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Intruder states in the no-core shell model

Calvin W. Johnson, SDSU

“This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Award Number DE-FG02-03ER41272 ”

PAINT Workshop @ TRIUMF, Feb 27, 2026



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Bruce Barrett
1939– 15 Mar 2025



Igal Talmi
31 Jan 1925 – 4 Feb 2026

PAINT Workshop @ TRIUMF, Feb 27, 2026

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Recognizes outstanding original contributions to the field of low-energy nuclear physics relevant to FRIB, performed by scientists early (≤ 6 years from *PhD*) in their careers.

Old(er) folks -- Nominate promising young researchers!
Young folks -- Ask your supervisors to nominate you!
See <https://frib.msu.edu/users/early-career>

**2025 Theory recipient:
Cole Pruitt**

Questions? Contact Andrea Richard (richarda1@ohio.edu)

**Nominations due
Monday March 2nd**





Outline of talk

- The intruders and islands of inversion
- ^{11}Li & ^{29}F as case studies in inversion

CWJ & M Caprio, arXiv:2511.00313

- ^{20}C as possible inversion (preliminary!)
- Shape coexistence and inversion using NCSM-Hartree-Fock (preliminary!)



Configuration-interaction shell model

Matrix formalism:

expand in some (many-body) basis

$$\hat{\mathbf{H}}|\Psi\rangle = E|\Psi\rangle$$

$$|\Psi\rangle = \sum_{\alpha} c_{\alpha} |\alpha\rangle$$

$$H_{\alpha\beta} = \langle\alpha|\hat{\mathbf{H}}|\beta\rangle$$

$$\sum_{\beta} H_{\alpha\beta} c_{\beta} = E c_{\alpha}$$

Disadvantage:

- not size-extensive, basis grow exponentially

Advantages:

- Excited states easy to generate
- **Direct access to wave functions allows for detailed analysis**

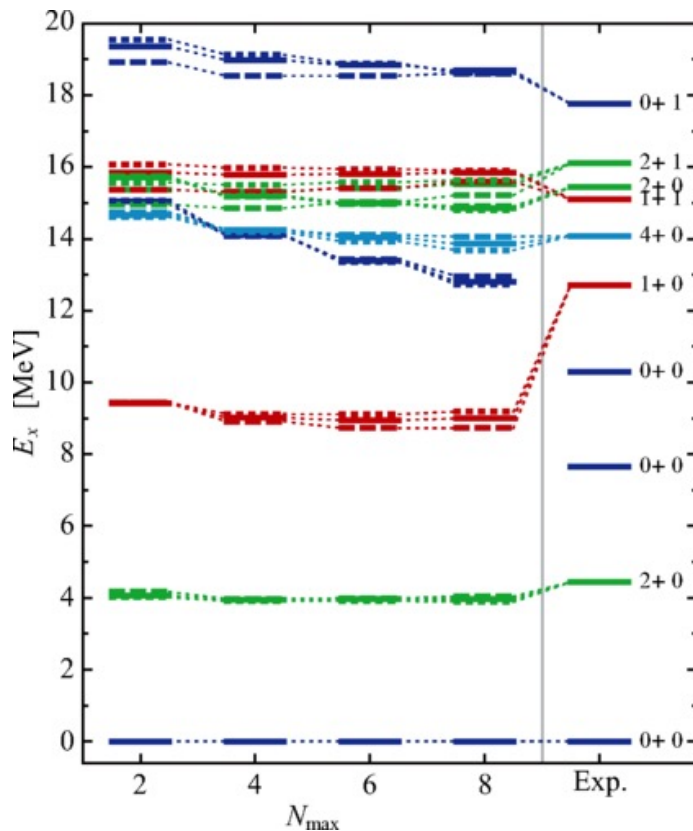
The NCSM has been a triumph!



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Maris *et al* PRC **90**, 014314 (2014)

^{12}C with chiral 2+3 body forces



We can reproduce
experimental data!
such as the g.s. band
of ^{12}C



TRIUMF NCSM has been a triumph!



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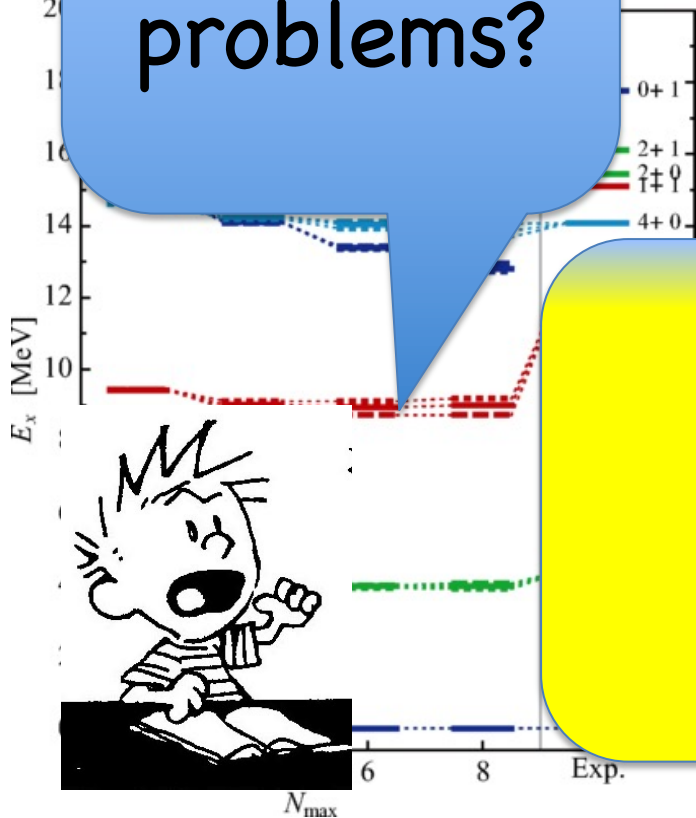
Maris

^{12}C w

So we've solved all problems?

We can reproduce experimental data! such as the g.s. band of ^{12}C

Not yet!

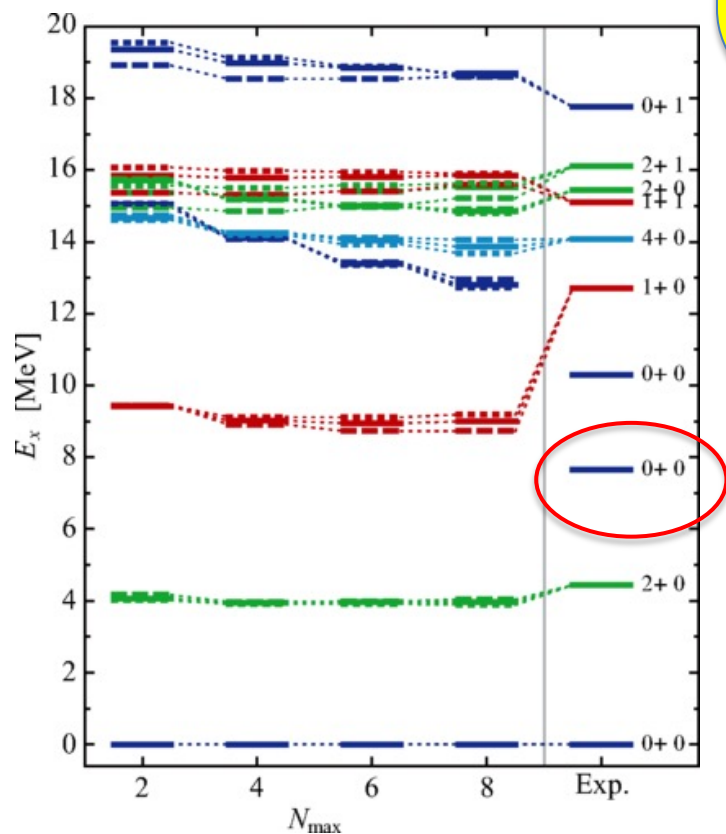




The Hoyle state in ^{12}C is a problem!

Maris *et al* PRC **90**, 014314 (2014)

^{12}C with chiral 2+3 body forces



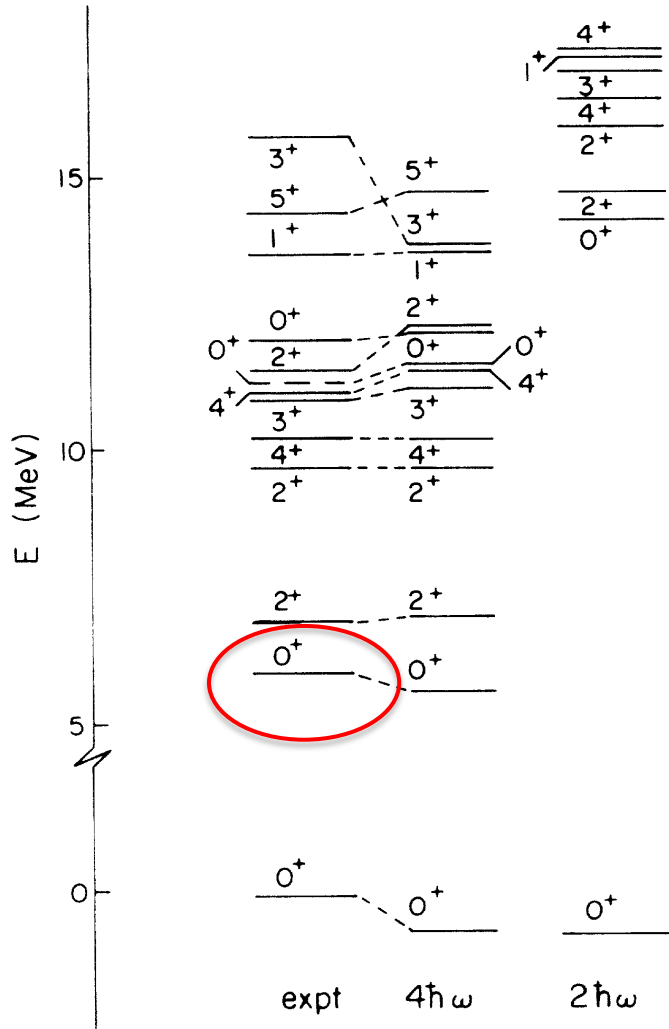
Hoyle state

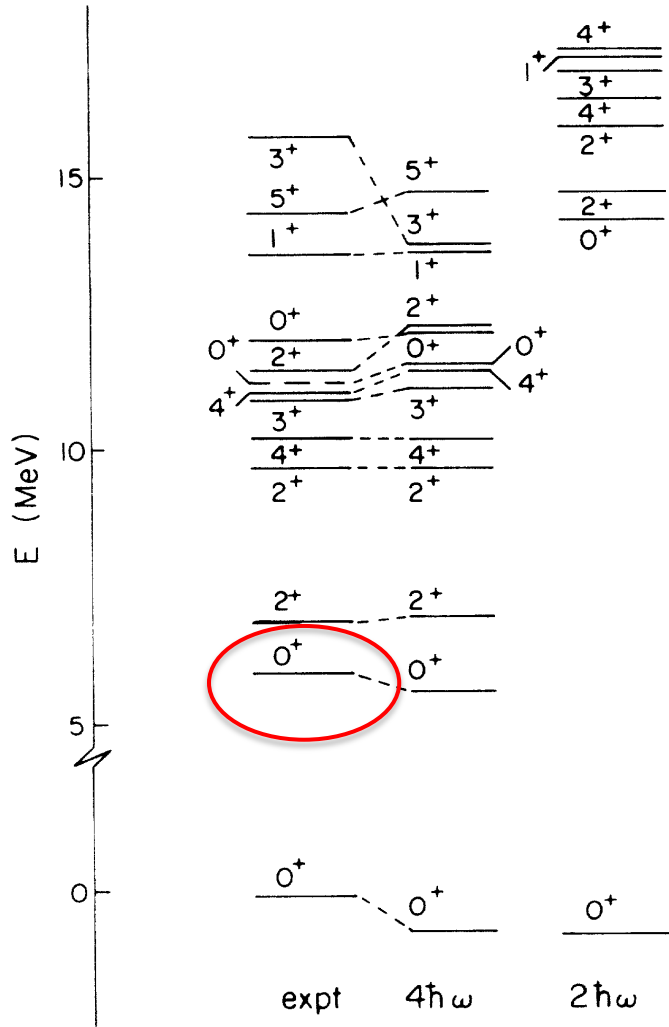


Haxton and Johnson, PRL **65**, 1325
(1990)



