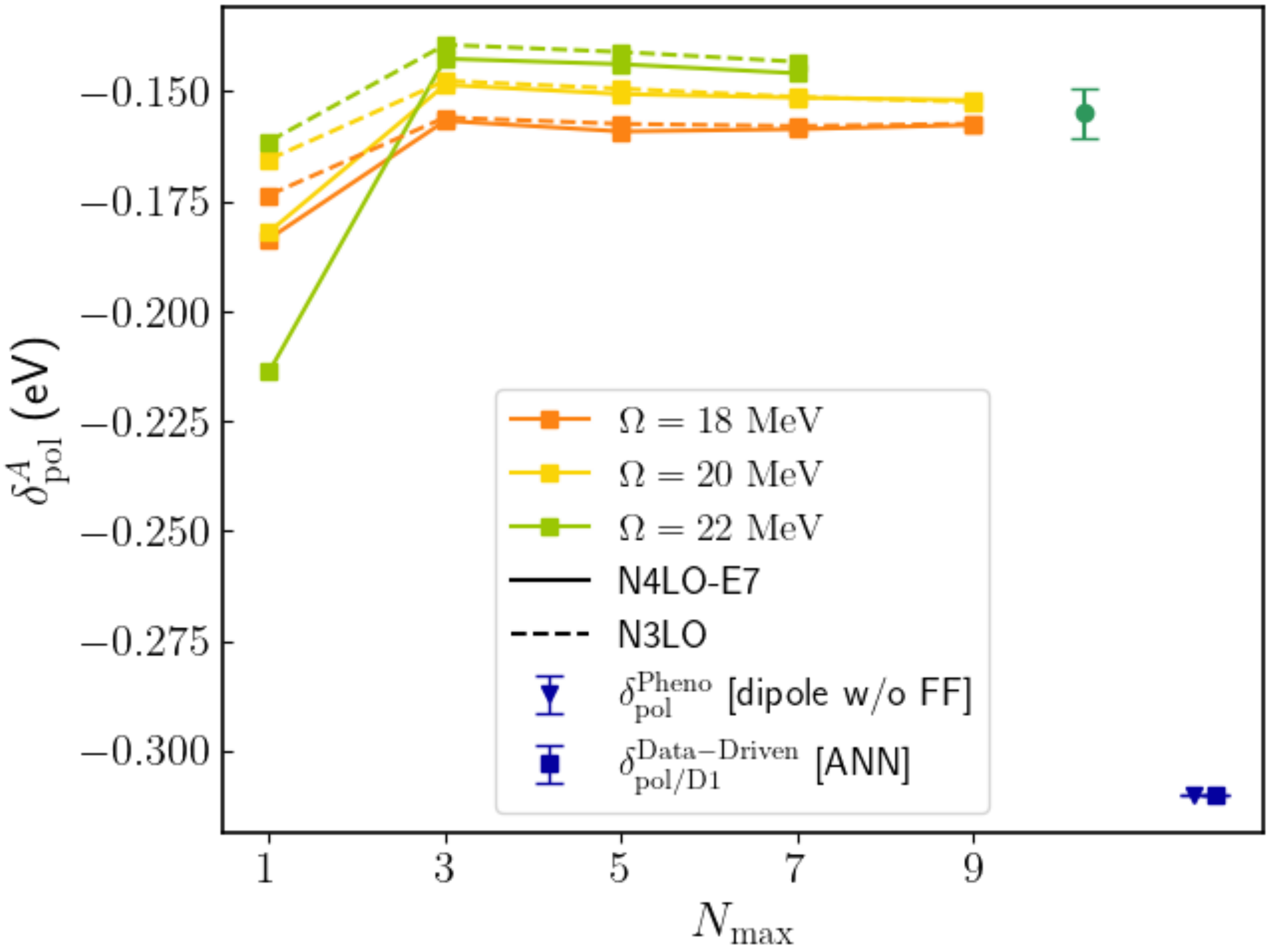

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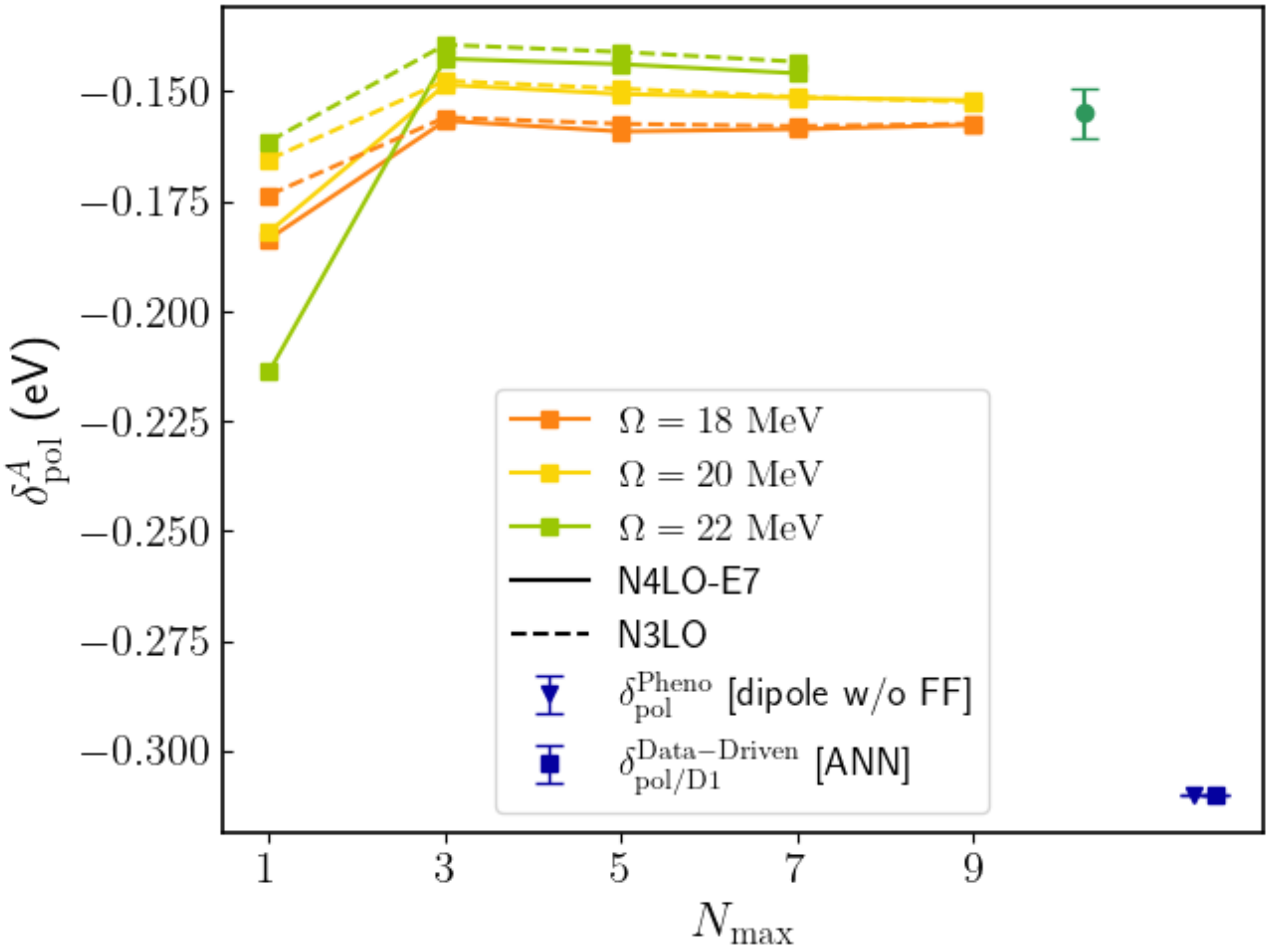

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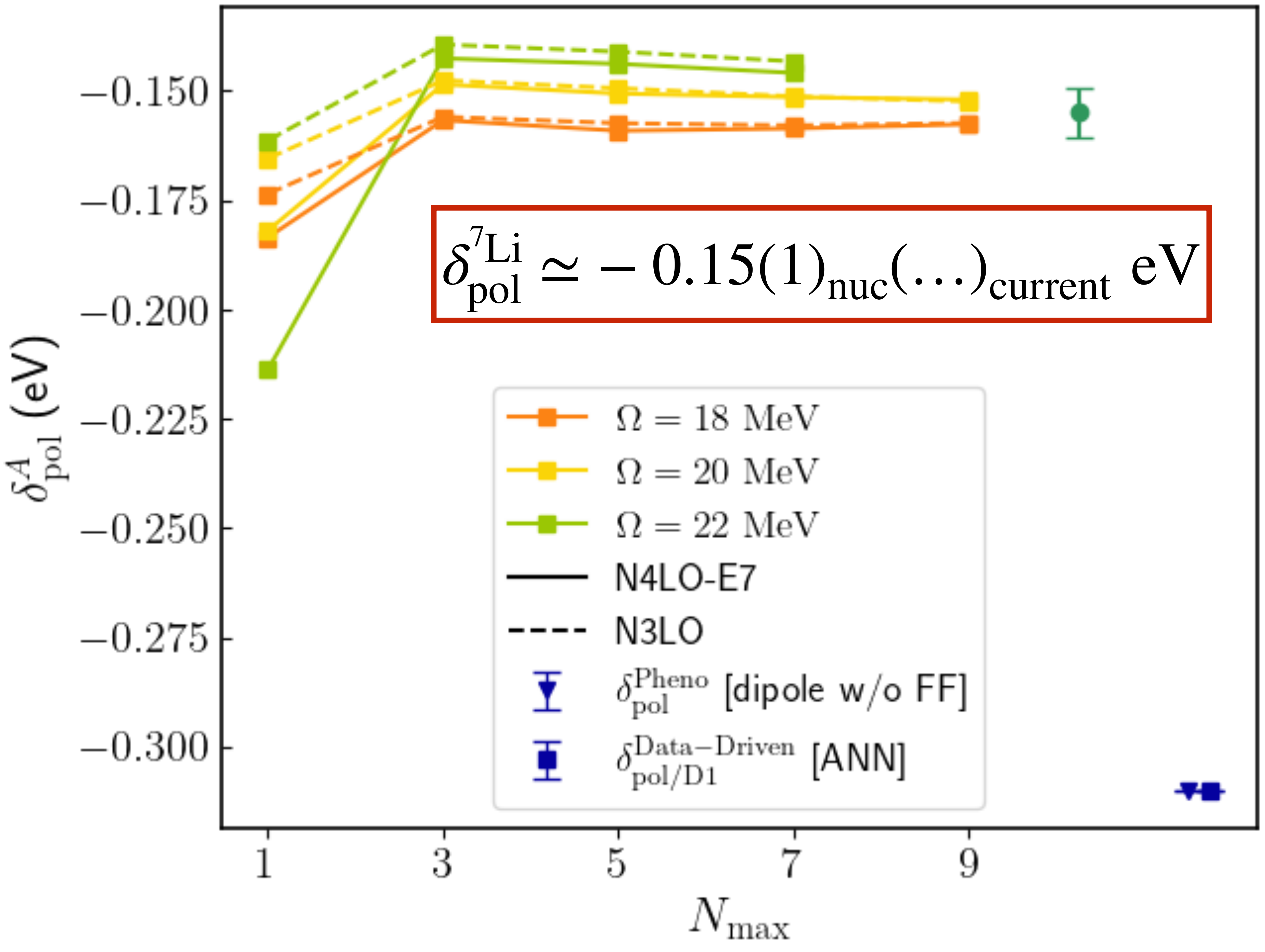

