



# EXPLORING QUARK MASS DEPENDENT THREE-NUCLEON FORCES IN MEDIUM MASS-NUCLEI

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arXiv:2512.20454

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**LOEWE**

Exzellente Forschung für  
Hessens Zukunft

# OUTLINE

## INTRODUCITON

- Enhanced 3N forces?
- Choice of interaction and calculational details

## FITTING STRATEGIES

- Comparison of fitting strategies
- Why is the effect of  $F_2$  large?

## ENERGY AND RADII OF CLOSED-SHELL NUCLEI

## CONCLUSIONS

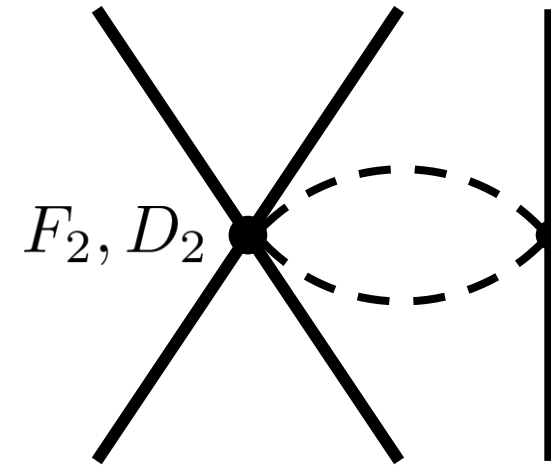
# ENHANCED 3N FORCES?

V. Cirigliano et al., Phys. Rev. Lett. 135, (2025)



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

- Using **KSW** power counting scheme, 3N topologies were identified that:
  - should contribute at N<sup>3</sup>LO rather than N<sup>5</sup>LO (enhanced by  $Q^{-2}$  compared to naive dimensional analysis)
  - include a quark-mass dependant  $m_\pi^2 (\bar{N}N)^2 \pi^2$  vertex ( $D_2$ ),
  - and a momentum dependent  $(\bar{N}N)^2 \pi^2$  vertex ( $F_2$ )
  - seem to play an important role in symmetric and neutron matter
- The suggested range is  $\pm 1/(5f_\pi^4) \rightarrow$  our normalization  $\pm 1/5 = \pm 0.2$

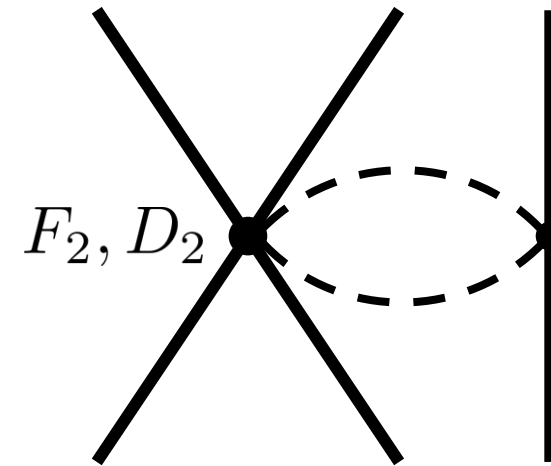
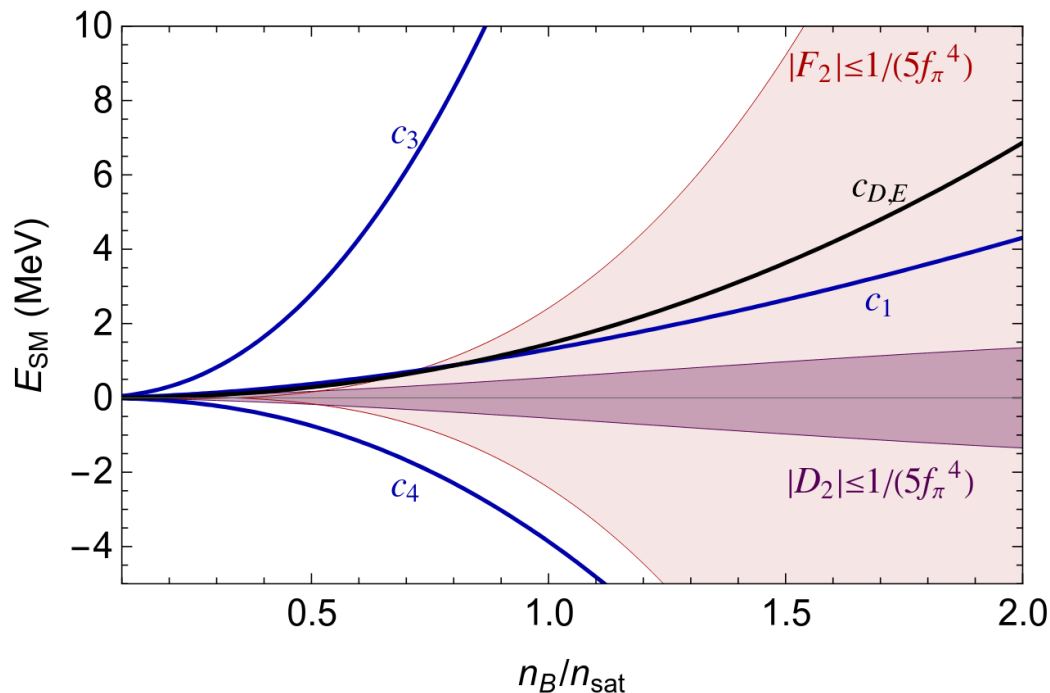