There's a similar state in ^{16}O

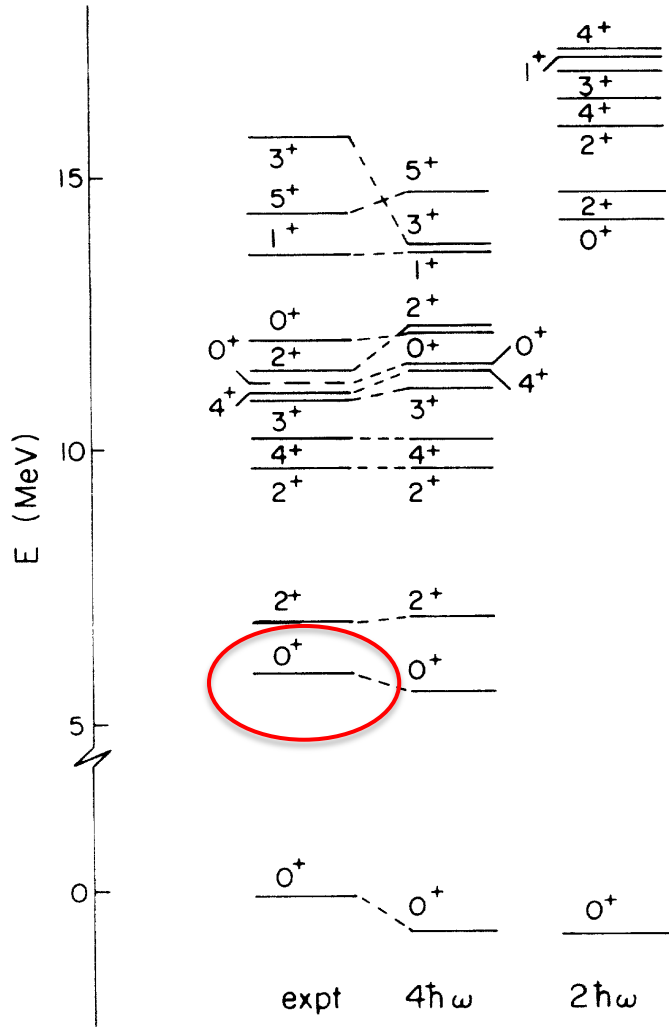




There's a similar state in ^{16}O

One can think of these as alpha-cluster states



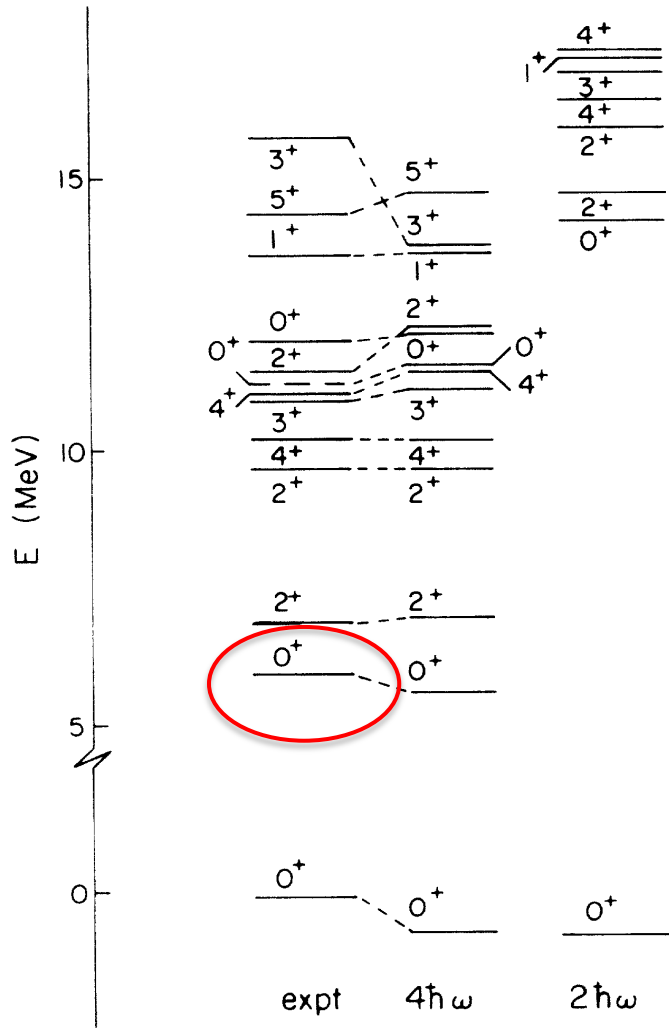


There's a similar state in ^{16}O

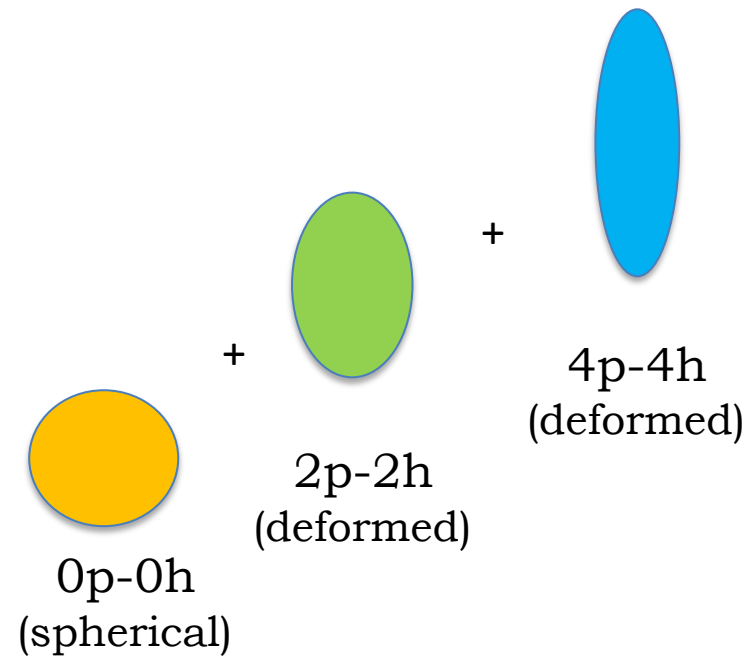
One can think of these as alpha-cluster states

Or as $np-nh$ states





Brown and Green, Nucl. Phys. **75**, 401 (1966)





These cluster states are not easy to reproduce in the NCSM.





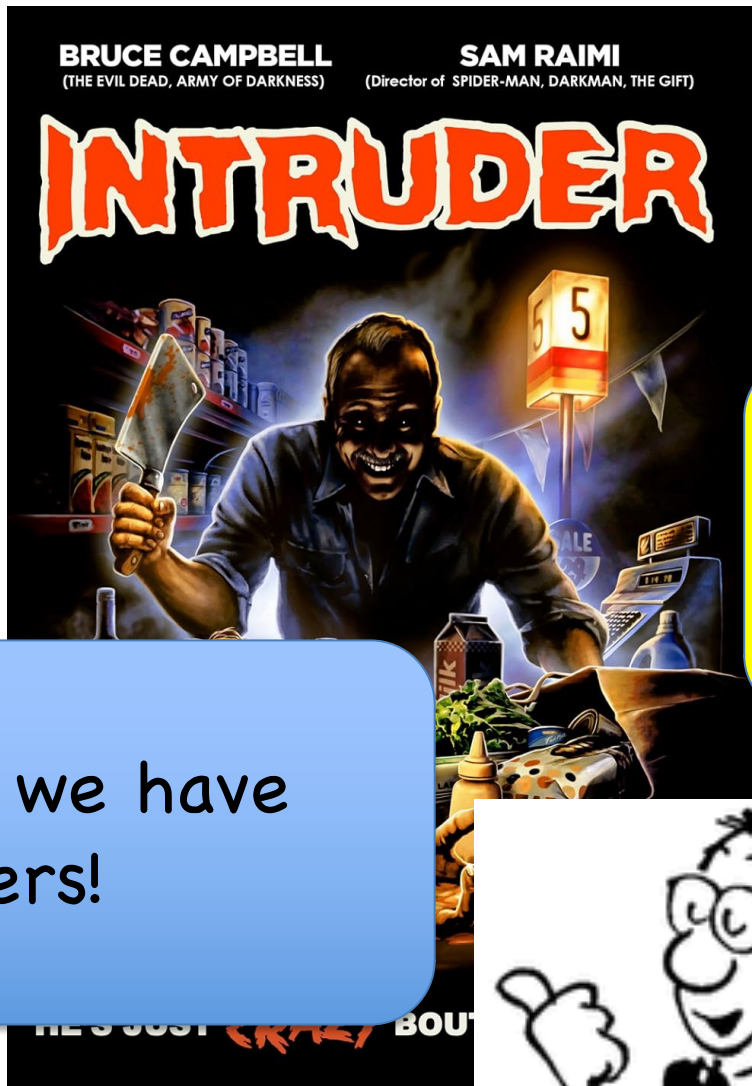
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So basically we have
intruders!





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Yikes! Intruders
are scary!

So basically we have
intruders!





Related to cluster states,
islands of inversions
and halo nuclei
form a similar **challenge** to
standard shell-model pictures





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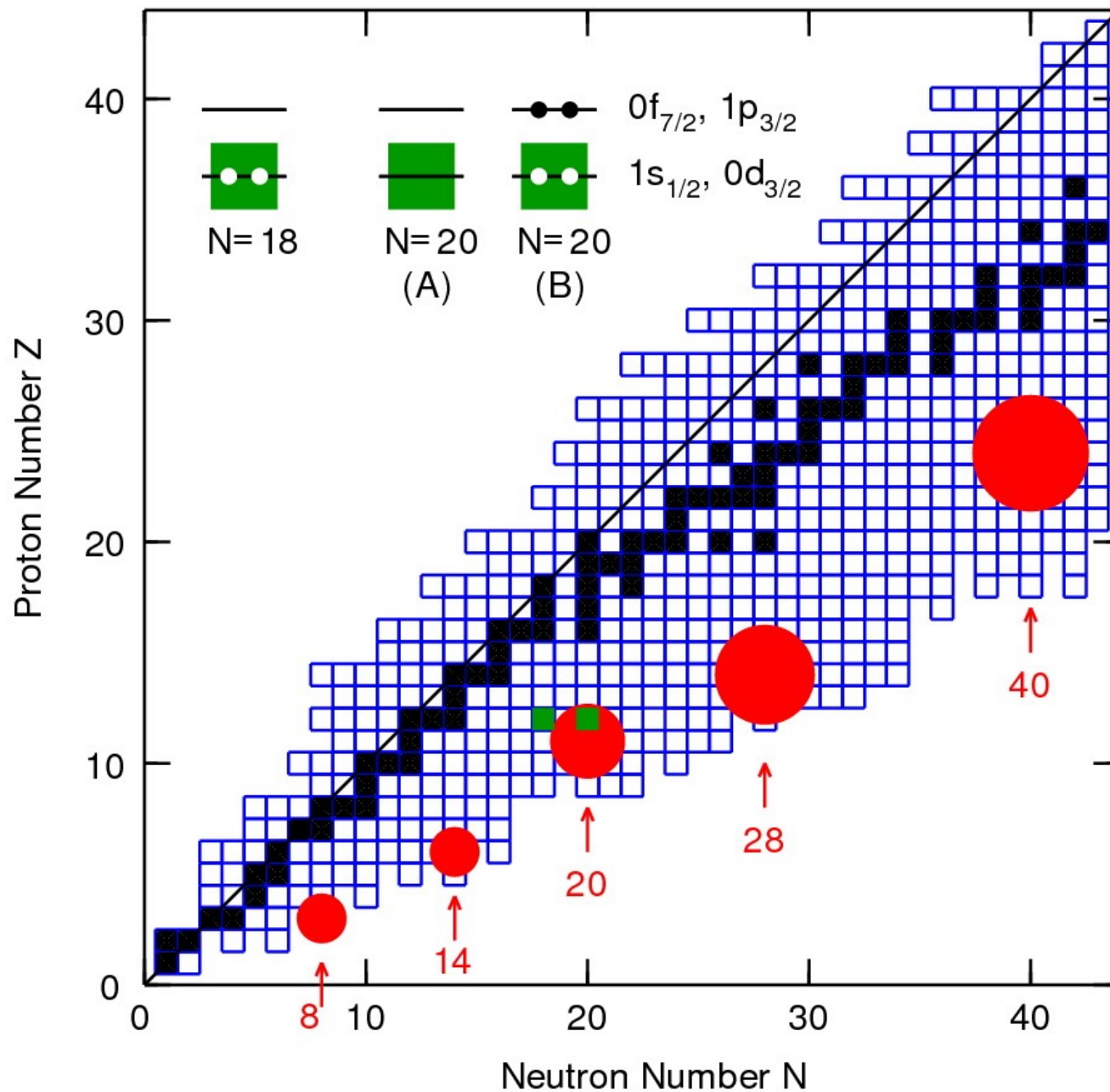