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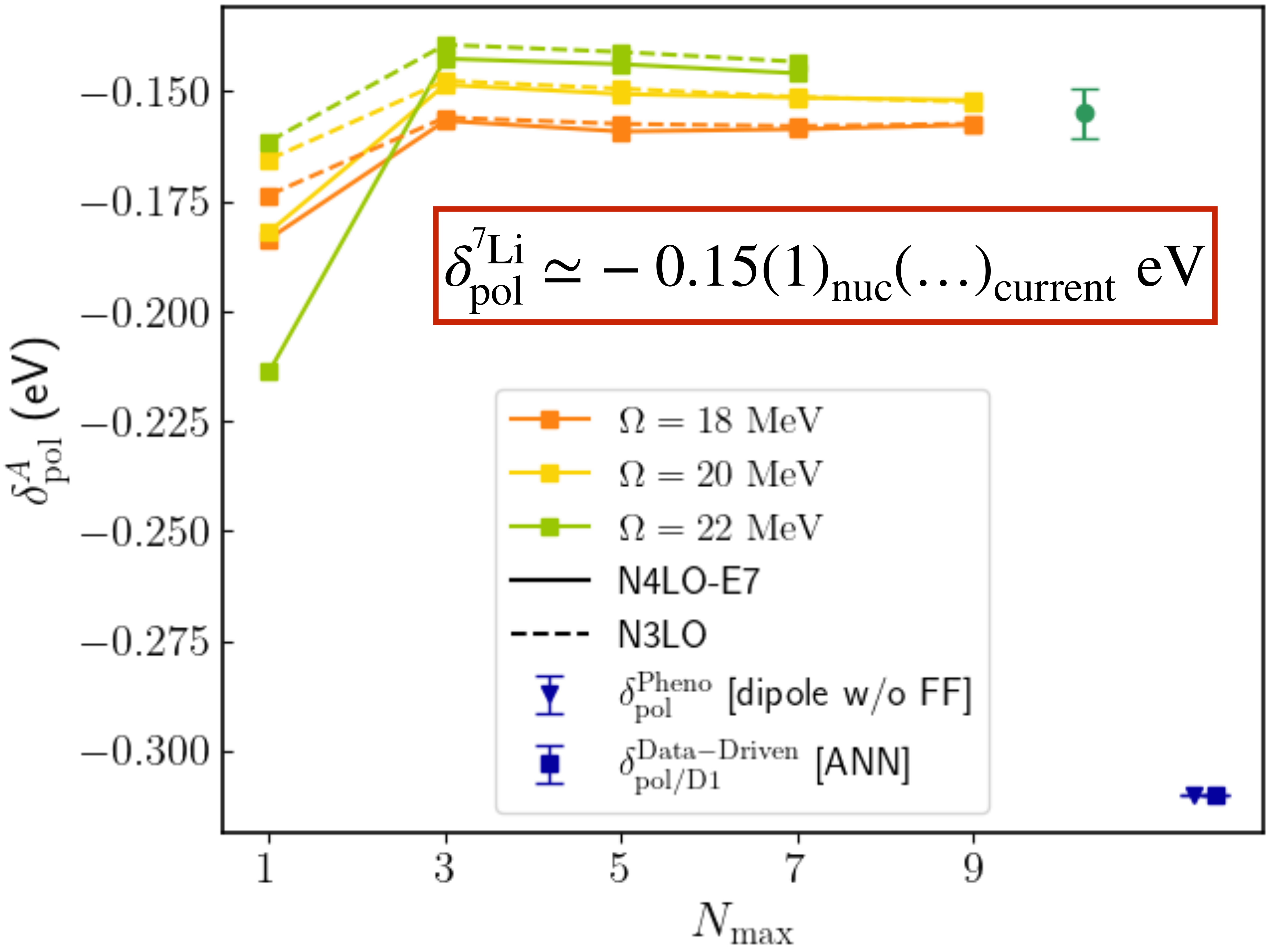

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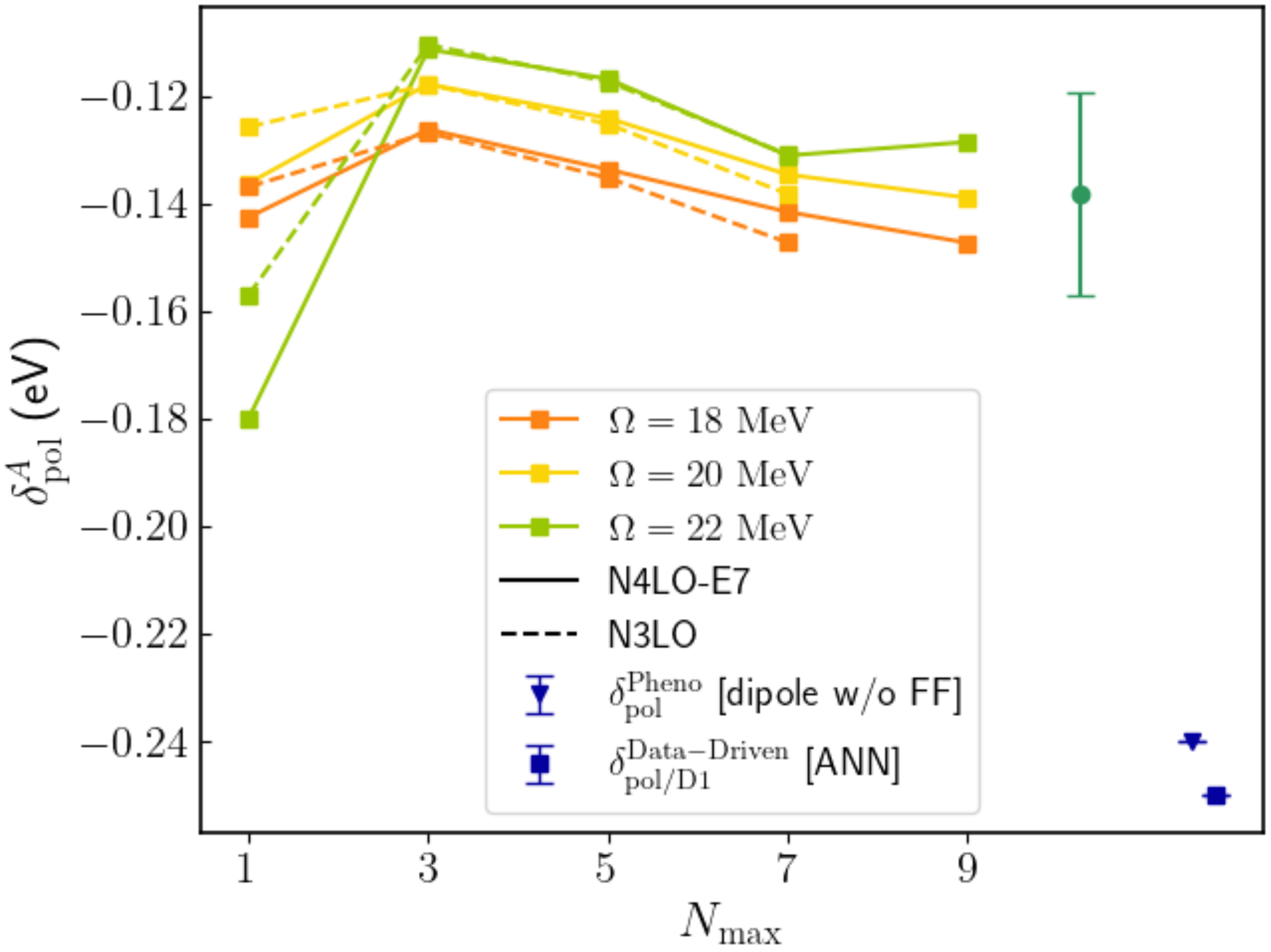
0.05 eV precision seems reachable for nuclear structure corrections!