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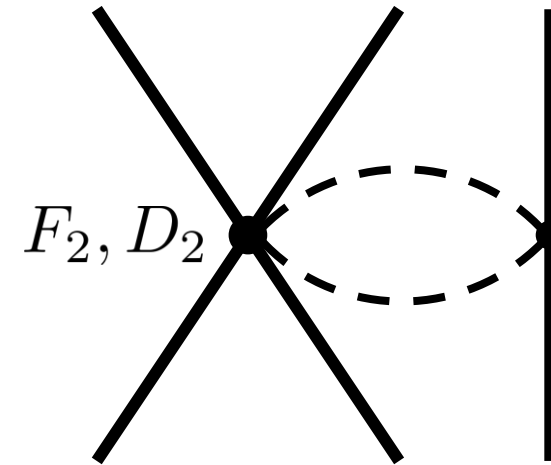
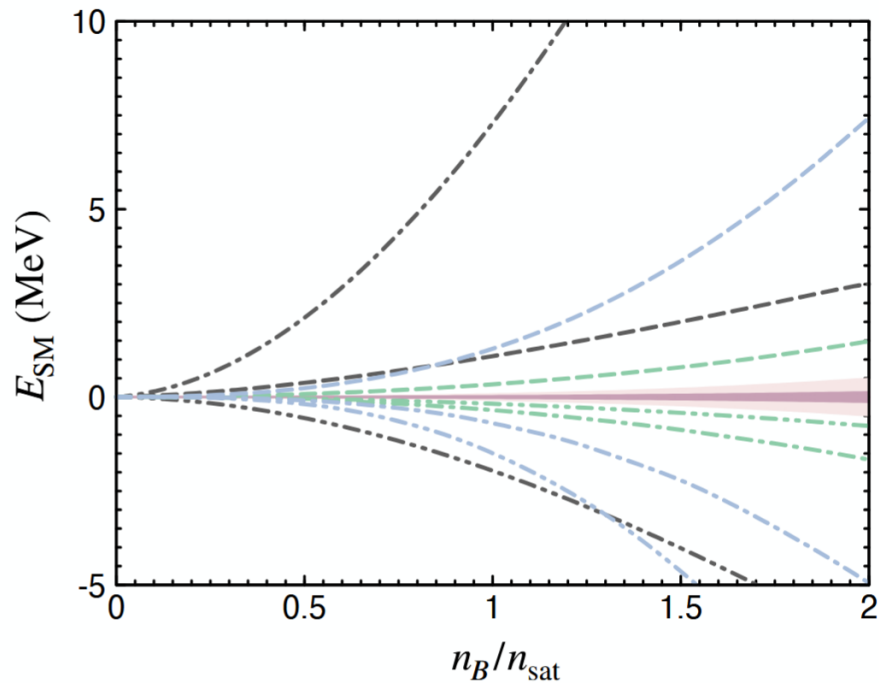


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- The effect is however scheme and regulator dependent
- By removing the dependence, the contributions are within the expectations of Weinberg PC and NDA.

E. Epelbaum et al., arXiv:2512.14117 [nucl-th] (2025)



# CHOICE OF INTERACTION & CALCULATION DETAILS

**INTERACTION: EMN** D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96, (2017).

- 2N and 3N treated at consistent chiral order → here: both N<sup>2</sup>LO and N<sup>3</sup>LO
- Bare ( $\lambda_{\text{SRG}} = \infty$ ,  $\Lambda_{2N} = \Lambda_{3N} = 450$  MeV) or SRG-evolved ( $\lambda_{\text{SRG}}^{2N} = 1.8$  fm<sup>-1</sup>,  $\Lambda_{3N} = 2.0$  fm<sup>-1</sup>  $\approx 394 \dots$  MeV)

LEC	$c_1$	$c_3$	$c_4$	$c_D$	$c_E$	$F_2$
N <sup>2</sup> LO/N <sup>3</sup> LO	-0.74/-1.20	-3.61/-4.43	2.44/2.67	<i>to fit</i>	<i>to fit</i>	<i>to fit</i>

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## CALCULATION DETAILS

- IMSRG(2),  $e_{\text{max}} = 14$ ,  $E_{3\text{max}} = 24$ ,  $\hbar\omega = 16$

# REFITTING

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- 2 fitting approaches (few-body only vs. including many-body data)



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3. Constrain  $c_D$
4. Compute observables



Approach	Constraint on $c_D$
few-body	${}^3\text{H}$ half-life ( <b>GT</b> )
many-body	${}^{16}\text{O}$ charge radius (and BE) ( <b><math>{}^{16}\text{O}</math></b> )