Figure:
Alex Brown





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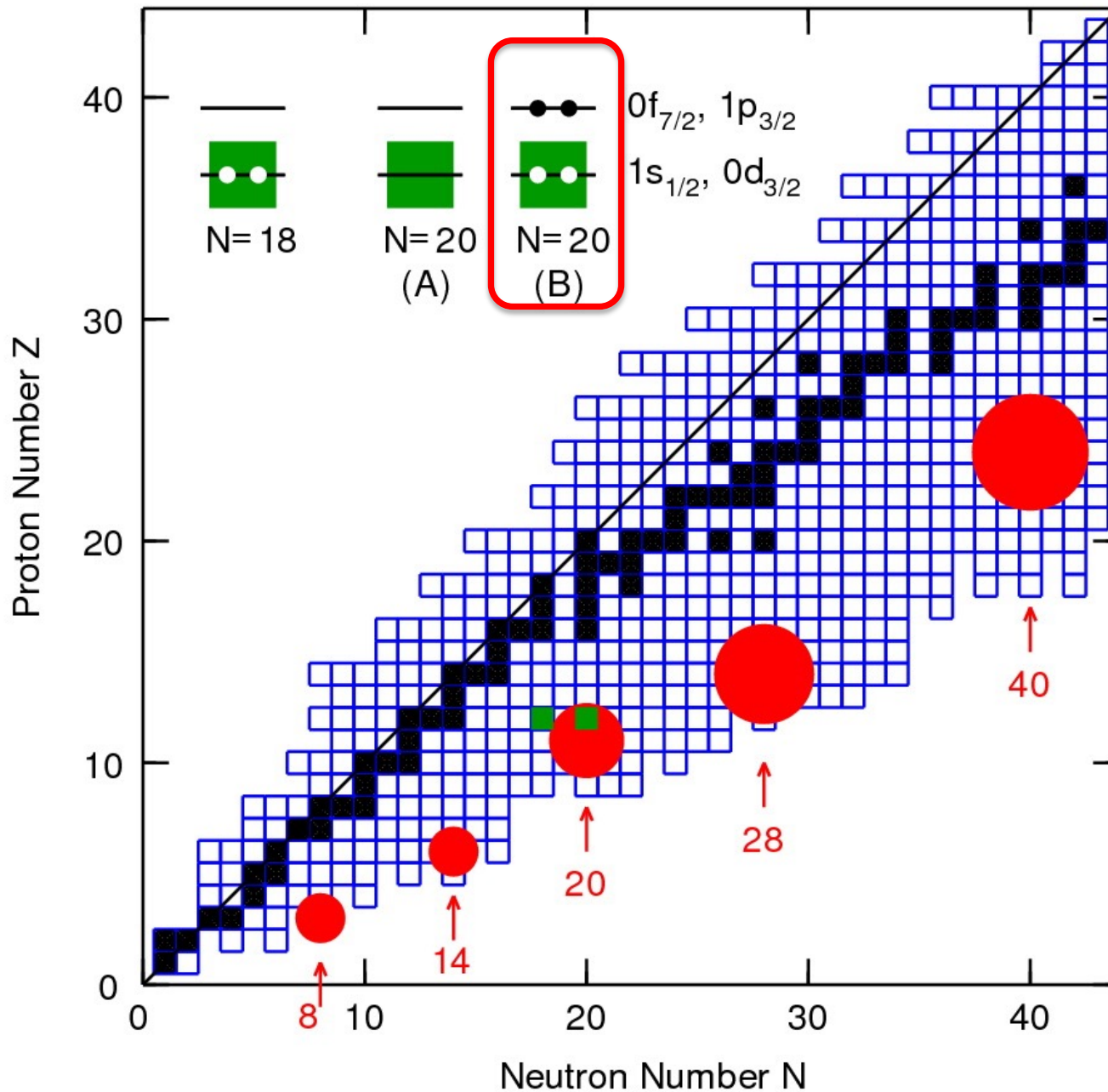


Figure:
Alex Brown





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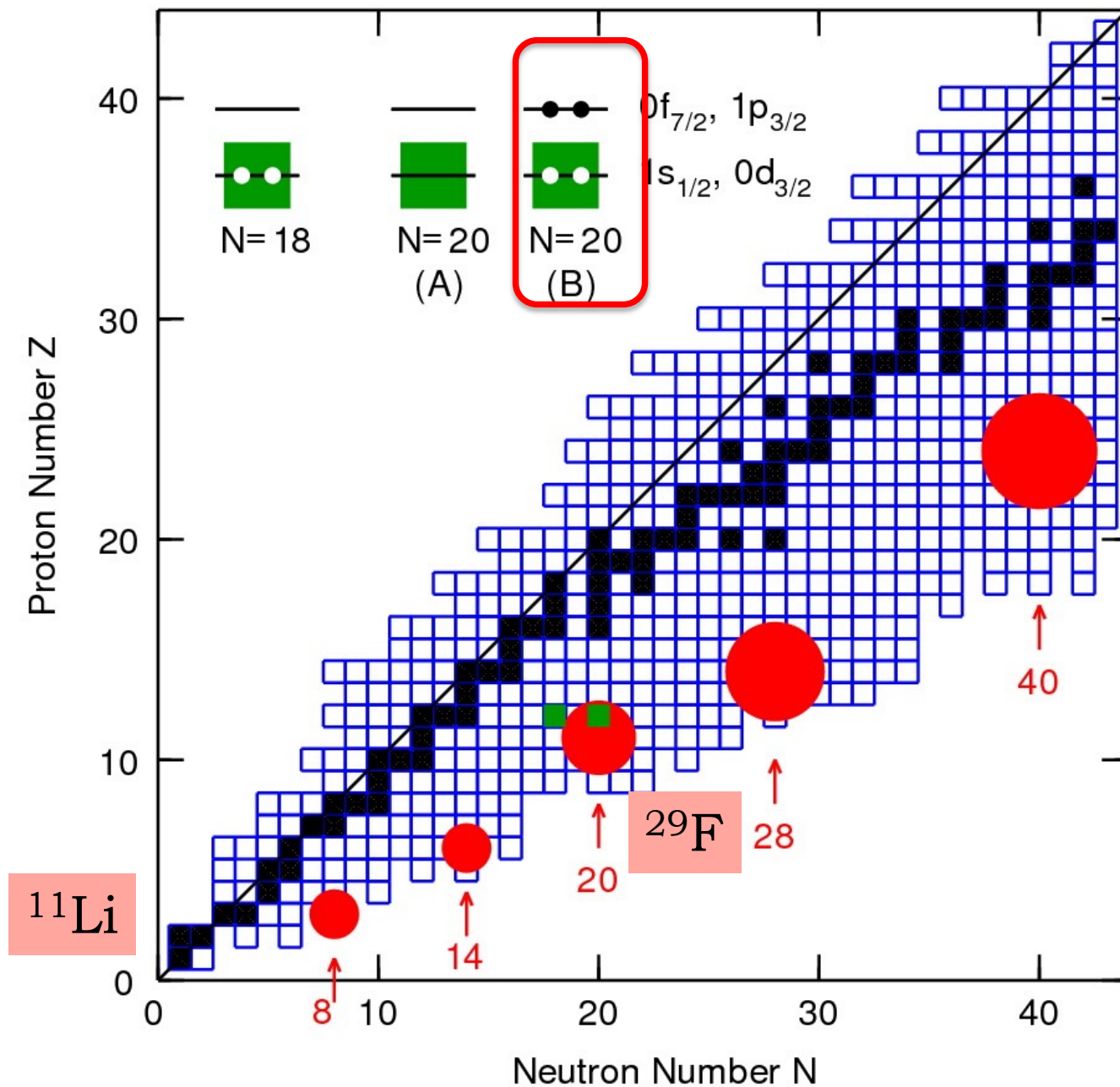


Figure:
Alex Brown



CASE STUDIES: ^{11}Li & ^{29}F



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CWJ & M Caprio,
arXiv:2511.00313



$N = 8$, 20 islands of inversion

CASE STUDY: ^{11}Li



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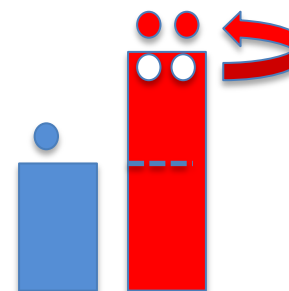
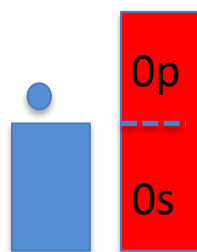
^{11}Li makes for an excellent case study:

- Example of “island of inversion”
- Halo or extended state; large deformation
- Small enough to be tackled numerically
- Testbed for techniques

CASE STUDY: ^{11}Li



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One proton outside a
filled shell
+ filled neutron shell