Model-space dependence for $^{6}\mathrm{Li}$



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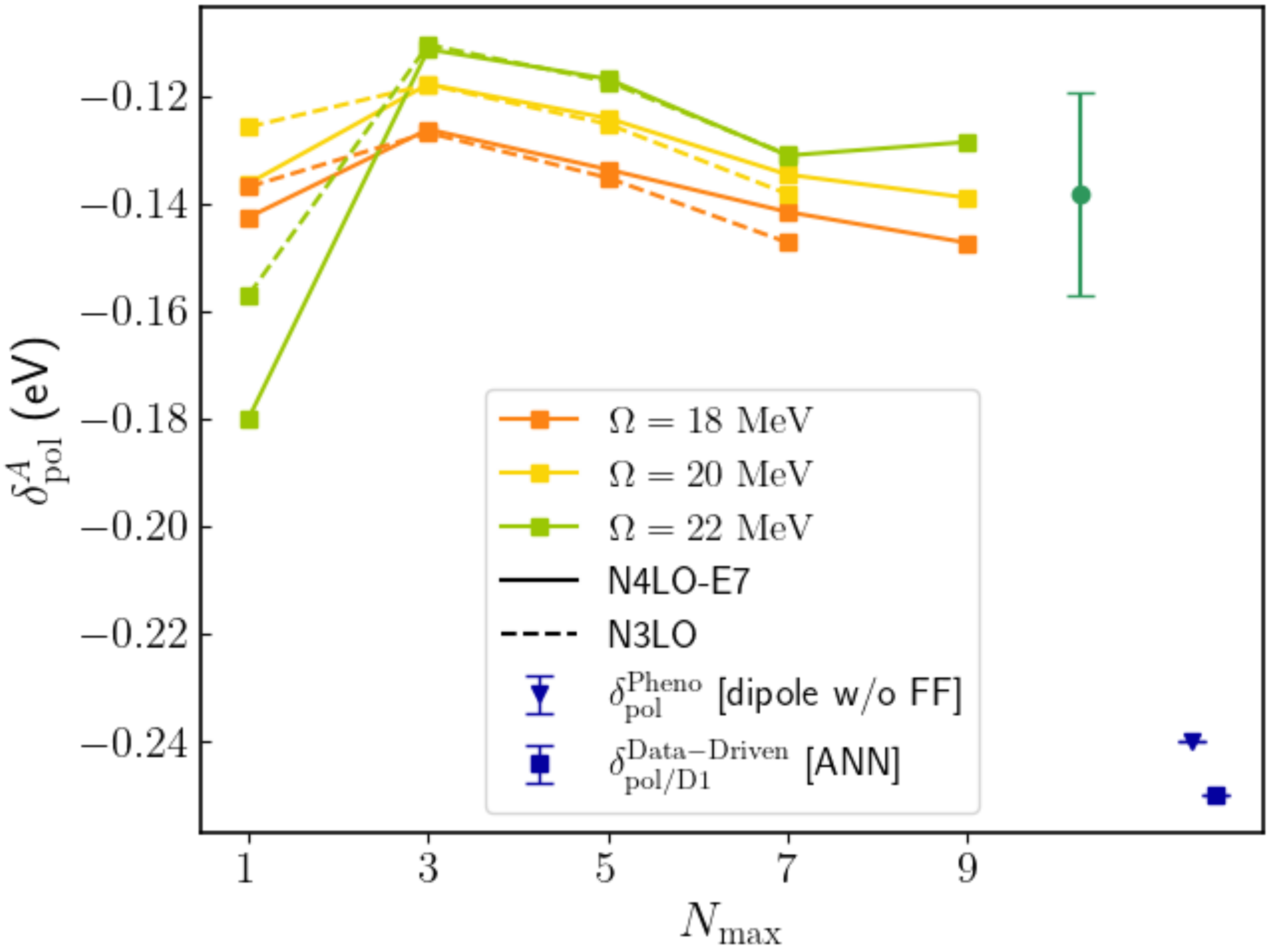

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➡ **Critical role of latest supercomputer generation**

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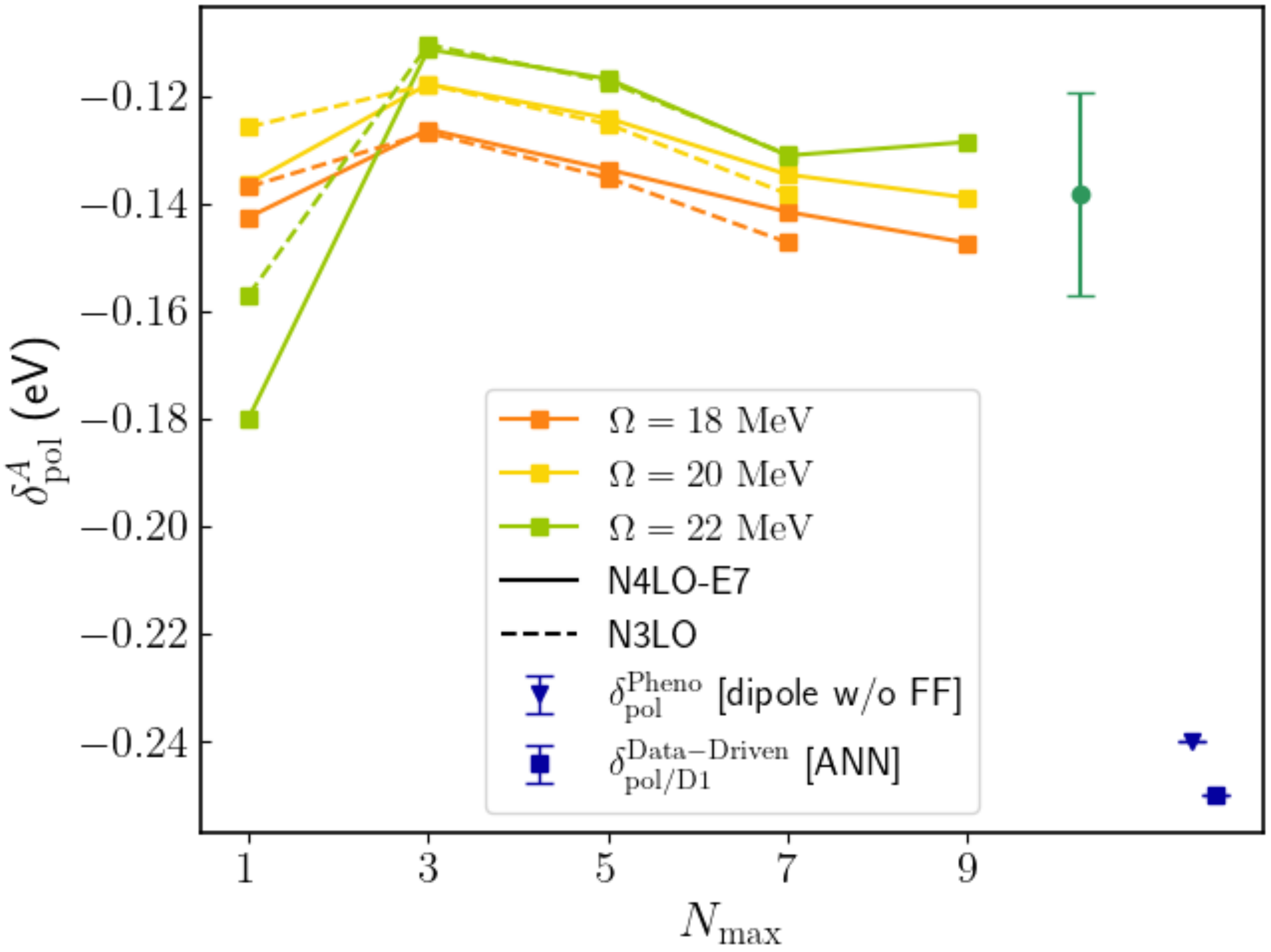

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- Uncertainty estimation
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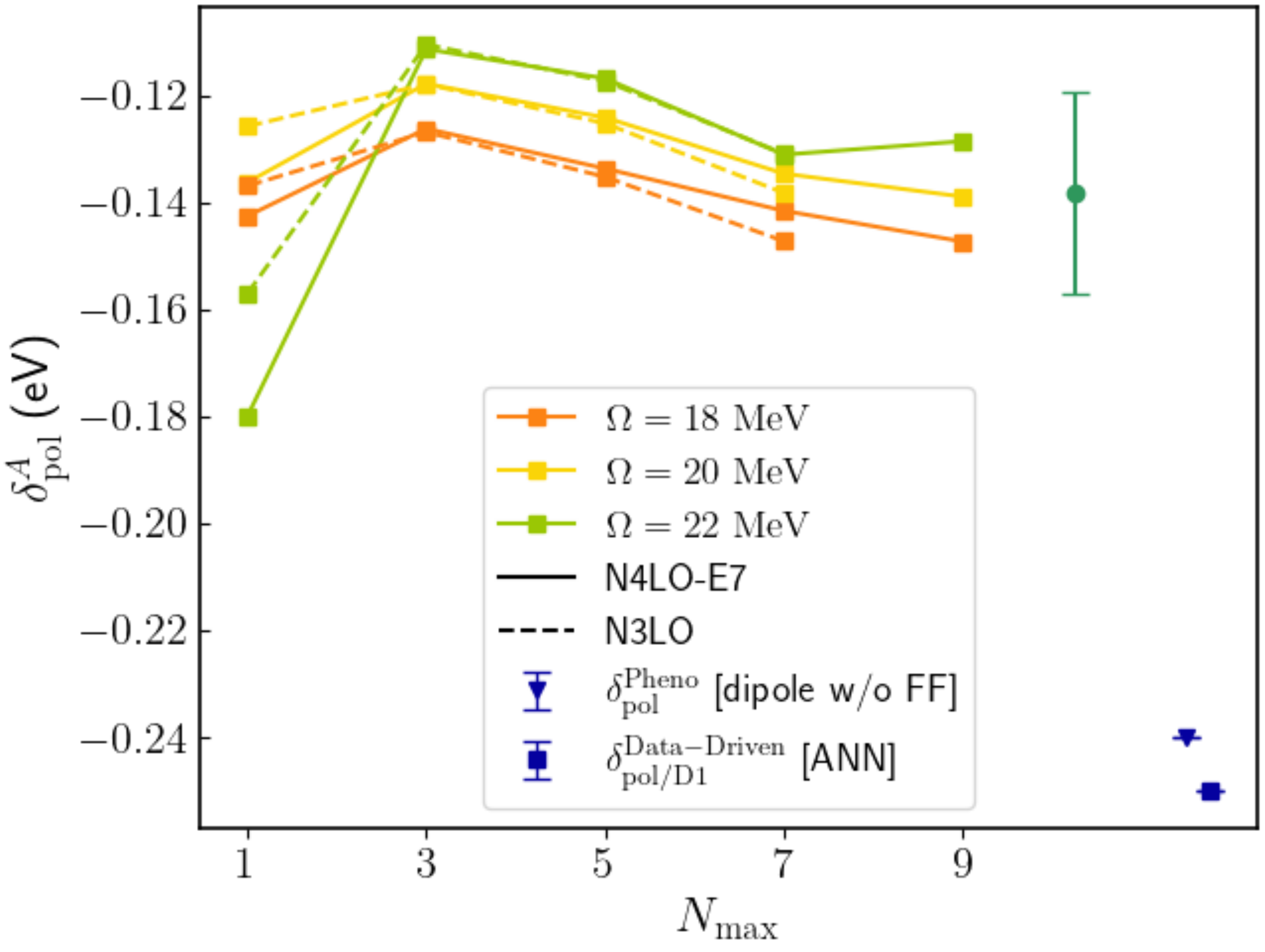