D. Gazit et al., Phys. Rev. Lett. 103, (2009); P. Klos et al Eur. Phys. J. A 53, (2017).

T. Hüther, et al. Phys. Lett. B (2020),  
P. Arthuis et al., arXiv:2411.00097 [nucl-th] (2024).

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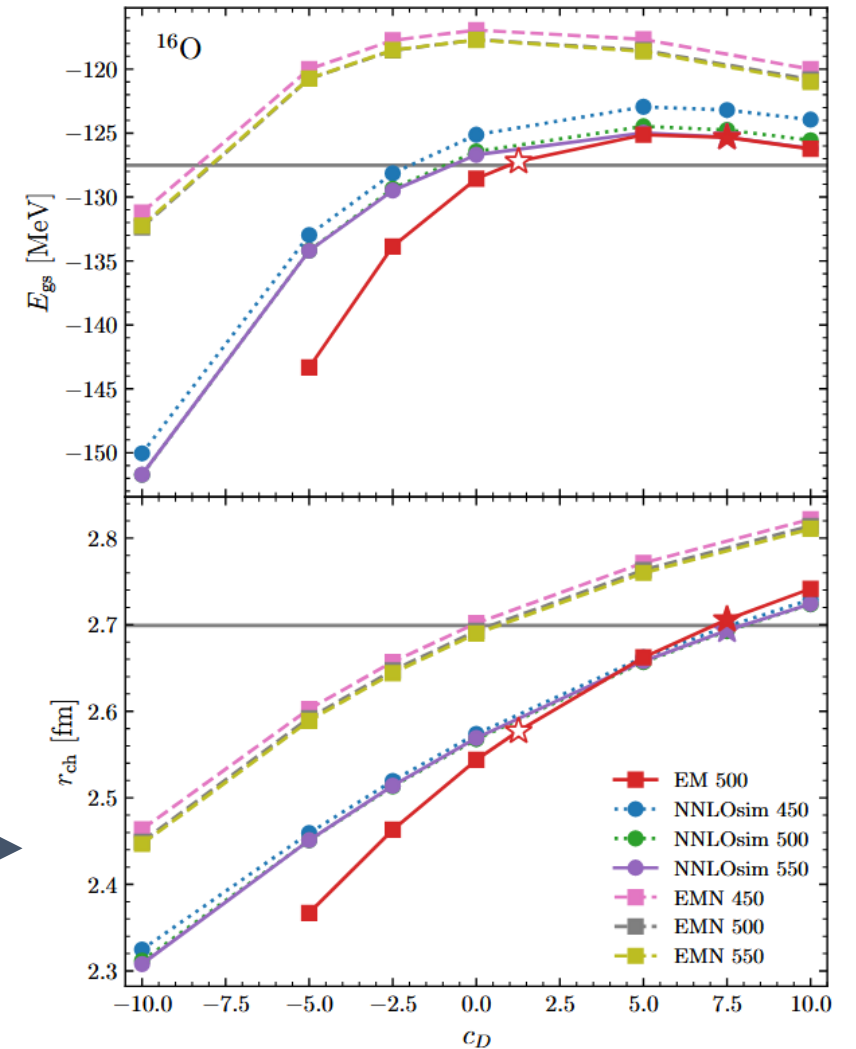


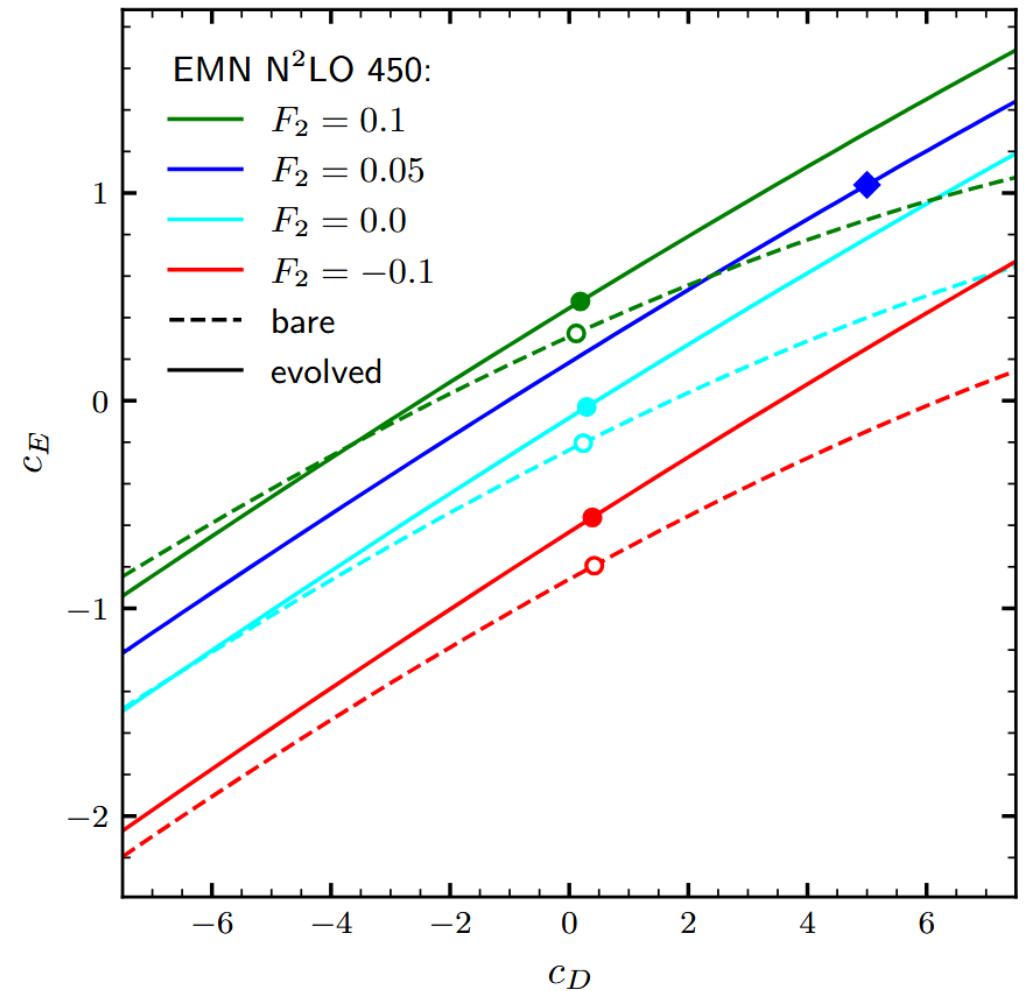
Figure: P. Arthuis et al., arXiv:2411.00097 [nucl-th] (2024)