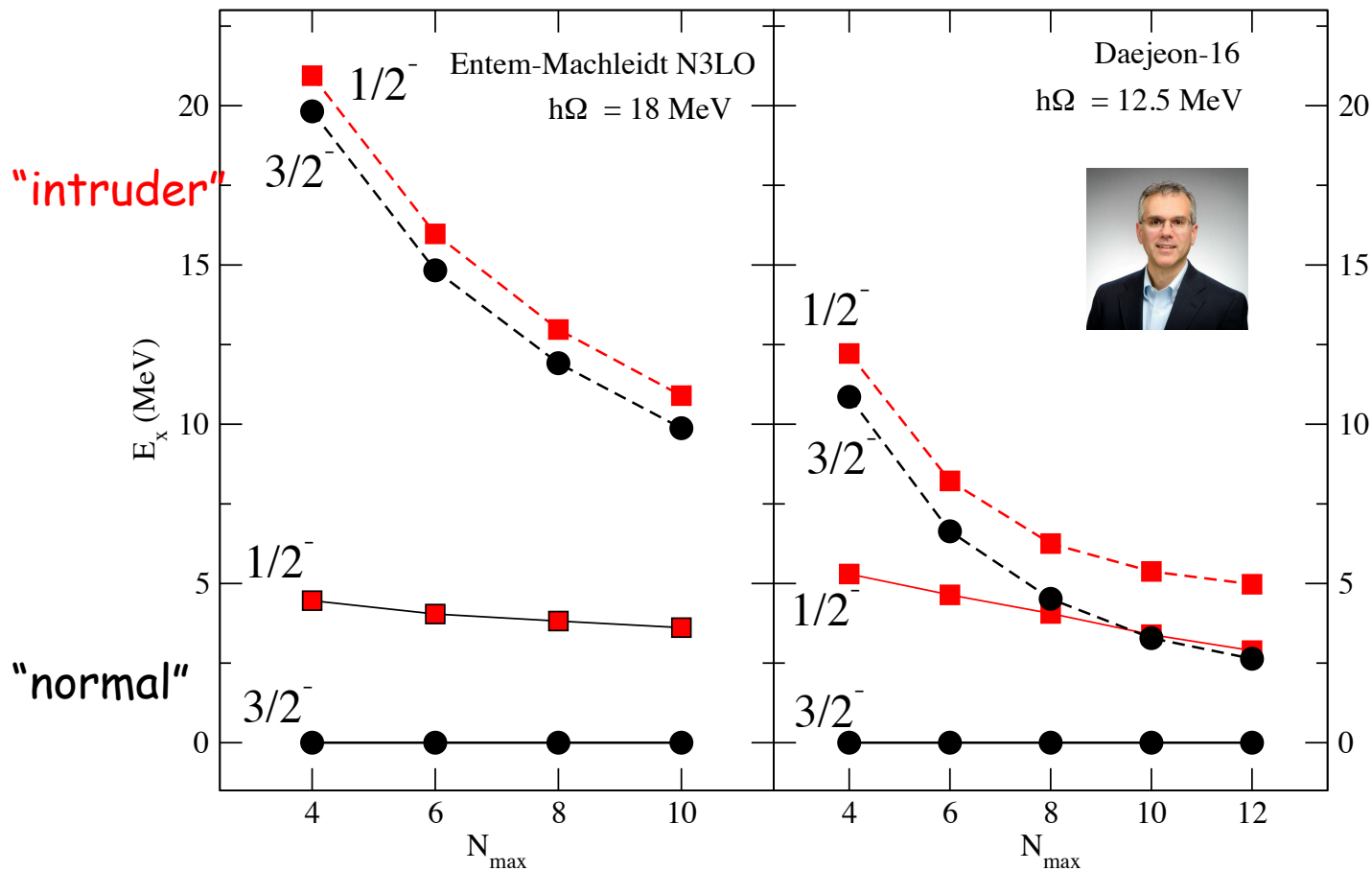
One proton outside a
filled shell
+ neutron 2p-2h

“island of inversion”

CASE STUDY: ^{11}Li



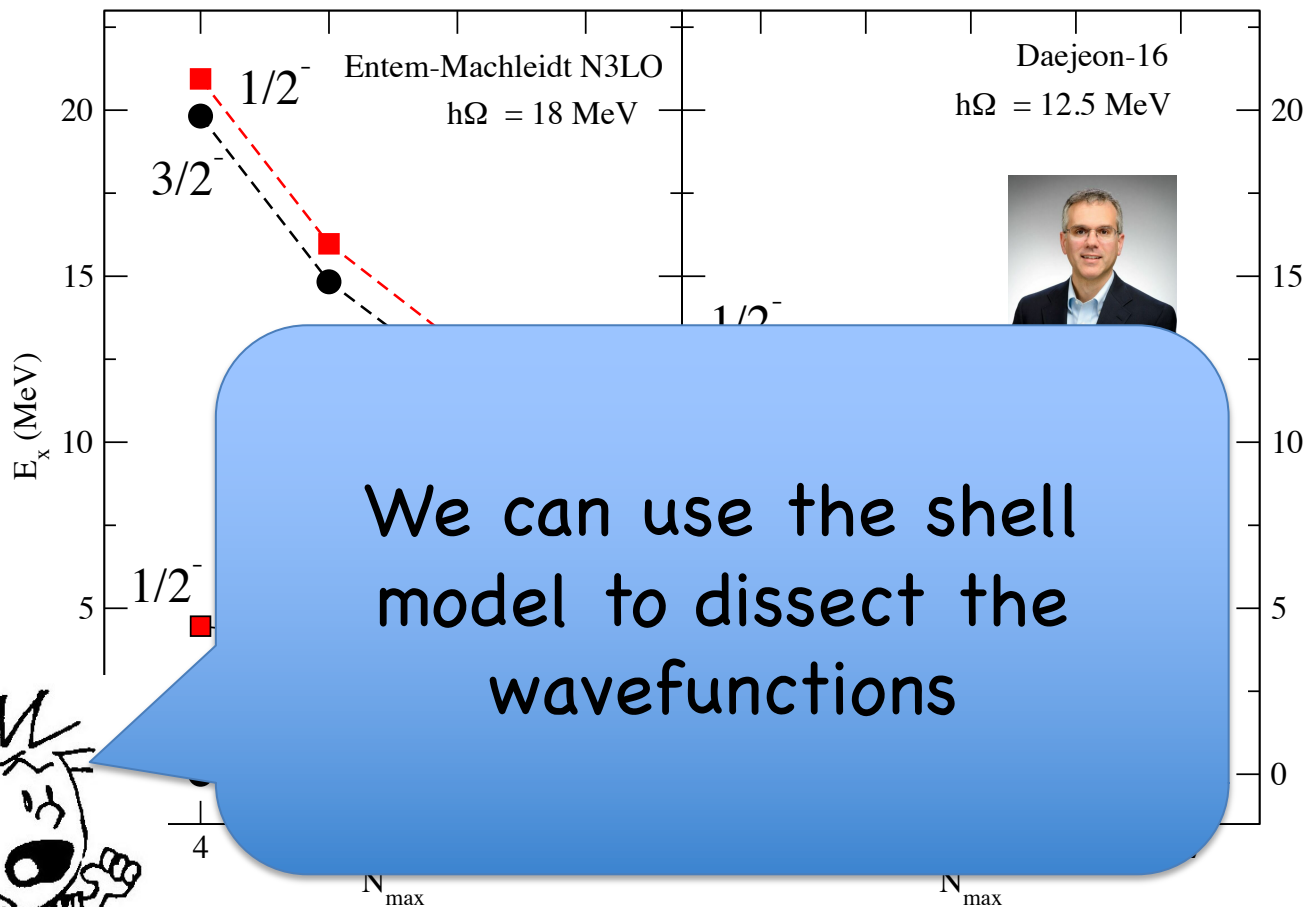
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CASE STUDY: ^{11}Li



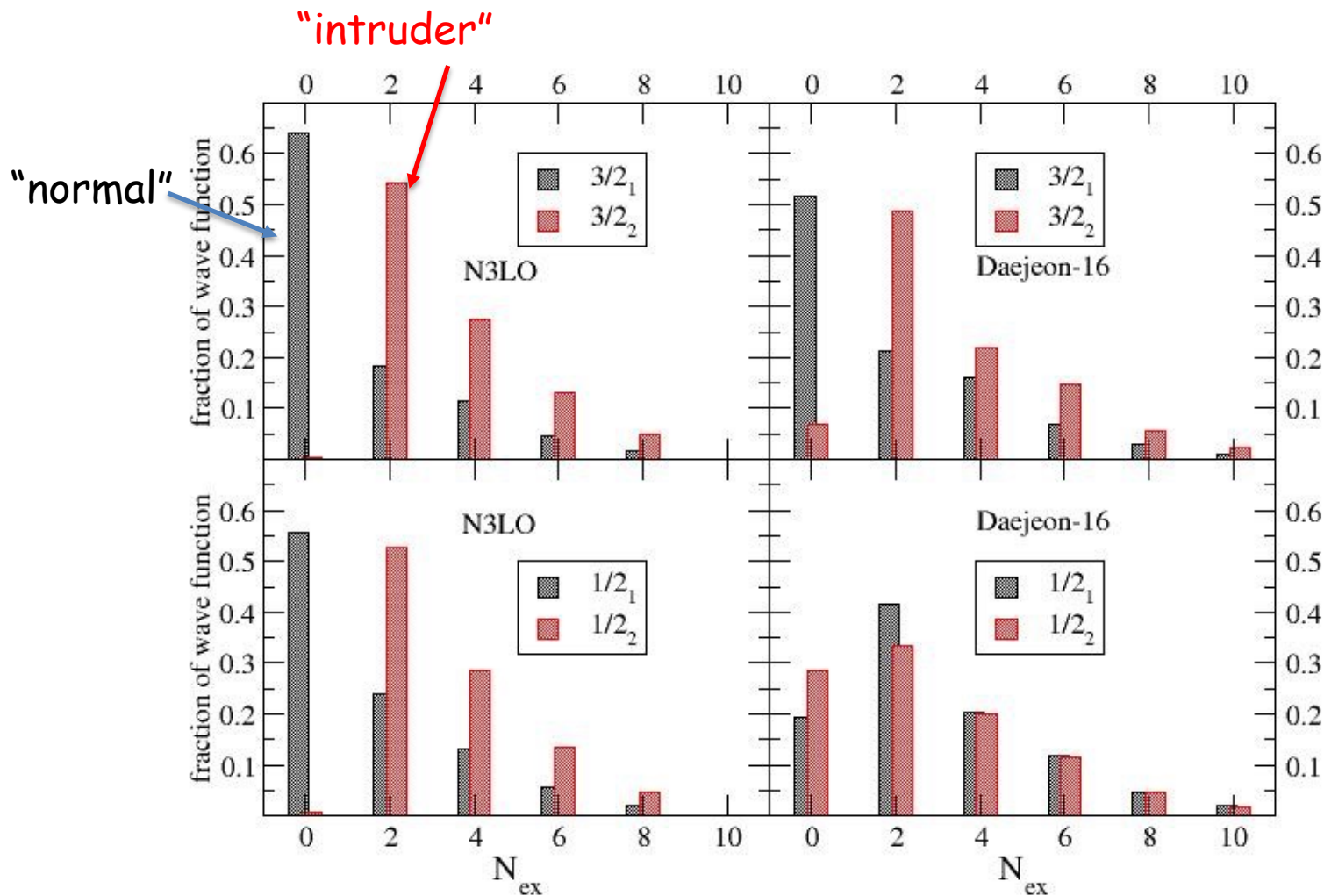
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CASE STUDY: ^{11}Li



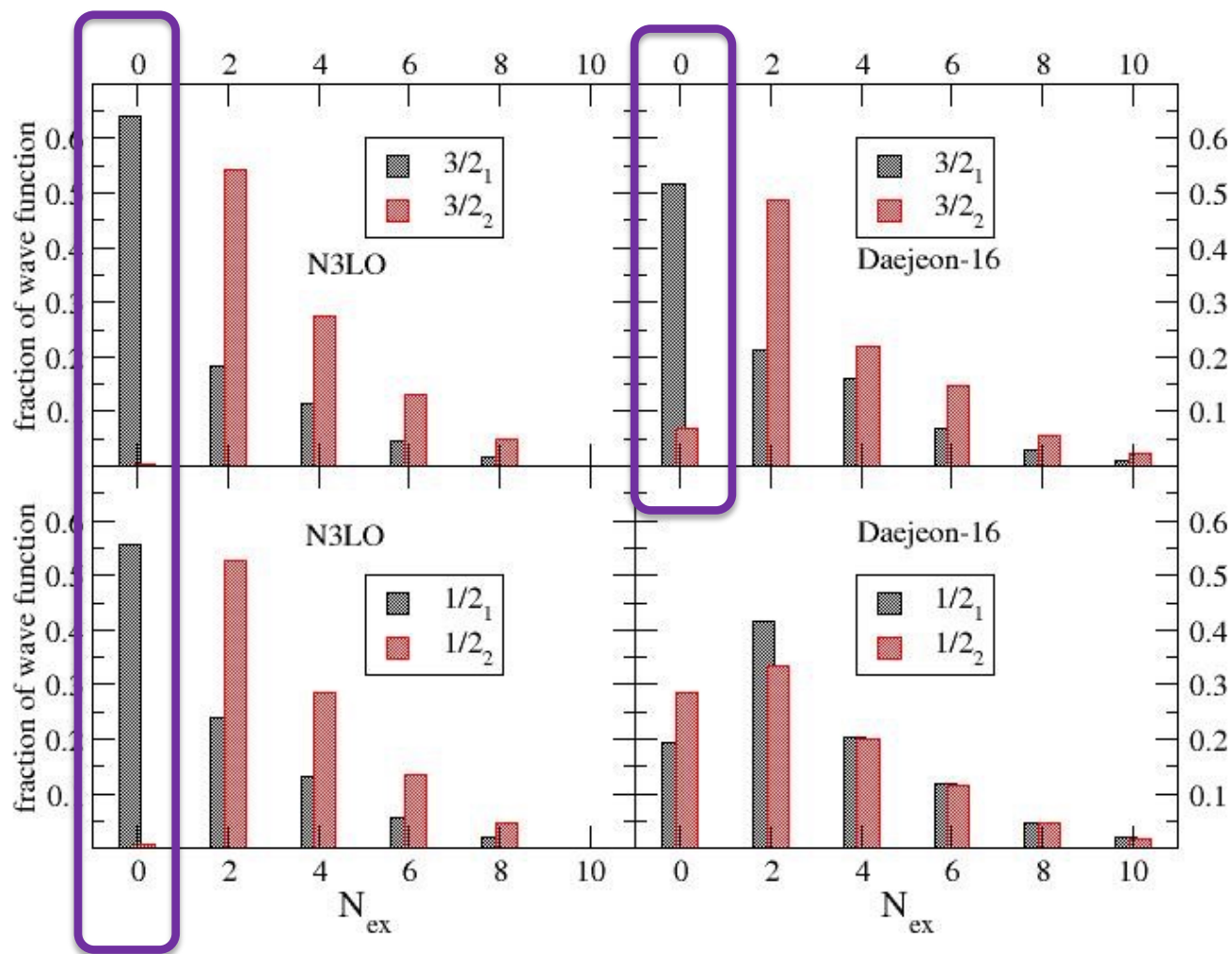
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CASE STUDY: ^{11}Li