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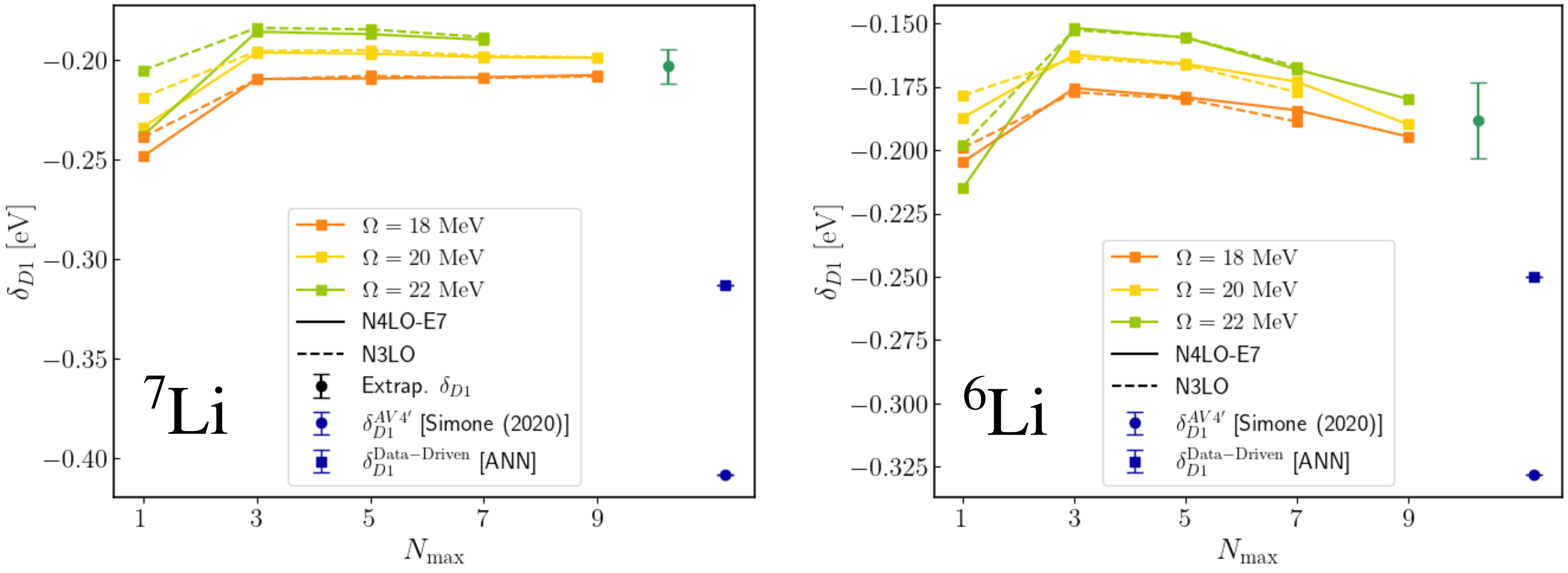
On-going results to be finalized : $\delta^{^6\mathrm{Li}}_{\mathrm{pol}} \sim -0.14\ (\ldots)_{\mathrm{nuc}}(\ldots)_{\mathrm{current}}$ eV

Benchmarking with pheno estimations



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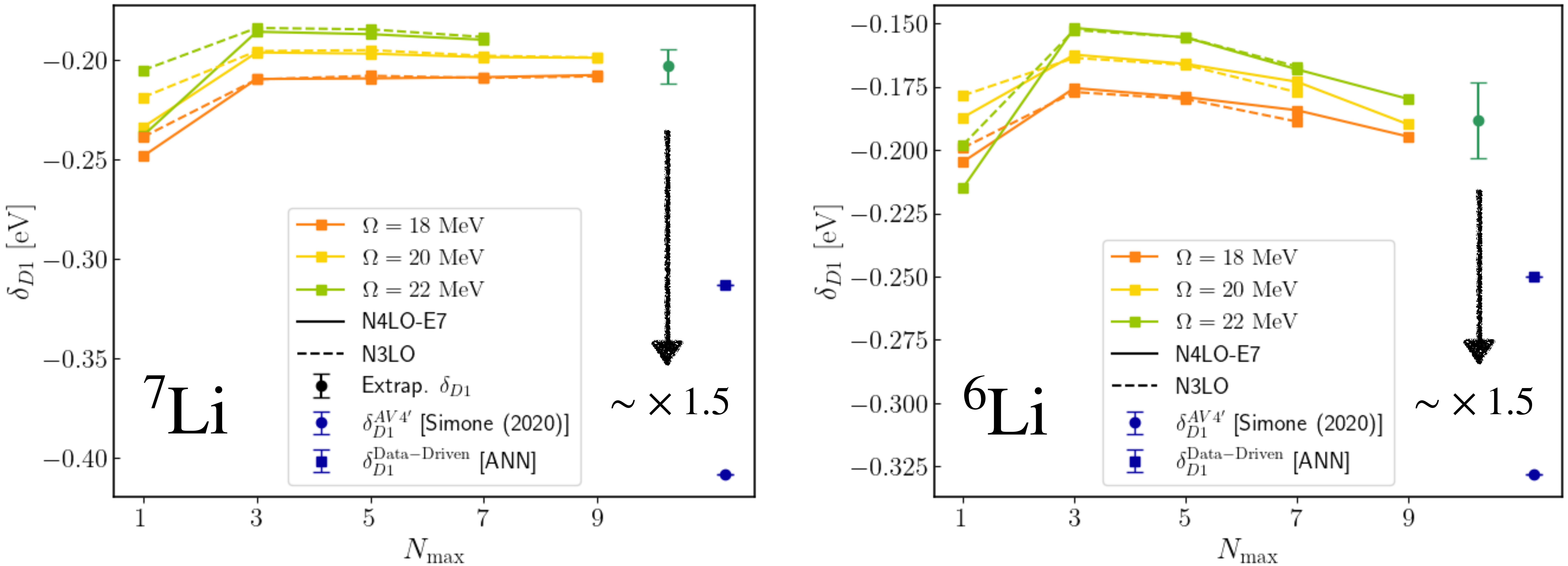

On-going investigation of discrepancy: *might* come from non-trivial (M_i, M) dependence

$$S_X(\omega, q) \equiv \sum_{J\geq 0} \sum_{N\neq 0} |\langle N | O_{X,J}(q) | \Psi \rangle|^2 \delta(E_N - E_0 - \omega)$$

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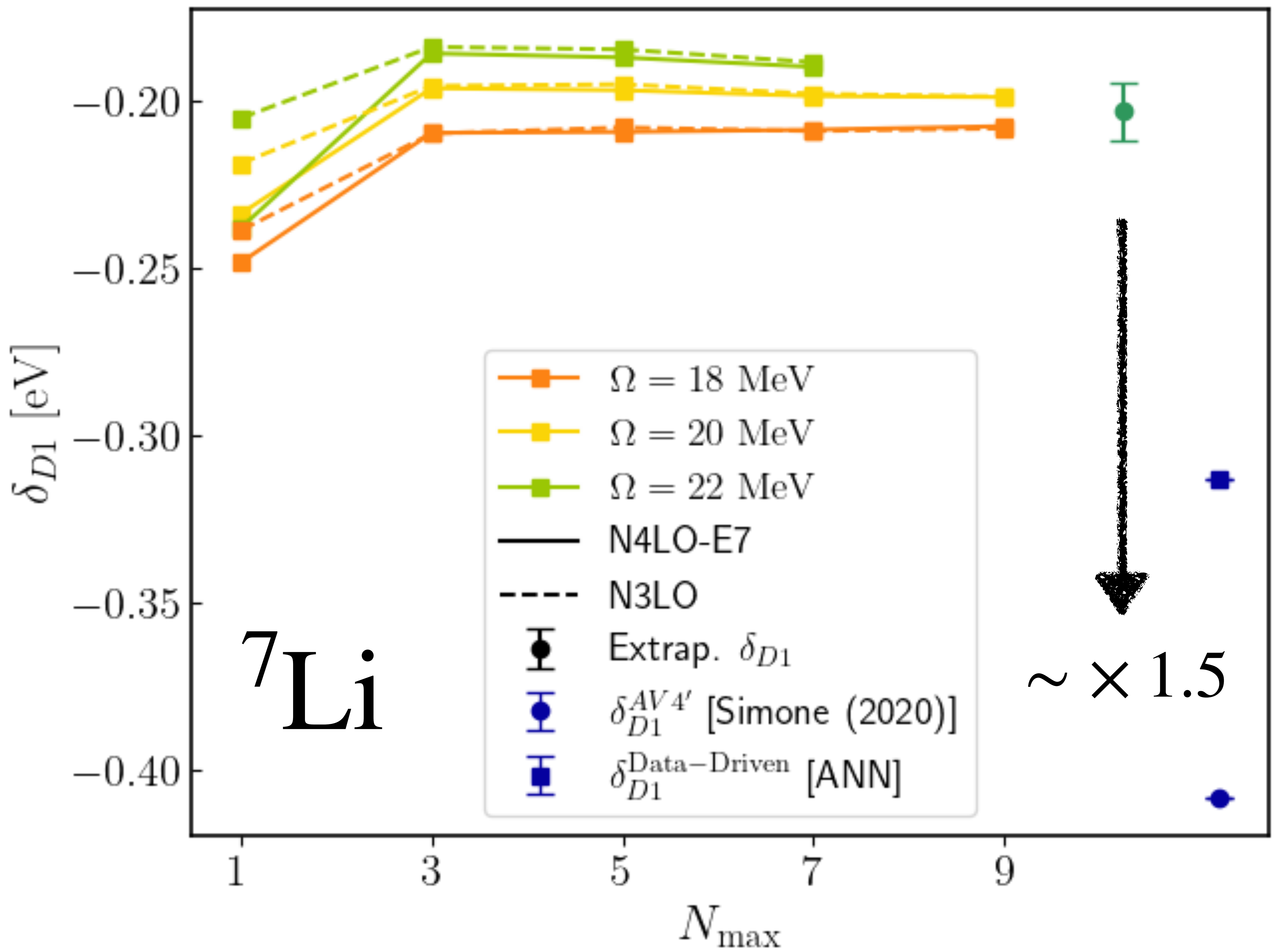