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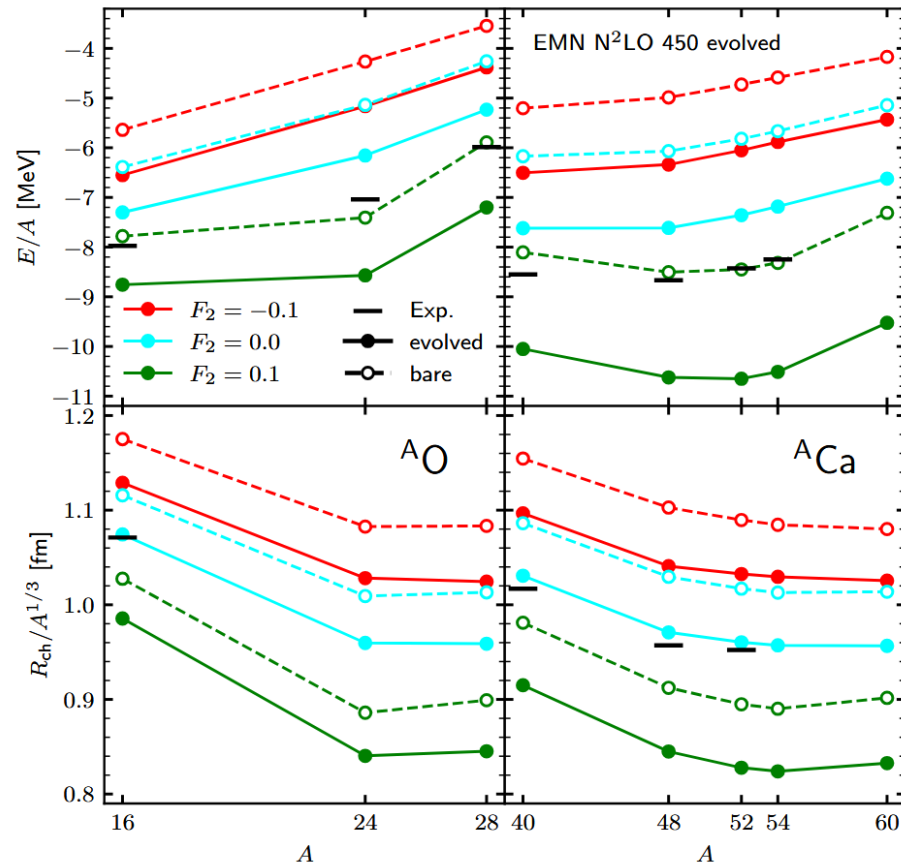


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# COMPARISON OF FITTING STRATEGIES

## GT STRATEGY

- Energies and radii cannot be reproduced at the same time
- To change energy and radius independently, one must be able to vary  $c_D$



Label	Order	$\lambda$ (fm <sup>-1</sup> )	$\Lambda_{3N}$	$F_2$	$c_D$	$c_E$	Fit
○	N <sup>2</sup> LO	$\infty$	450 MeV	-0.1	0.421	-0.794	GT
○	N <sup>2</sup> LO	$\infty$	450 MeV	0.0	0.235	-0.204	GT
○	N <sup>2</sup> LO	$\infty$	450 MeV	0.1	0.118	0.324	GT
●	N <sup>2</sup> LO	1.8	2.0 fm <sup>-1</sup>	-0.1	0.389	-0.562	GT
●	N <sup>2</sup> LO	1.8	2.0 fm <sup>-1</sup>	0.0	0.294	-0.030	GT
●	N <sup>2</sup> LO	1.8	2.0 fm <sup>-1</sup>	0.1	0.188	0.478	GT