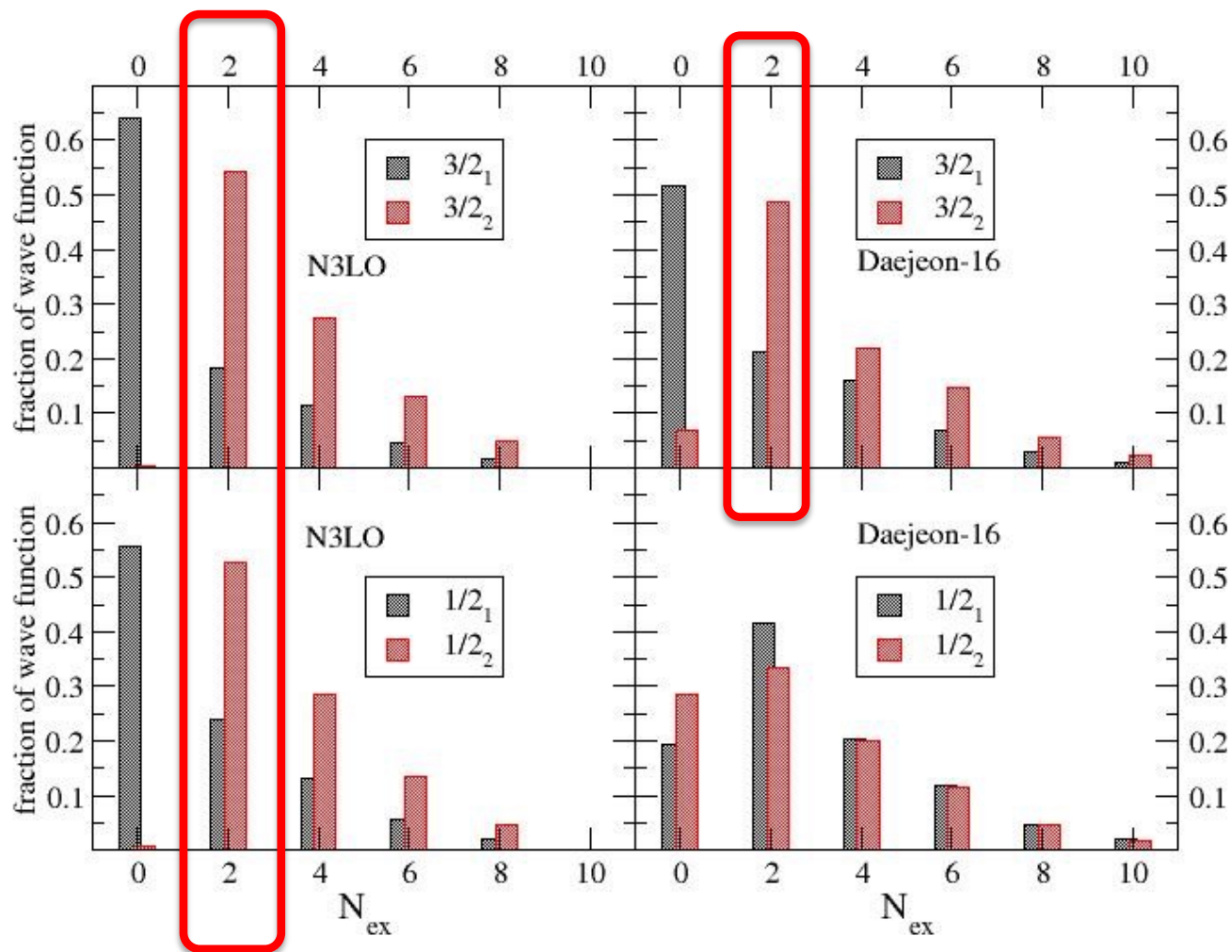
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"intruder" CASE STUDY: ^{11}Li



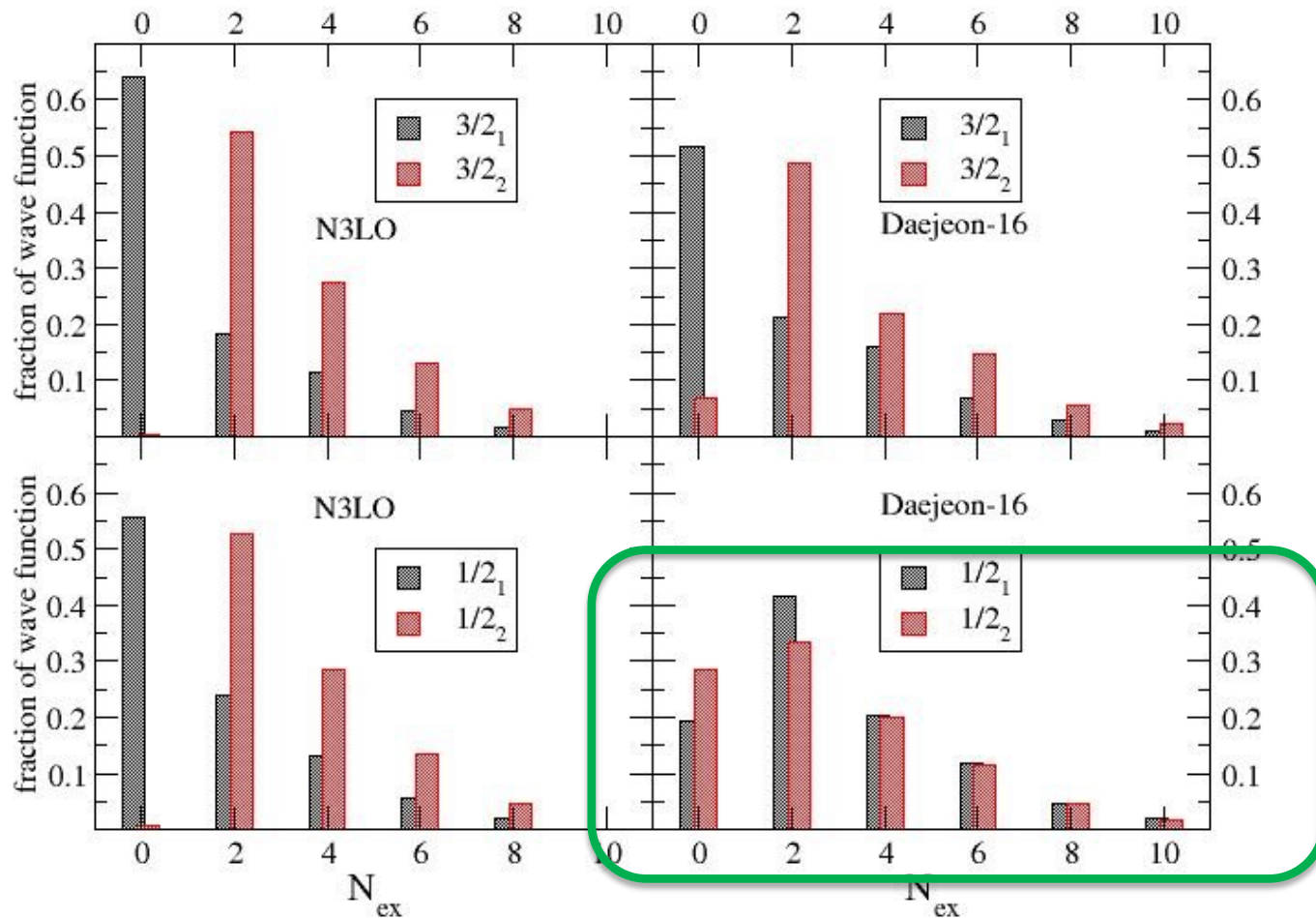
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CASE STUDY: ^{11}Li



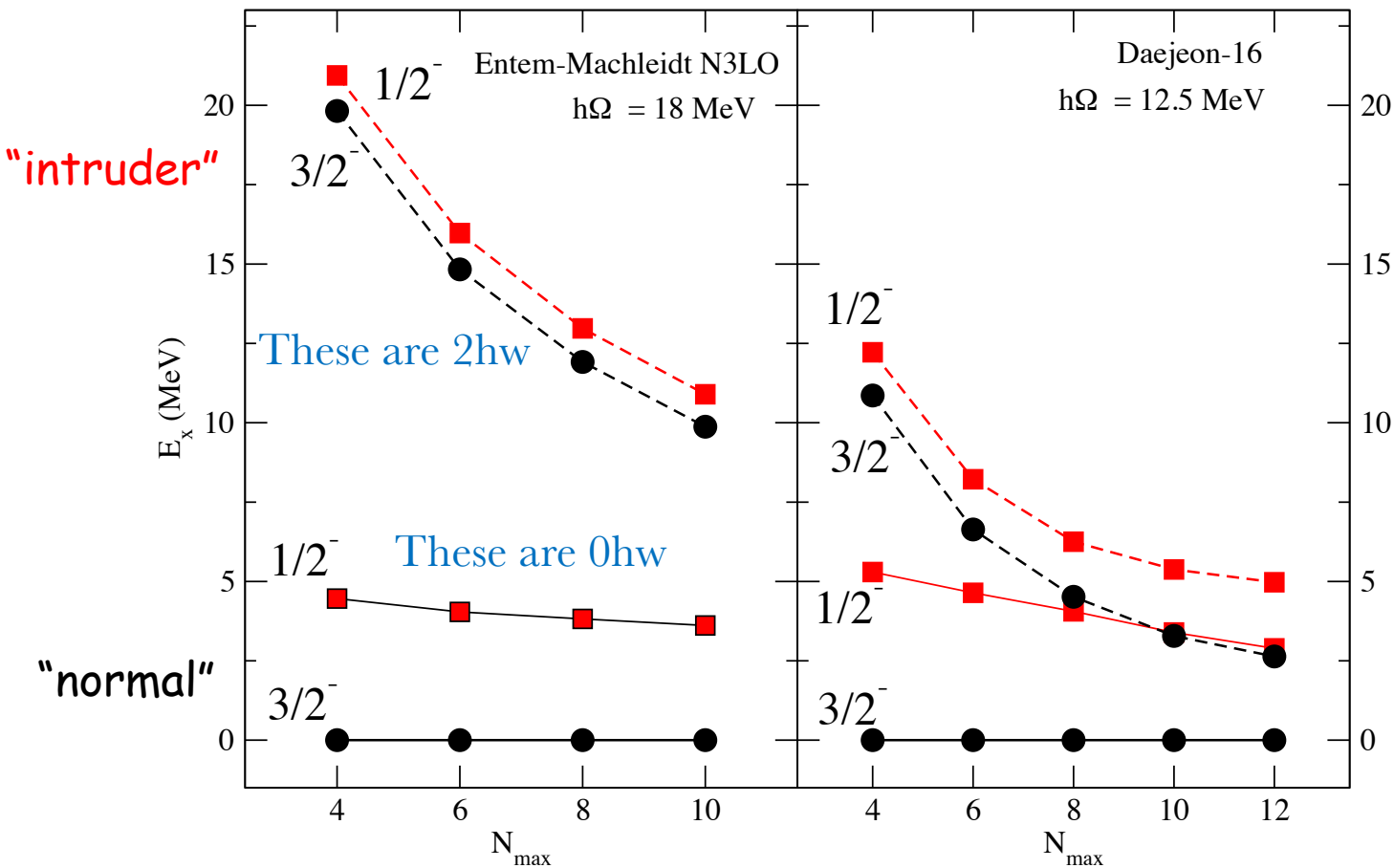
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CASE STUDY: ^{11}Li