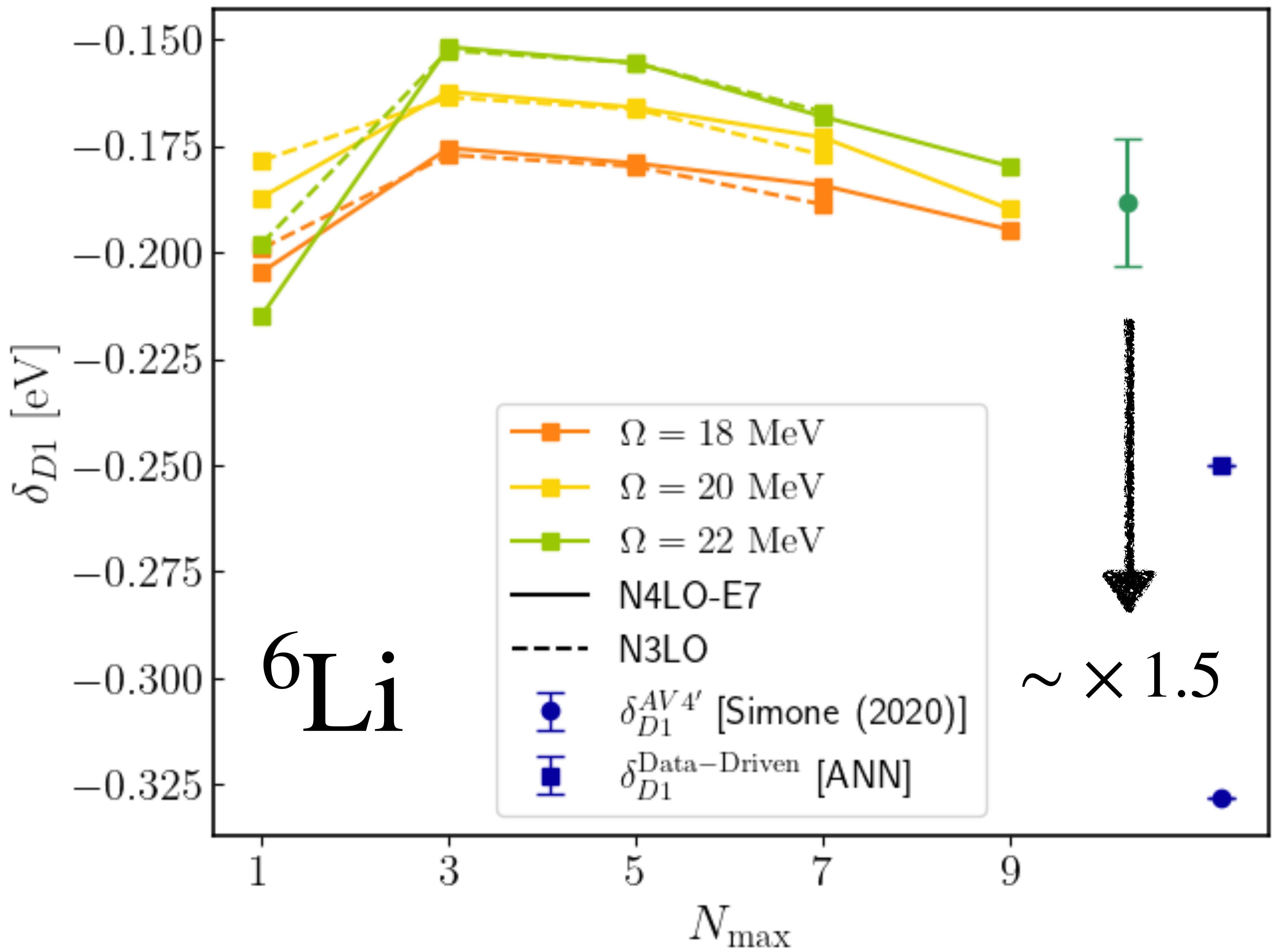

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NCSM calculations $\times (2J_i + 1)(2J + 1)$

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Conclusion

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Summary

- Muonic atoms: a precision probe for nuclear physics
  - Radii extraction: reference point + isotope-shift
  - Precise reference point: muonic atoms

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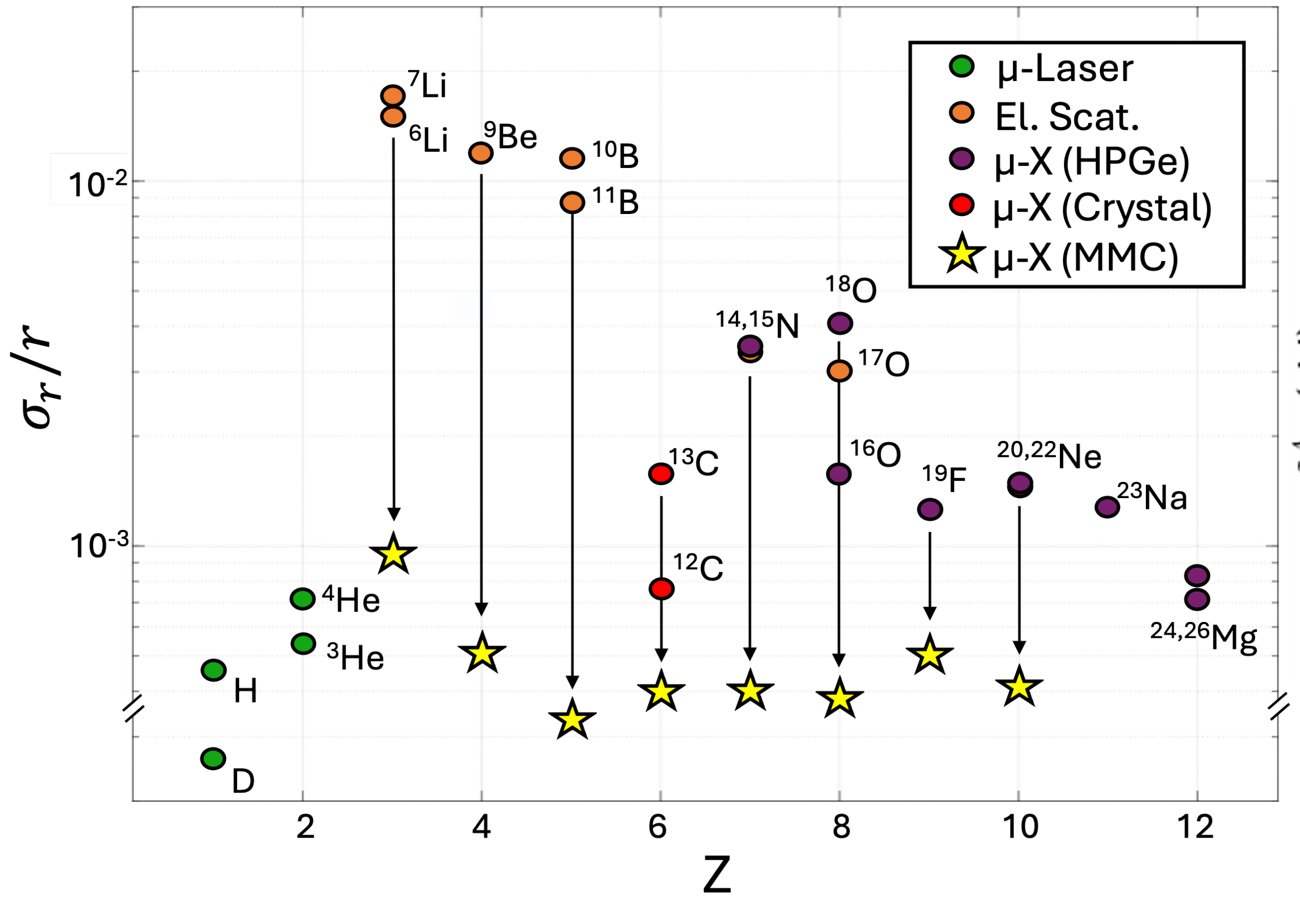