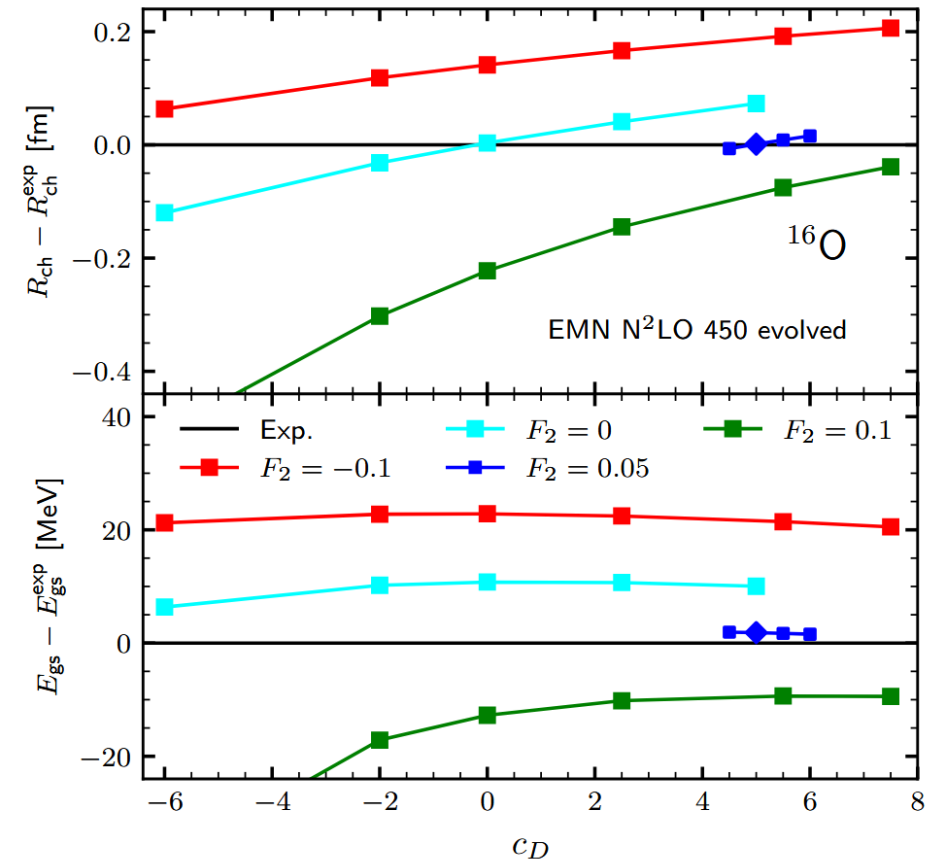
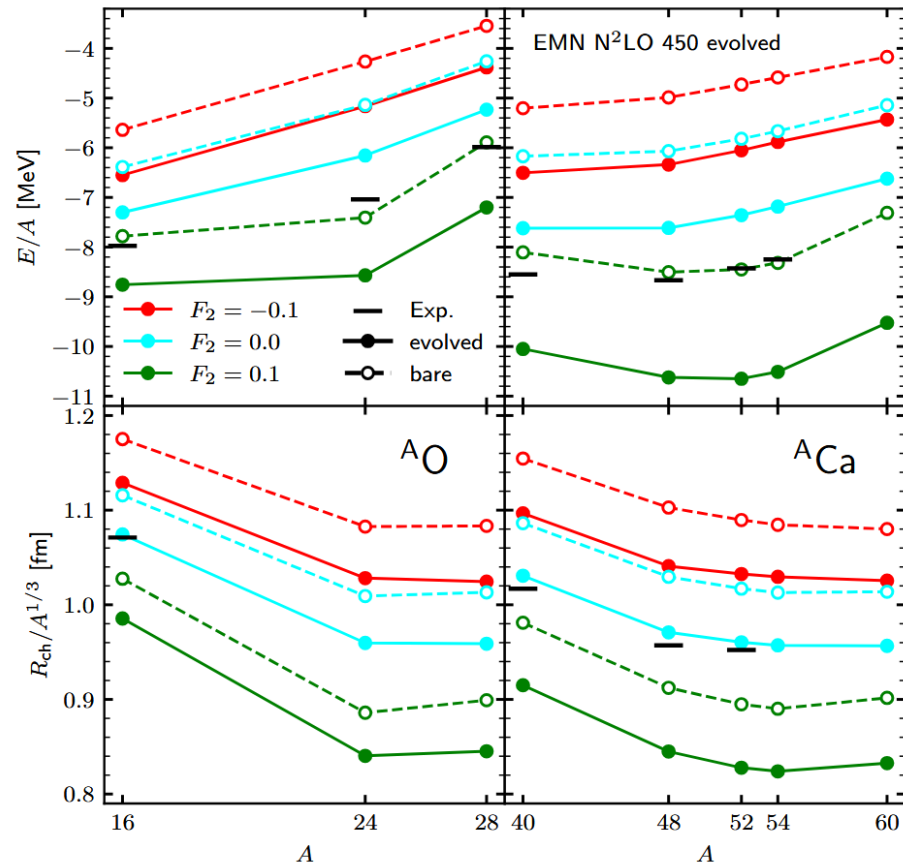
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## $^{16}\text{O}$ STRATEGY

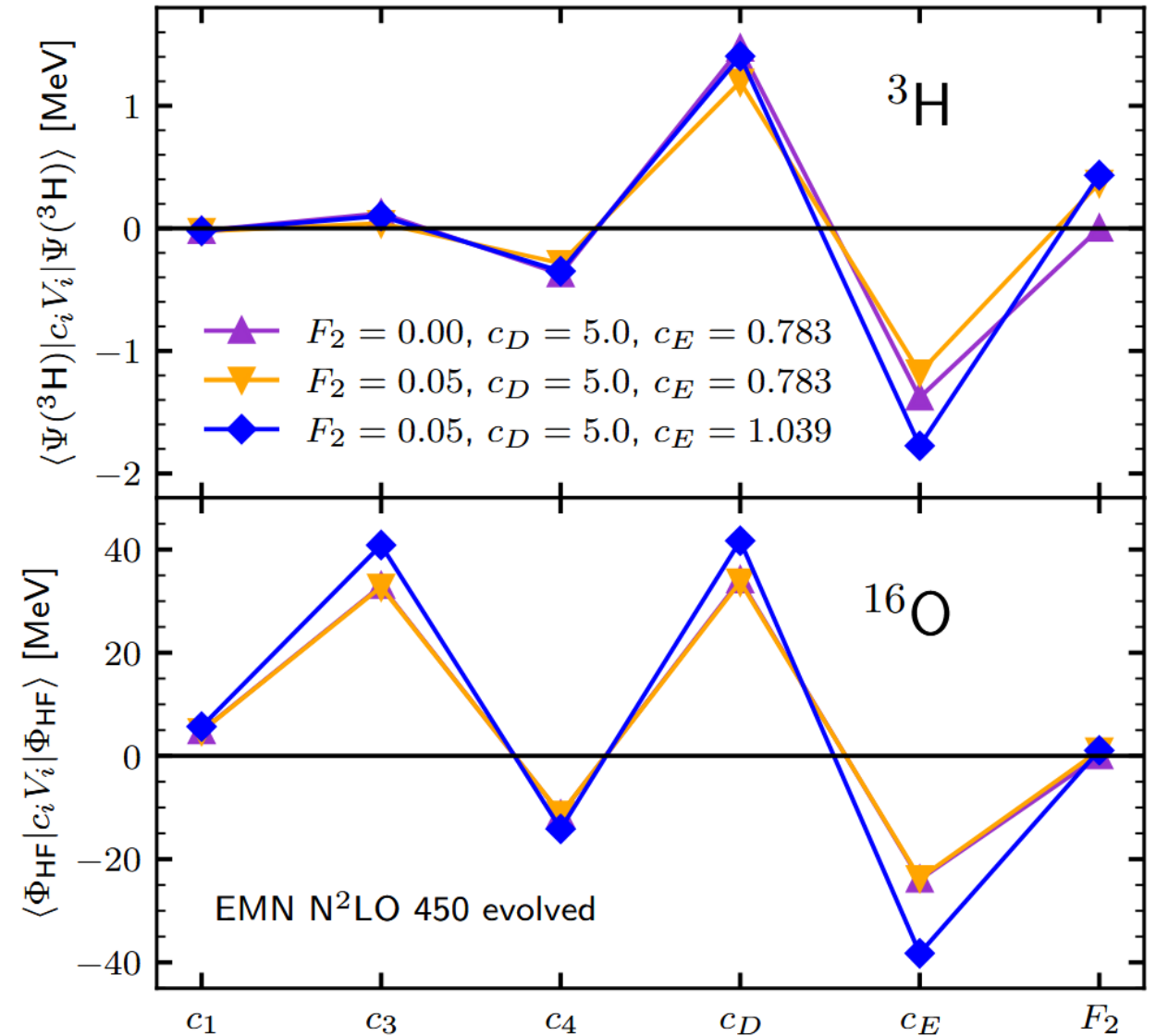
- Energies and radii can be reproduced at the same time
- Works similarly well for both chiral orders for the same  $F_2$