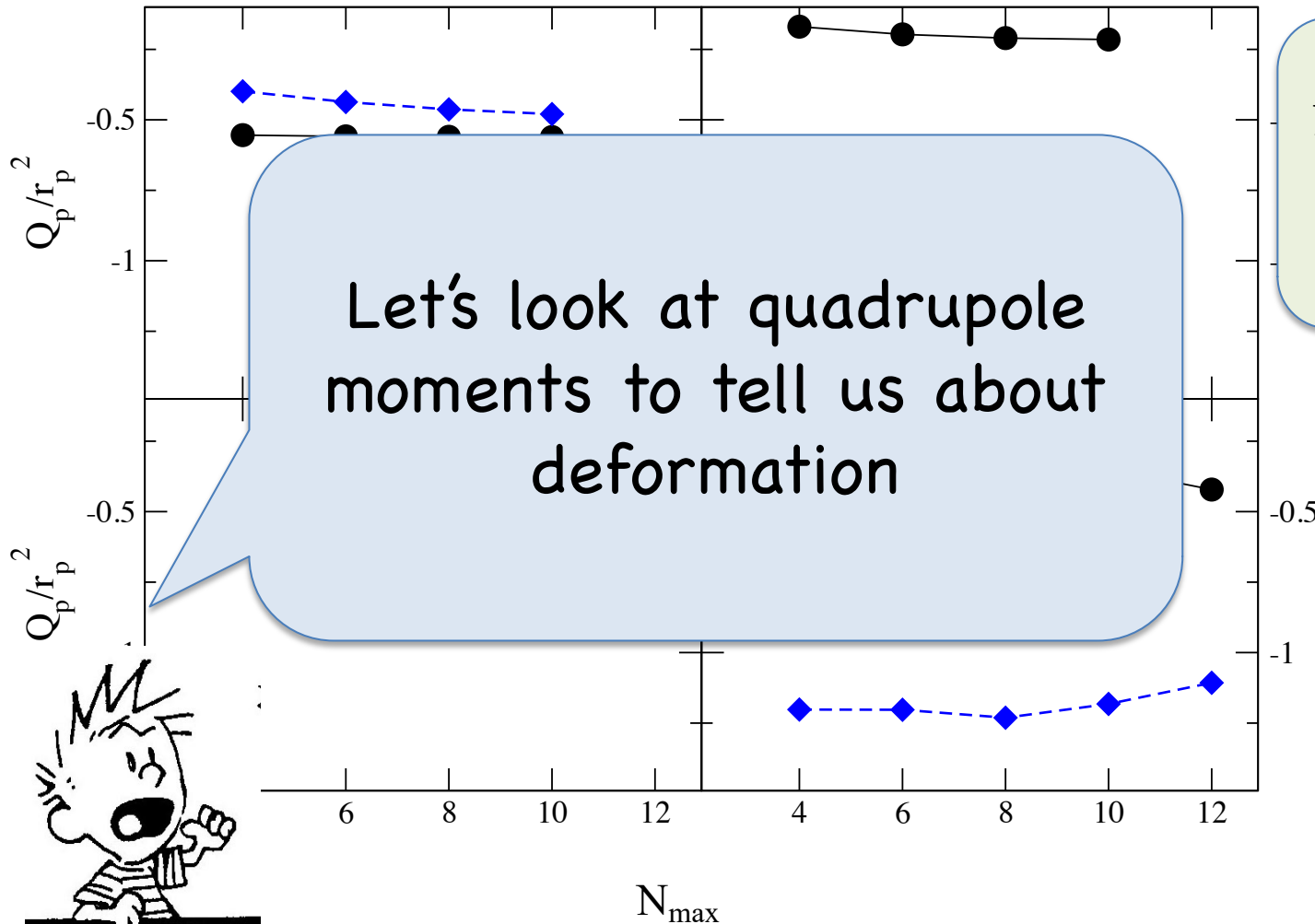
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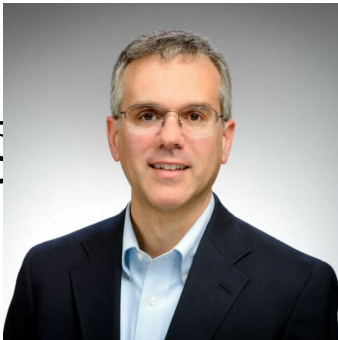
CASE STUDY: ^{11}Li



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Divide by radius² for more robust results



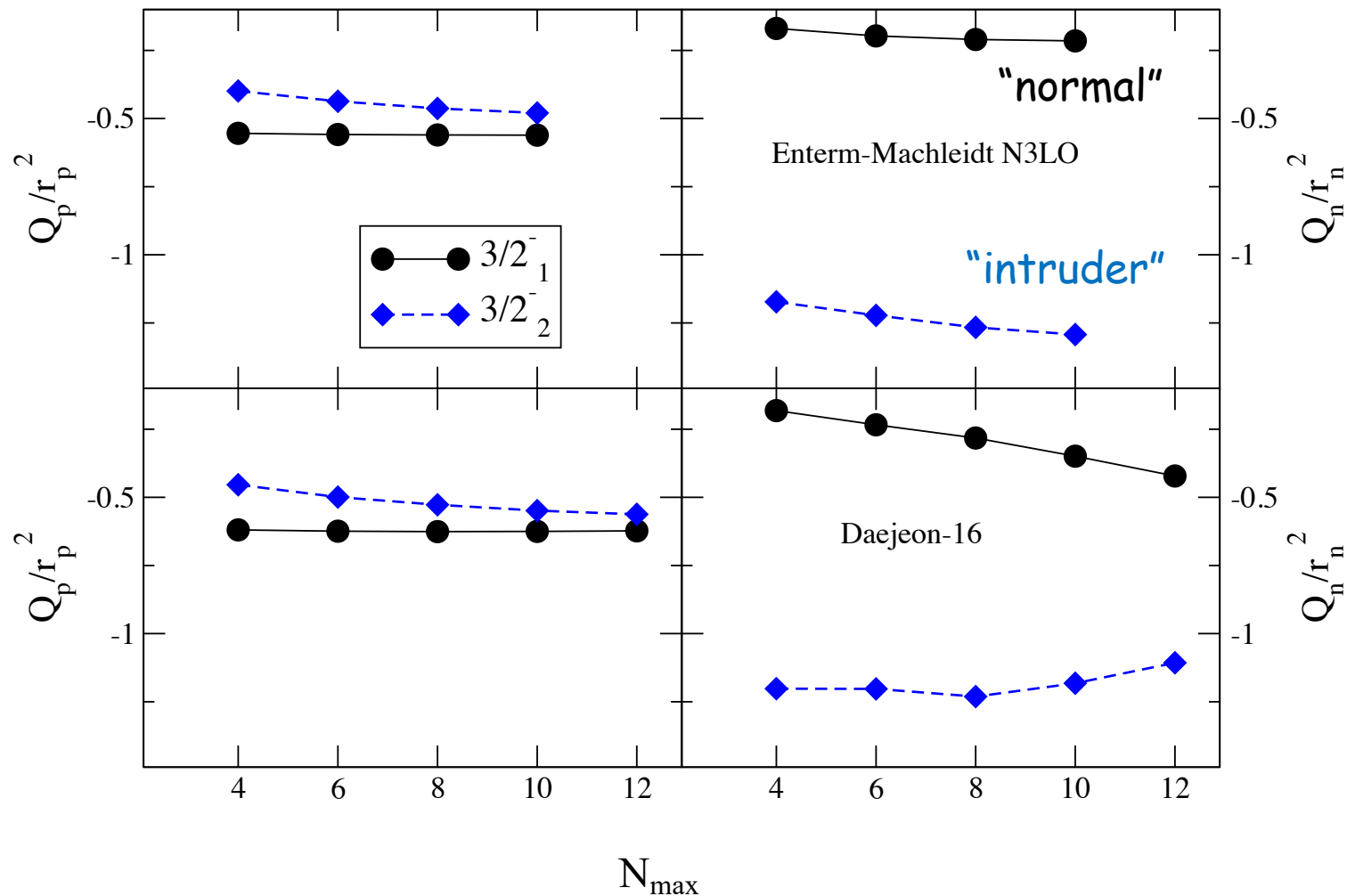
Mark Caprio



CASE STUDY: ^{11}Li



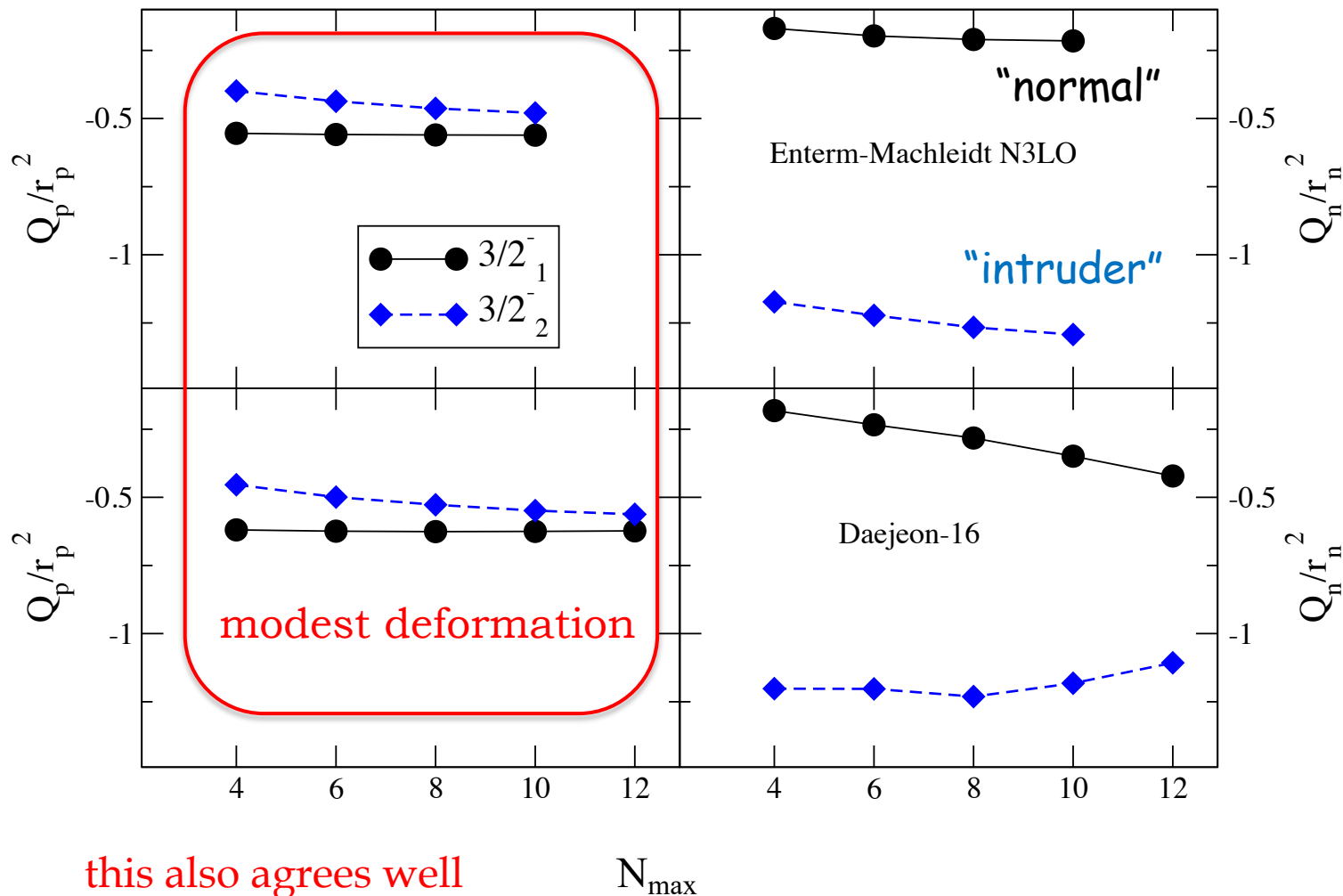
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CASE STUDY: ^{11}Li