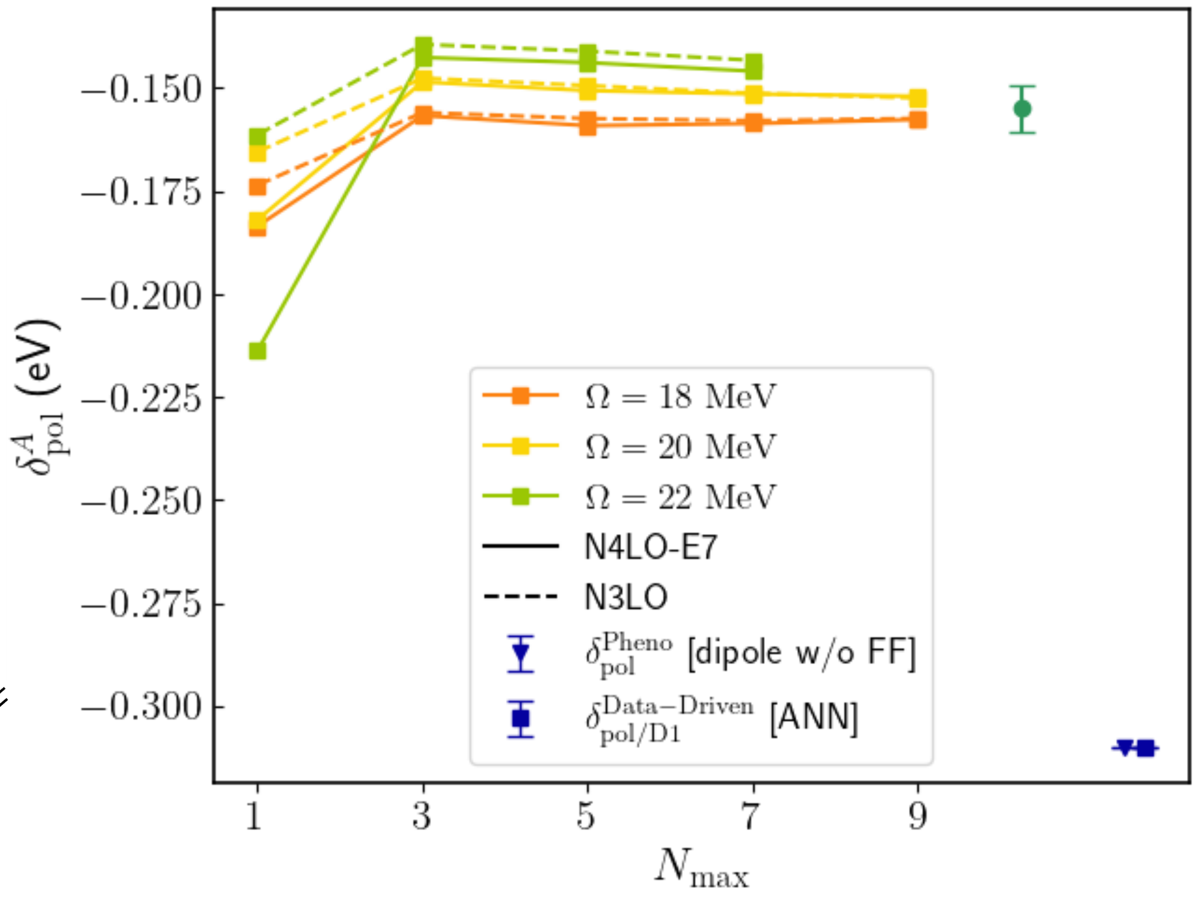

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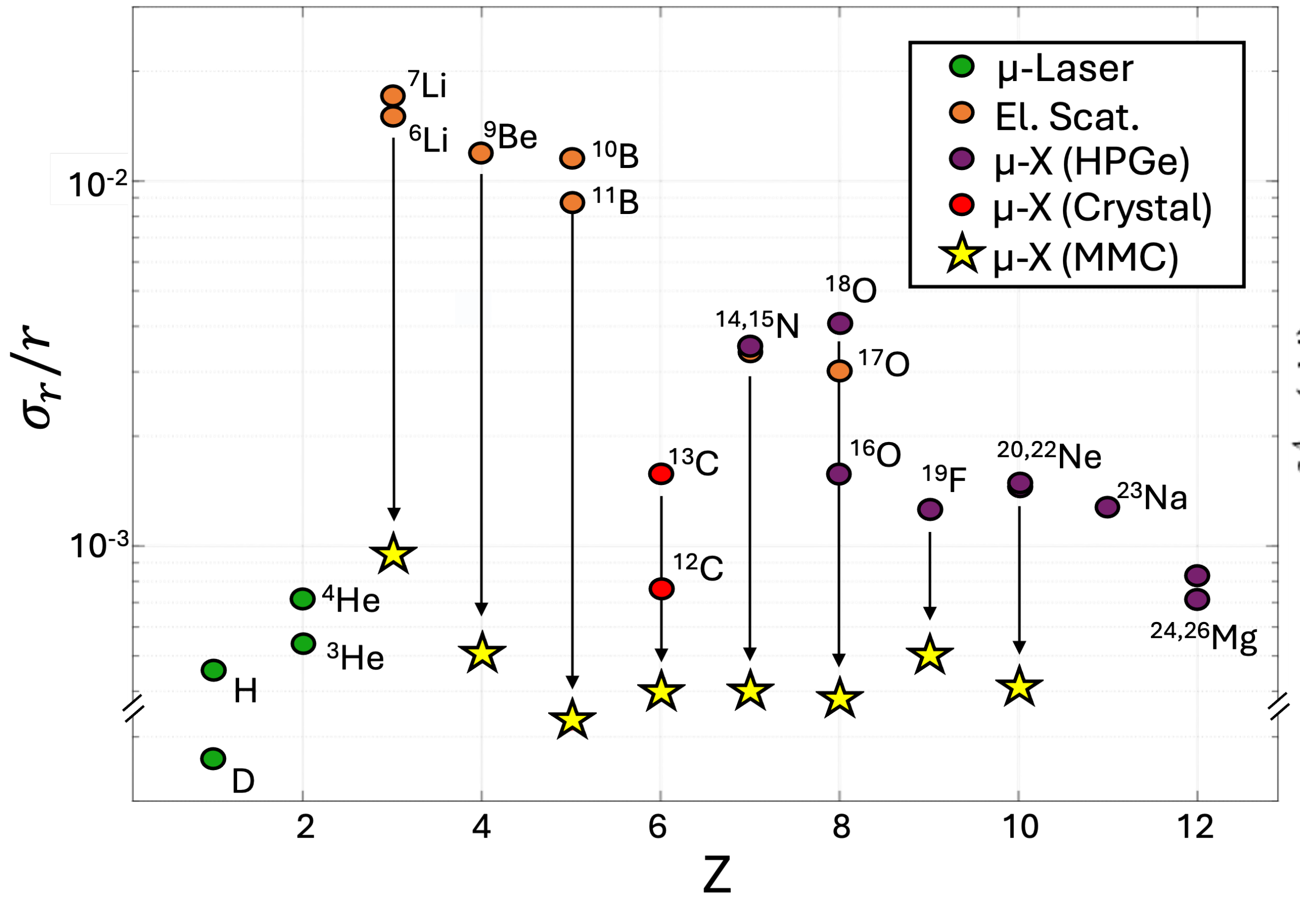
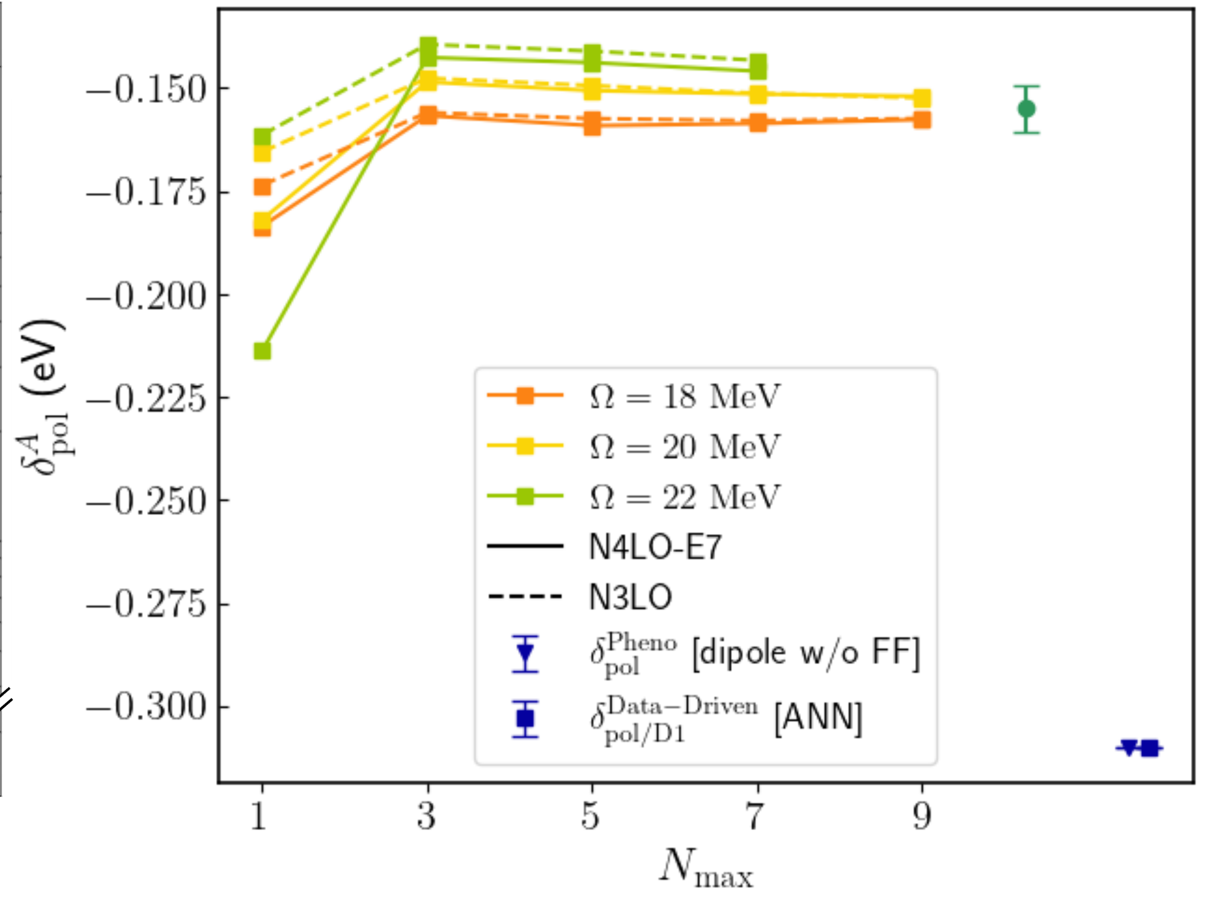
  - ➡ **QUARTET**: 10x exp. improvement for $Z \lesssim 10$
- **Nuclear polarization: reaching precision ab initio**
  - Critical nuclear theory input for: $E_{1S} \rightarrow \langle r_c^2 \rangle$
  - **Theory Goal**: 0.1 eV precision on 1S δ_{TPE}





Conclusion

Summary

- **Muonic atoms: a precision probe for nuclear physics**
  - Radii extraction: reference point + isotope-shift
  - Precise reference point: muonic atoms
  - ➡ **QUARTET**: 10x exp. improvement for $Z \lesssim 10$