$$F_2^{\text{evolved}} = 0.05, F_2^{\text{bare}} = 0.15$$

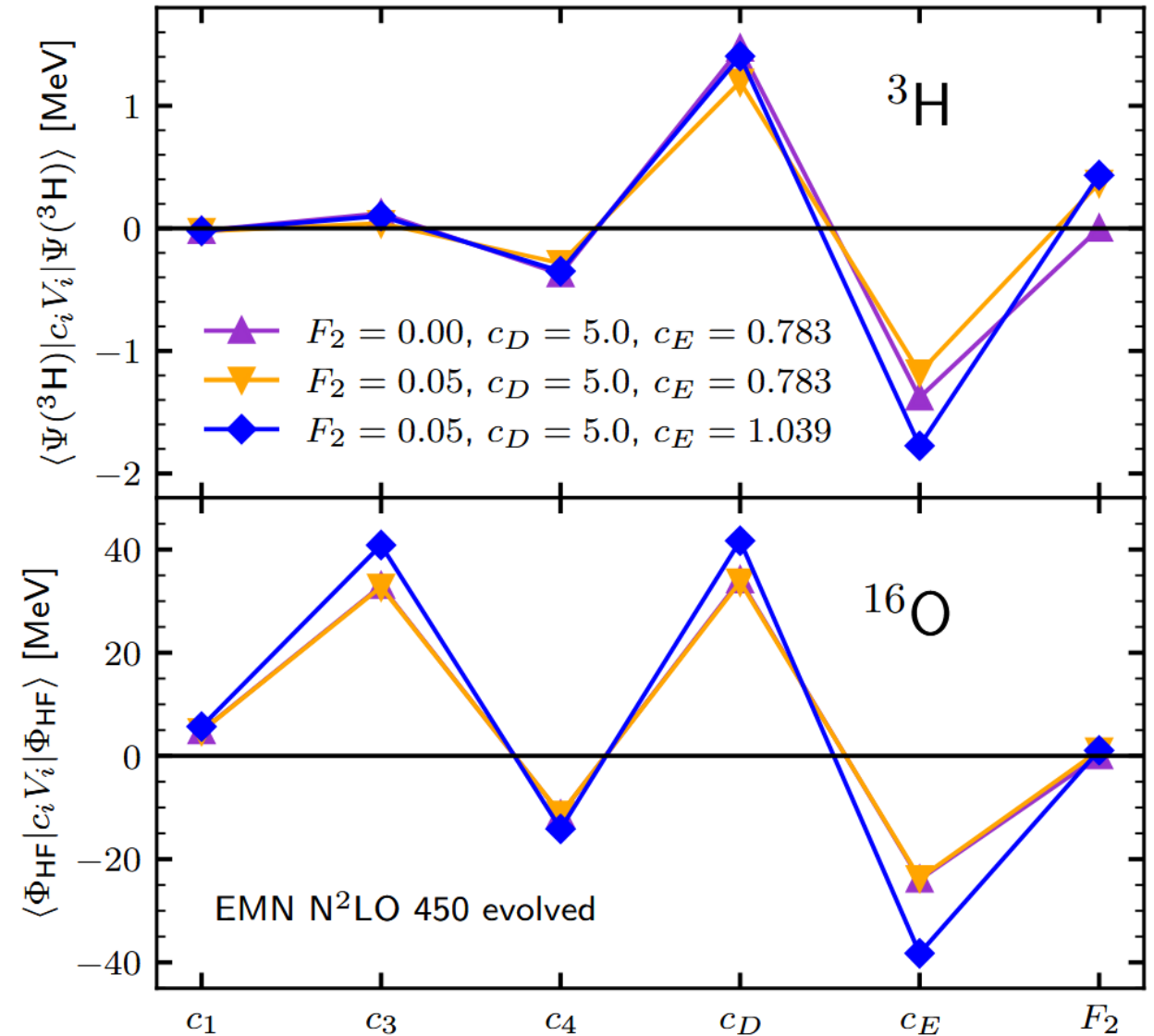
# WHY IS THE EFFECT OF $F_2$ SO LARGE?