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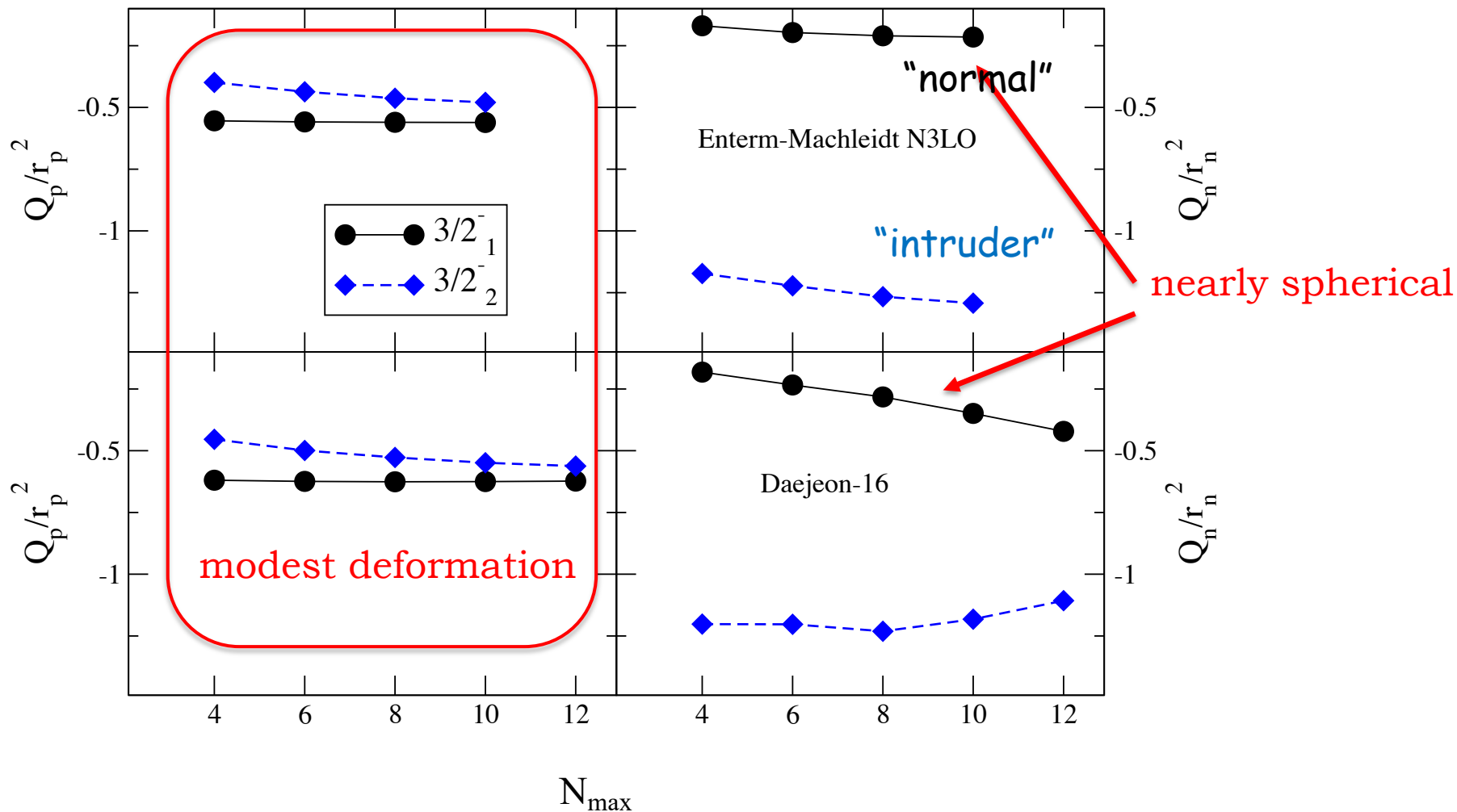


this also agrees well with experiment

CASE STUDY: ^{11}Li



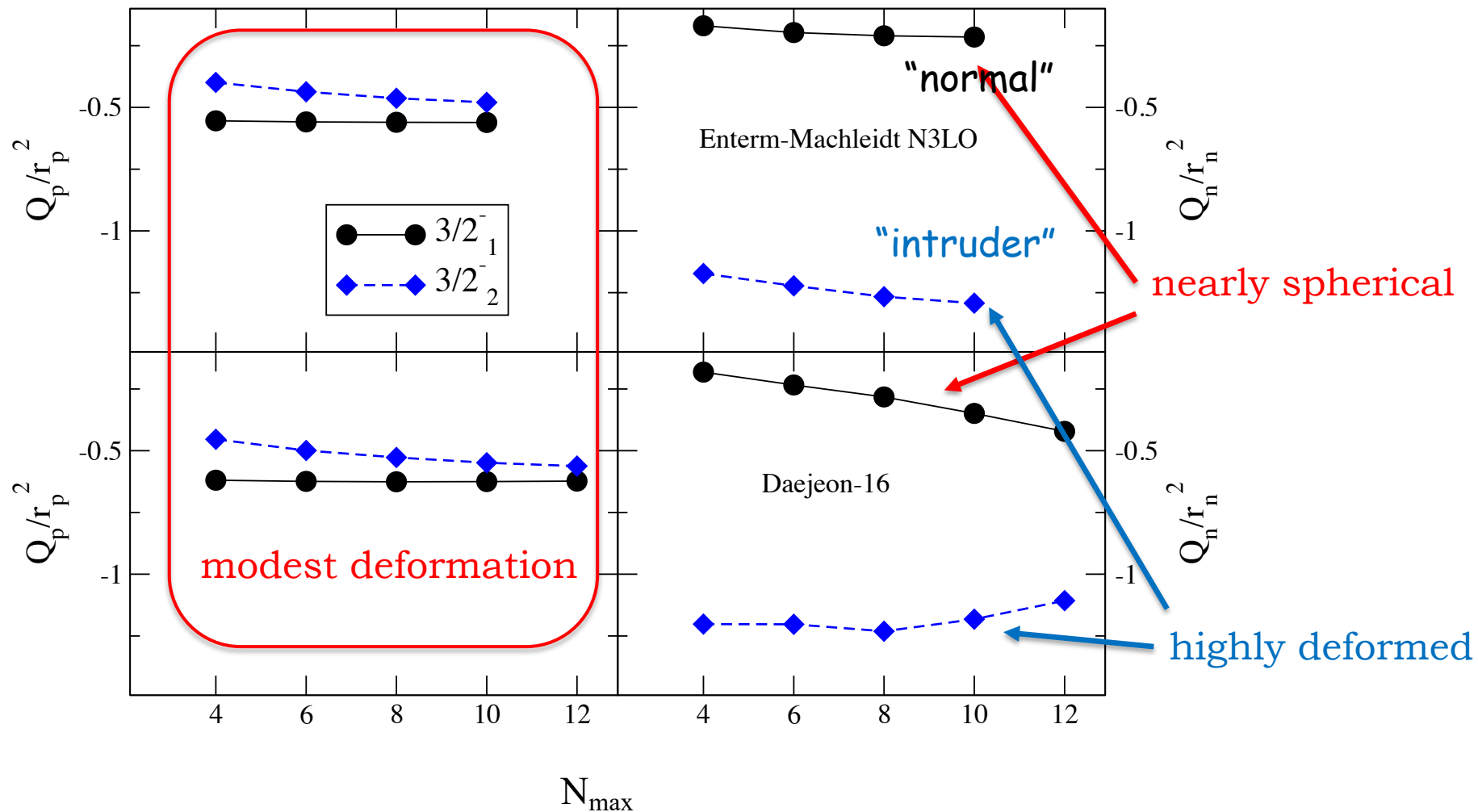
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CASE STUDY: ^{11}Li



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CASE STUDY: ^{11}Li



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We can use the shell
model to dissect the
wavefunctions