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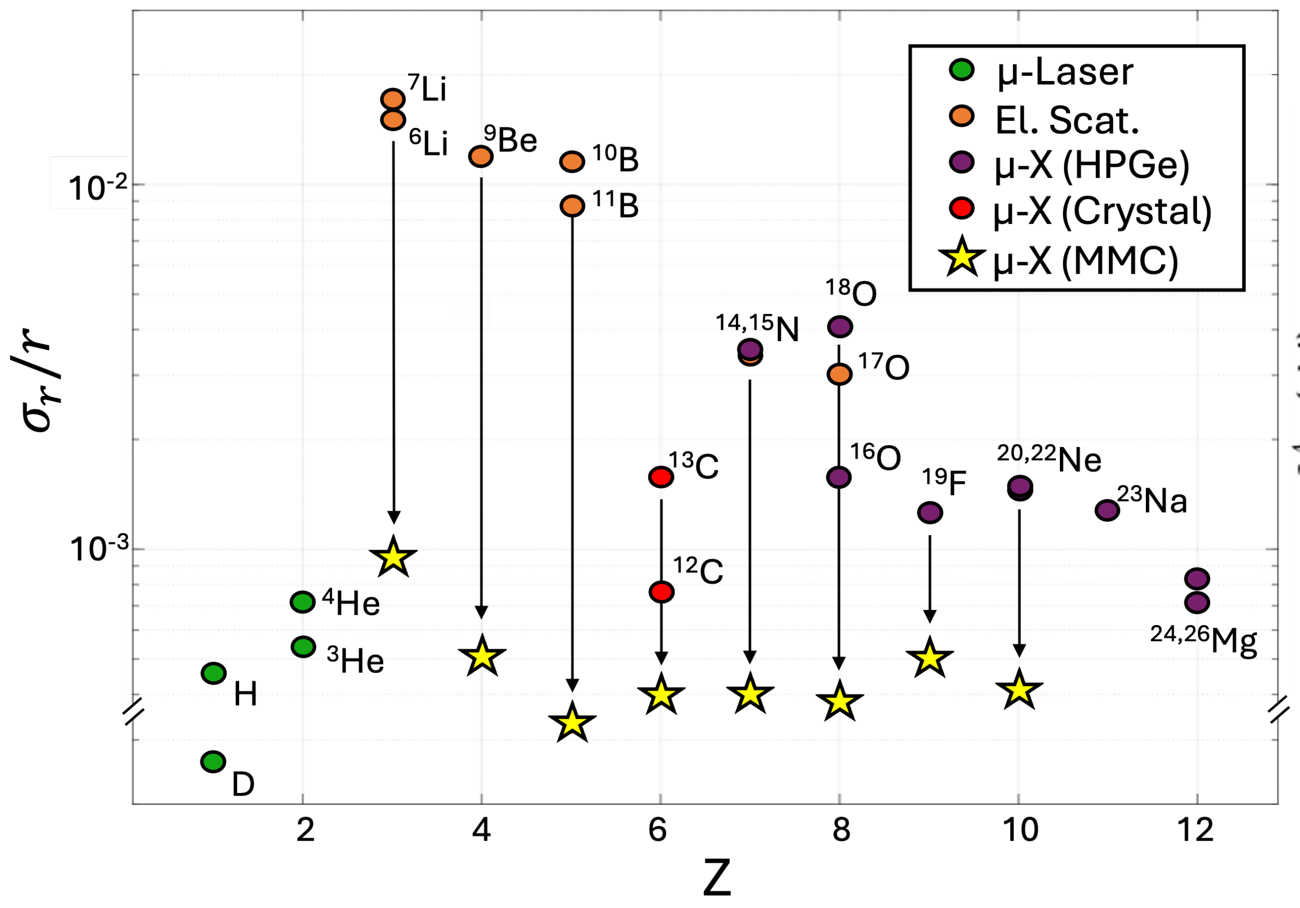

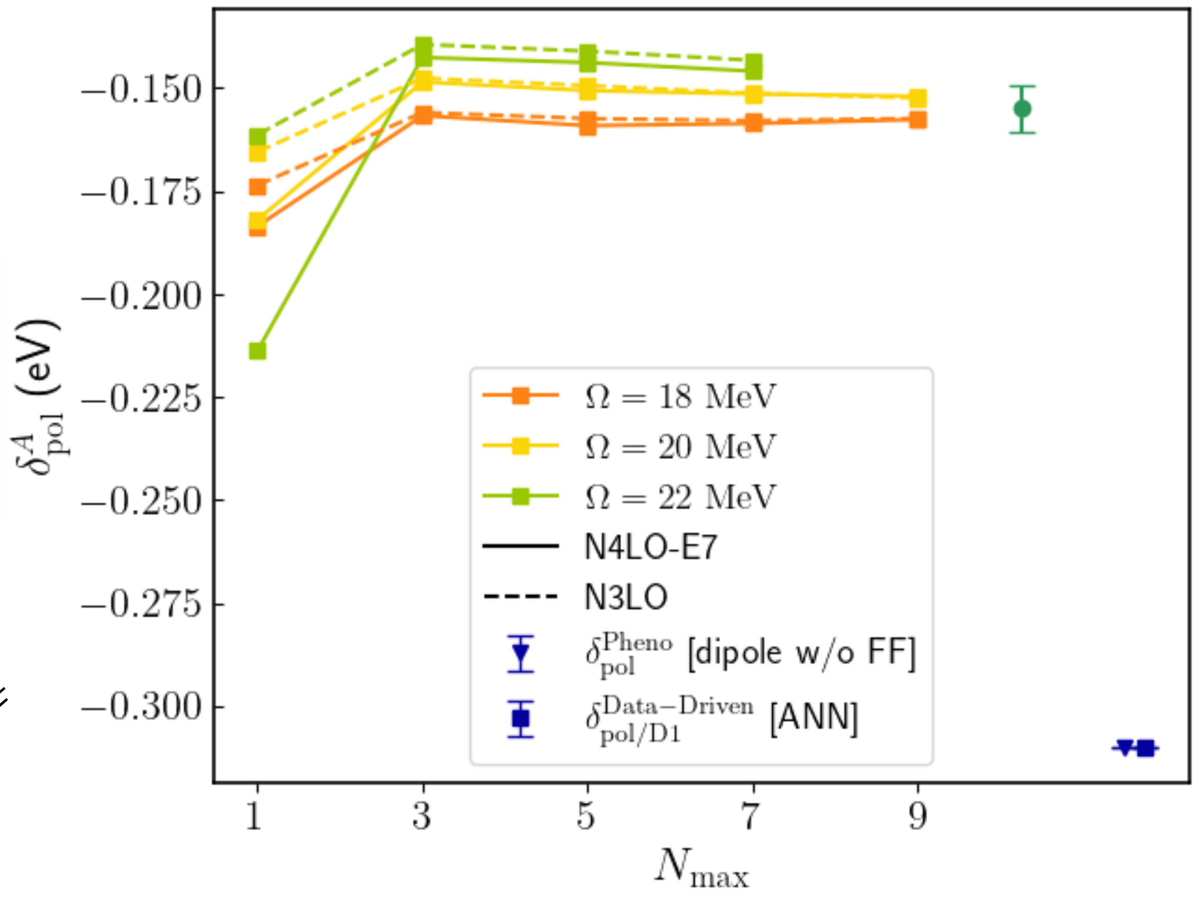

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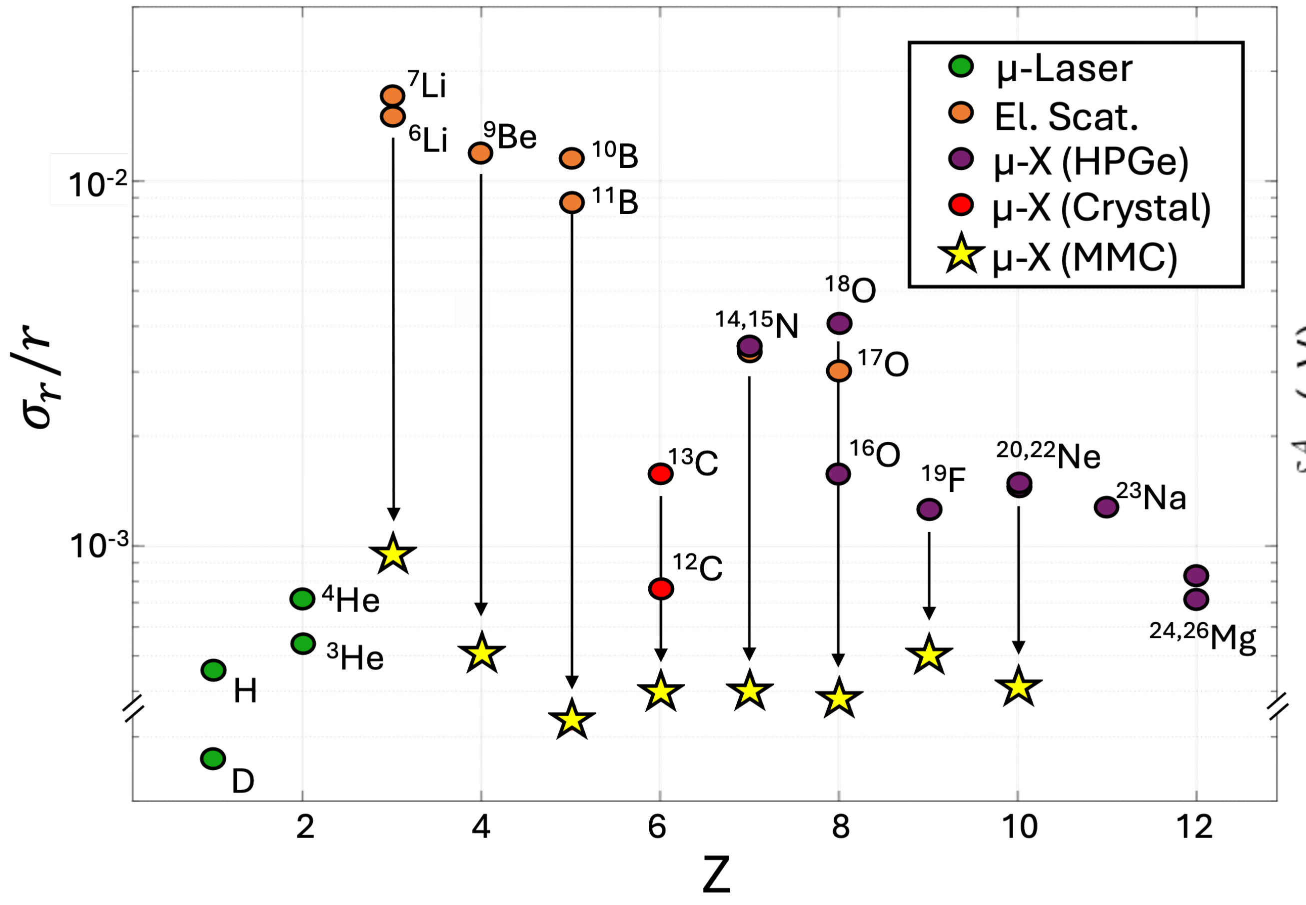