- Is it safe/consistent to promote individual topologies?
  - case study:  $c_D$  and  $c_E$  exhibit fine cancellations
  - counter terms to  $F_2$  might need to be promoted
- $F_2$  has a small contribution to the energy but fitting rearranges other topologies significantly

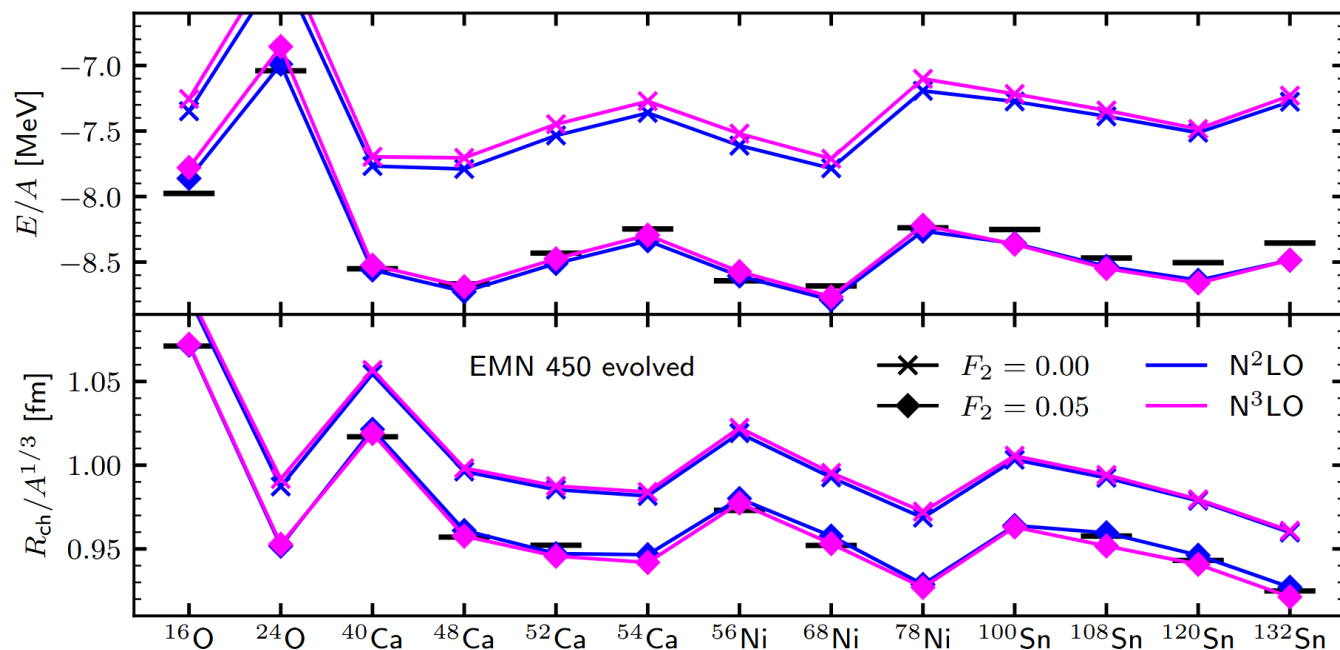


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- $F_2$  has a small contribution to the energy but fitting rearranges other topologies significantly
- It is the refitted  $c_E$  that affects the observables
- At the cost of reproducing triton data,  $c_D = 5.0$ ,  $c_E = 1.039$  would be an example of EMN interaction with great reproduction of oxygen properties

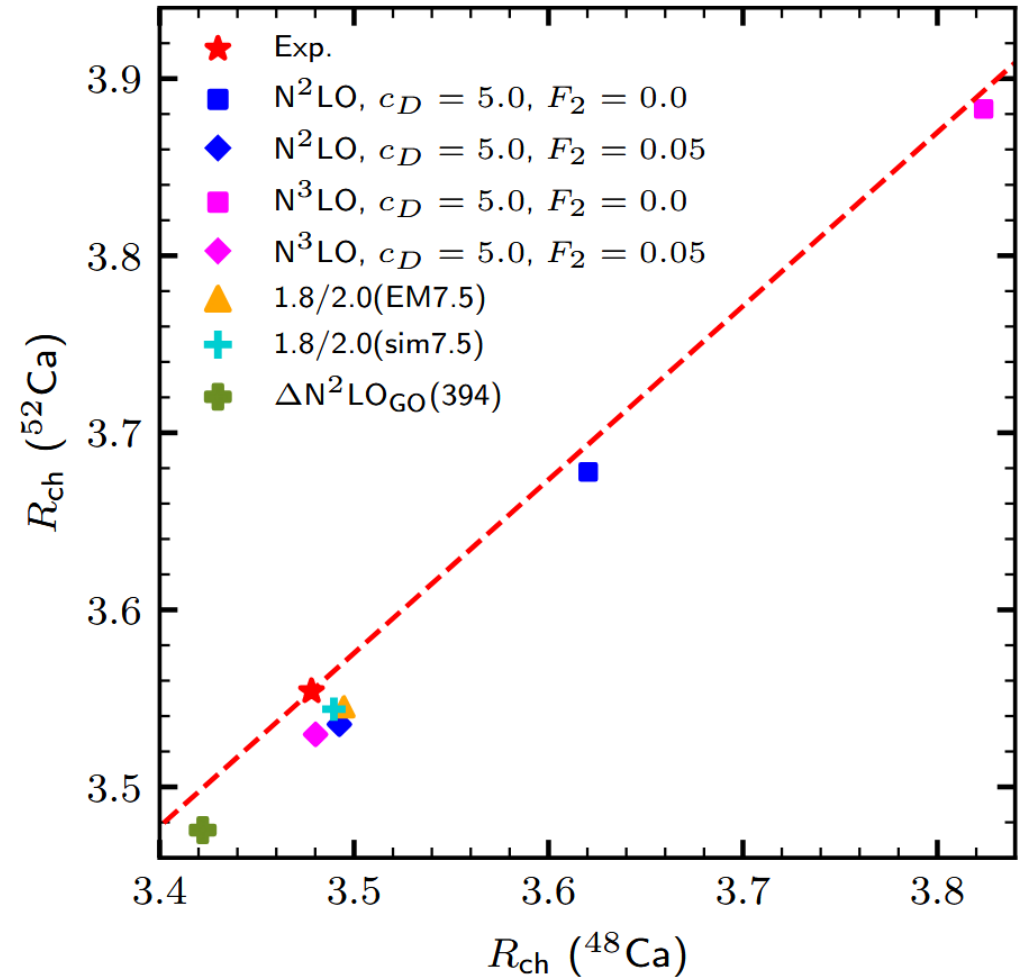
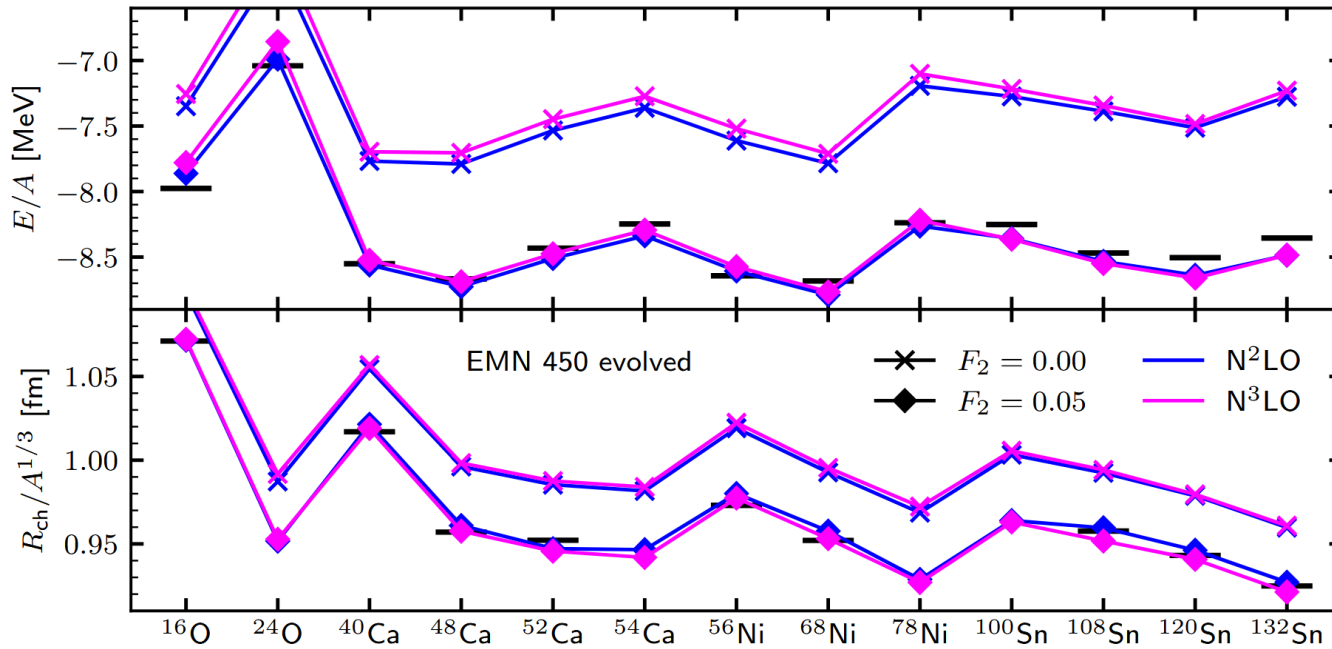


# RESULTS FOR $^{16}\text{O}$ - $^{132}\text{Sn}$



- Both energies and charge radii are very well reproduced at both chiral orders
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- No significant changes in trends as a function of mass and proton-to-neutron ratio
- Inclusion of  $F_2$  cannot resolve the theoretical discrepancy to the experimental value of  $\delta\langle R_{\text{ch}}^2 \rangle^{48,52}$

# CONCLUSIONS

- We investigated the impact of new 3N forces on the properties of medium-mass nuclei.
- Many-body energies and radii are noticeably impacted by  $F_2 \rightarrow$  but due to refit of  $c_E$ .
- Employing the  $^{16}\text{O}$  fitting strategy and adding  $F_2$  to the EMN interactions brings it in a very good agreement with the experimental results along the chain of closed-shell nuclei.
- While  $F_2$  improves the EMN interaction, this is mainly due to shifts in  $c_D$  and  $c_E$  rather than new physics hiding in  $F_2$ .

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# THANK YOU FOR YOUR ATTENTION!