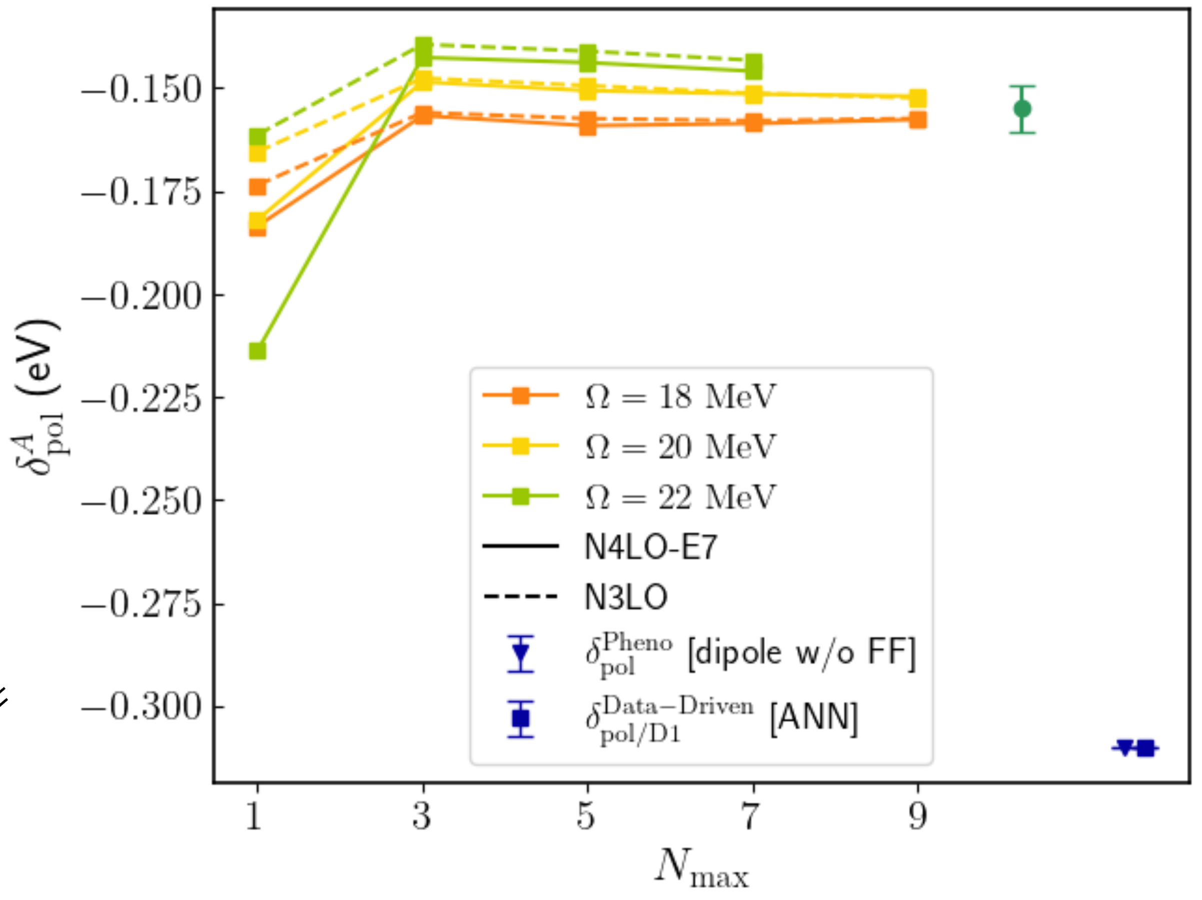

Outlook

- **Completing on-going ab initio calculation**
  - Complete (M_i, M) dependence + uncertainty analysis
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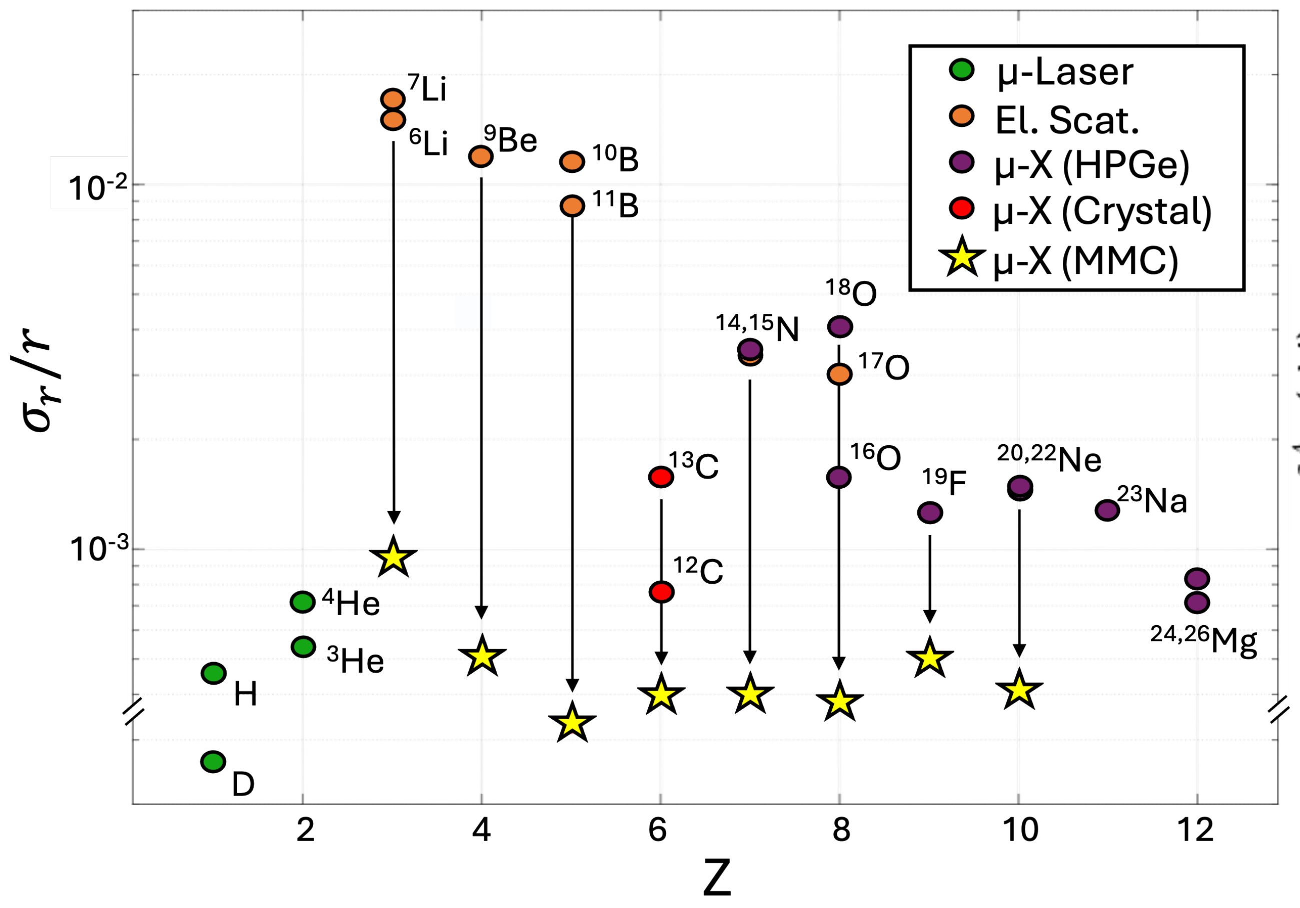

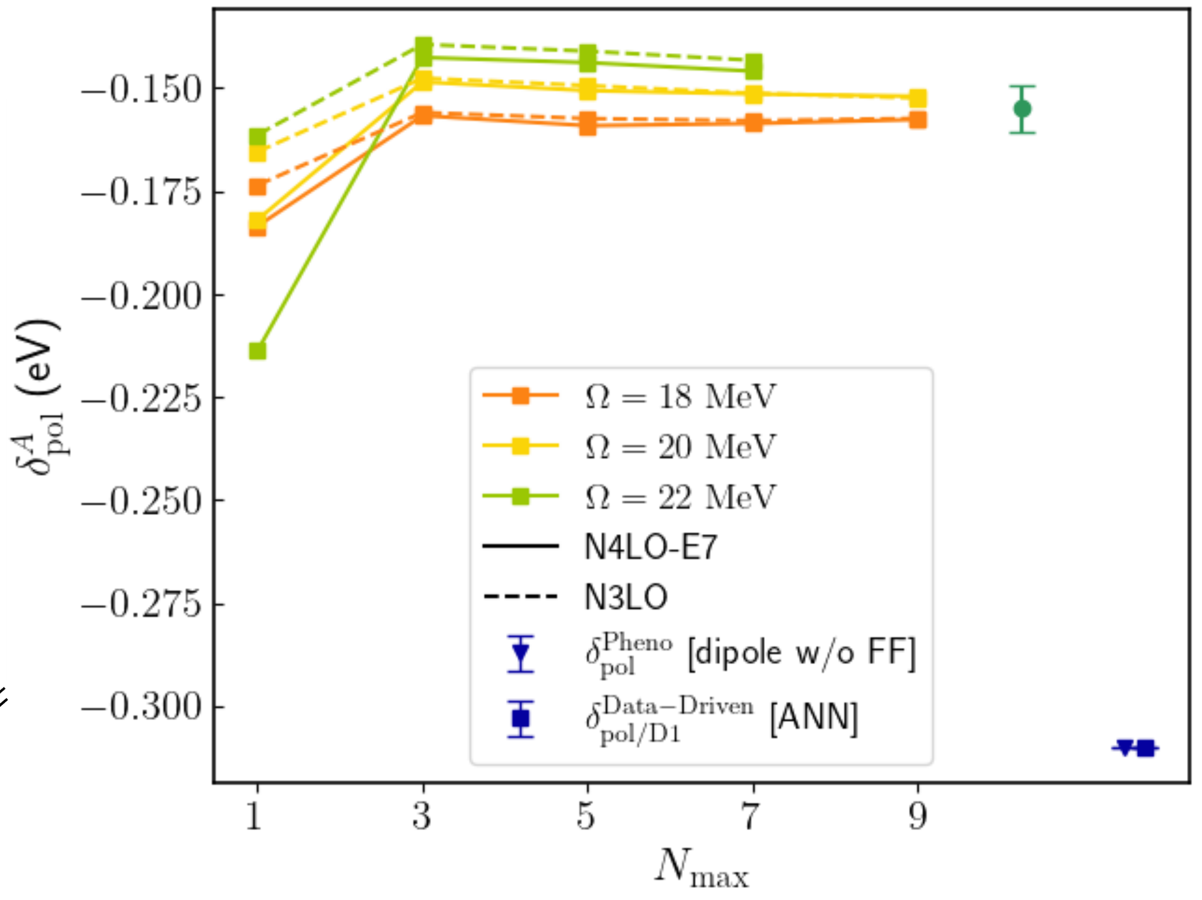

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  - Complete (M_i, M) dependence + uncertainty analysis
  - Extension to ^{9}Be, $^{10-11}$B $\Rightarrow$ **See Diego's poster!**
- Towards better controlling theoretical uncertainty
  - Shifting from pheno towards EFT approach
  - EFT based on **potential-NRQED** for $Z > 1$

[Peset et al., EPJA (2015)]

THANK YOU!

STRONGINT Group

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QUARTET Collaboration

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Mikhail Gorchtein — **Tim Egert**