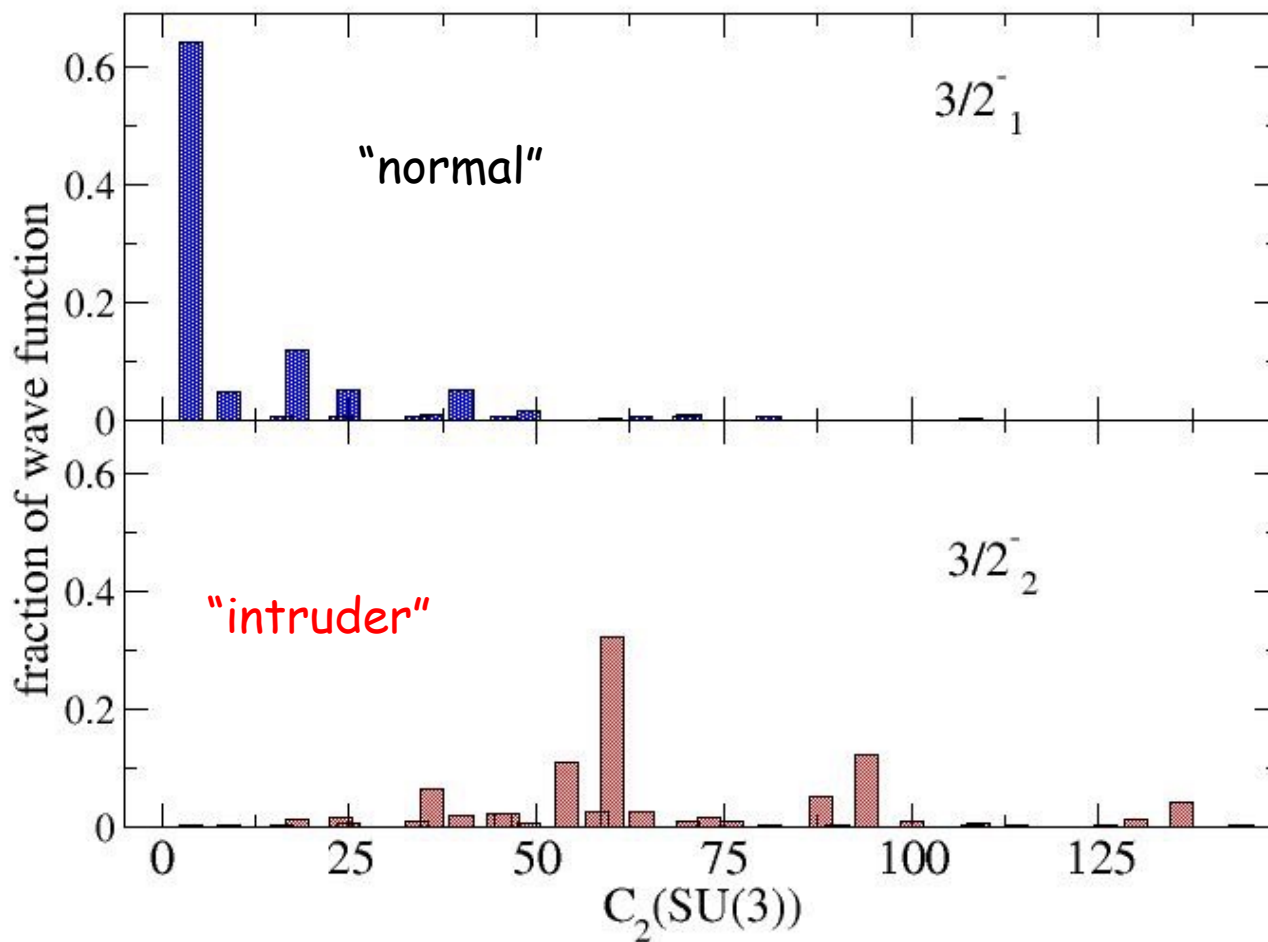


CASE STUDY: ^{11}Li



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→ more deformed



Group-theoretical Decomposition

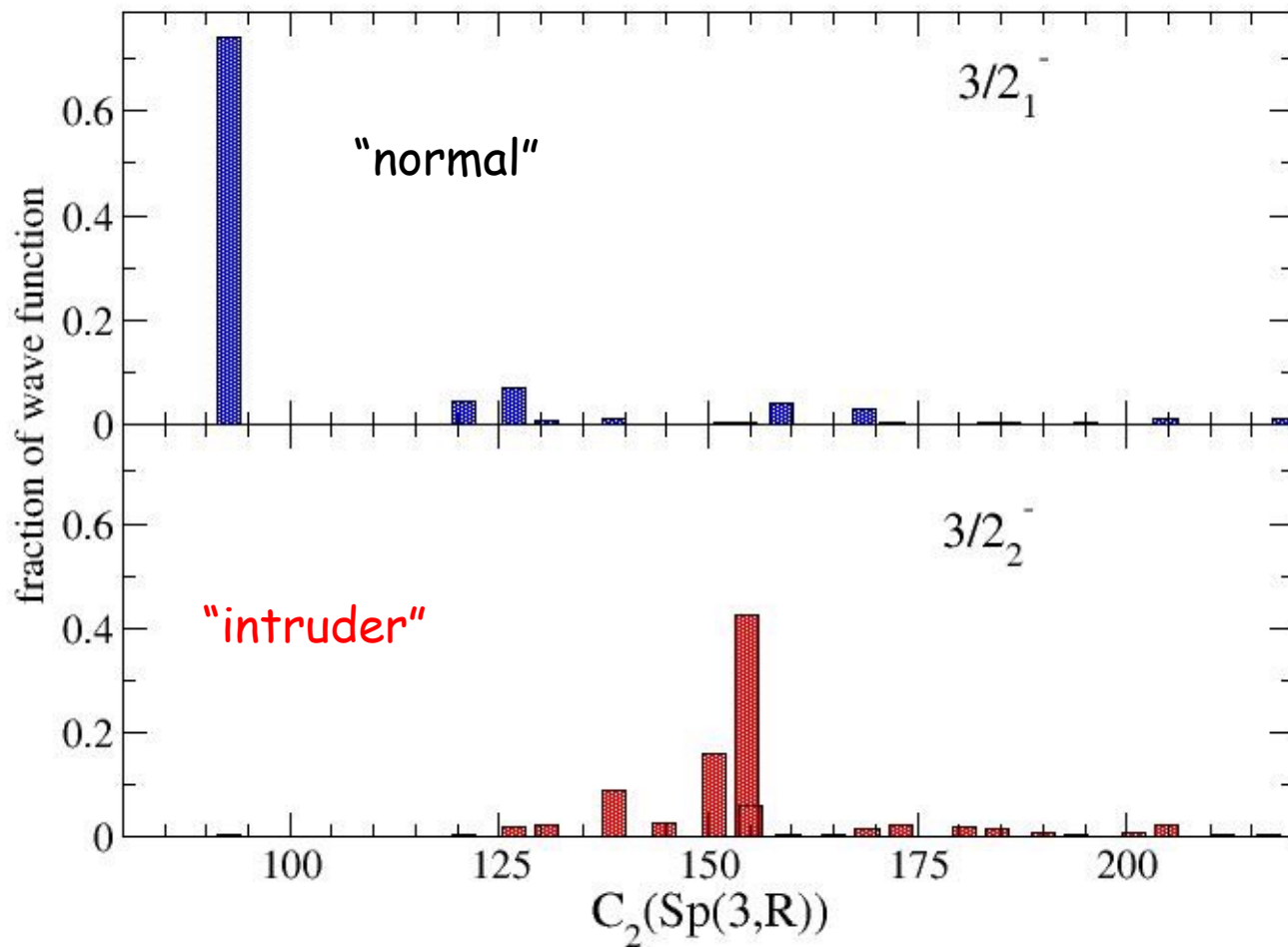
Elliot SU(3)

CASE STUDY: ^{11}Li



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→ more deformed



Group-
theoretical
Decomposition

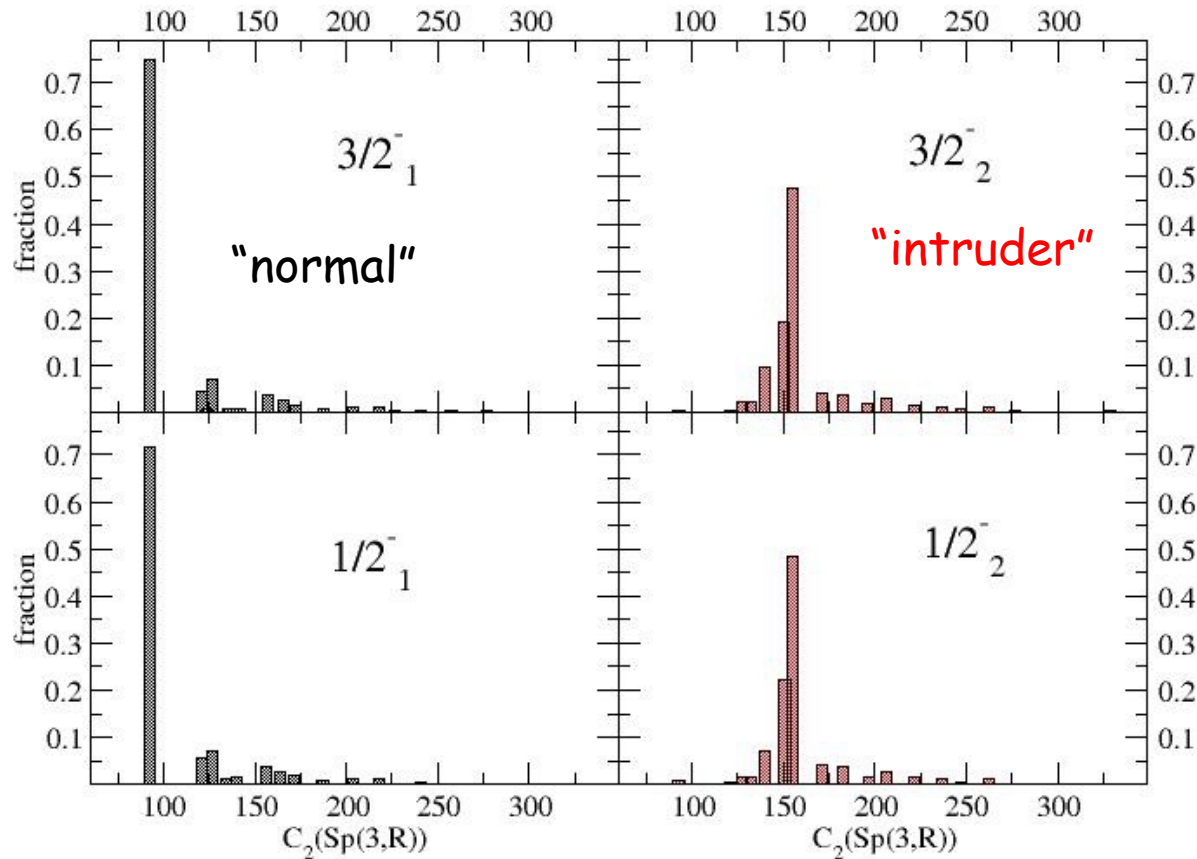
Symplectic
 $\text{Sp}(3,\mathbb{R})$

CASE STUDY: ^{11}Li



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→ more deformed

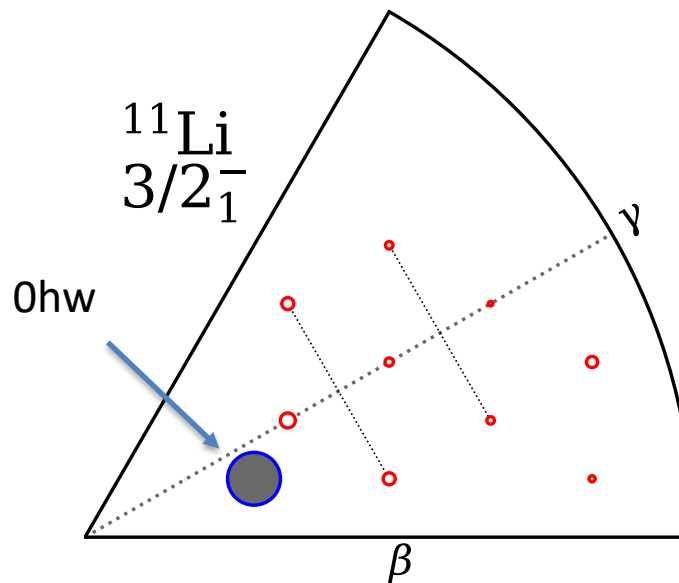
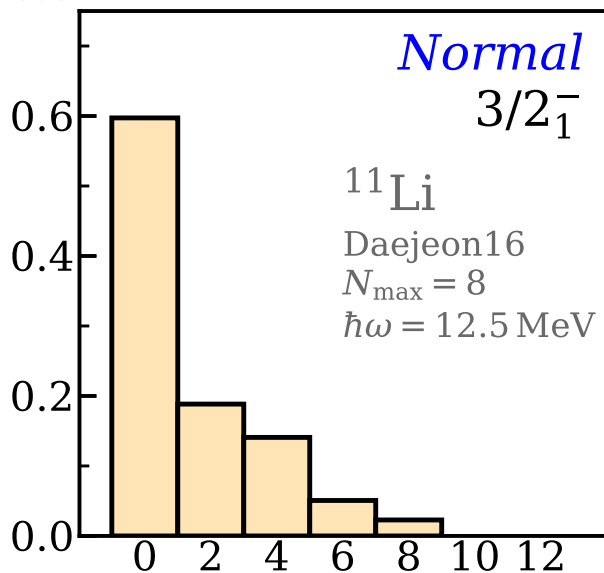
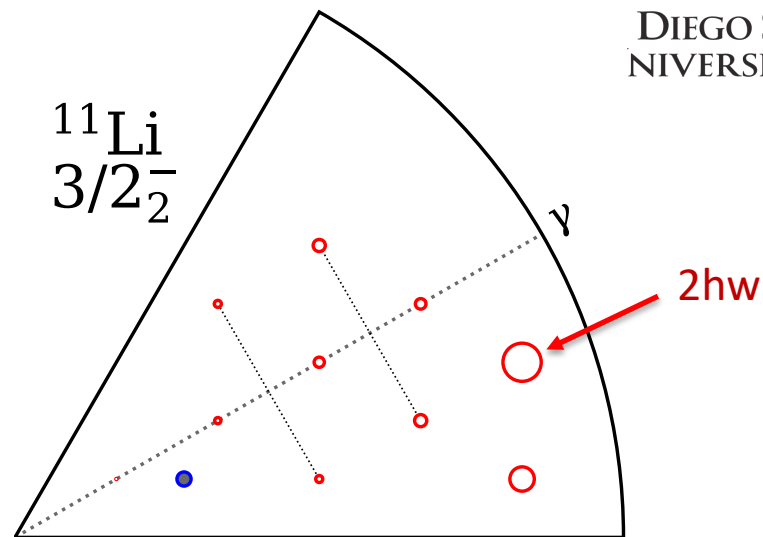
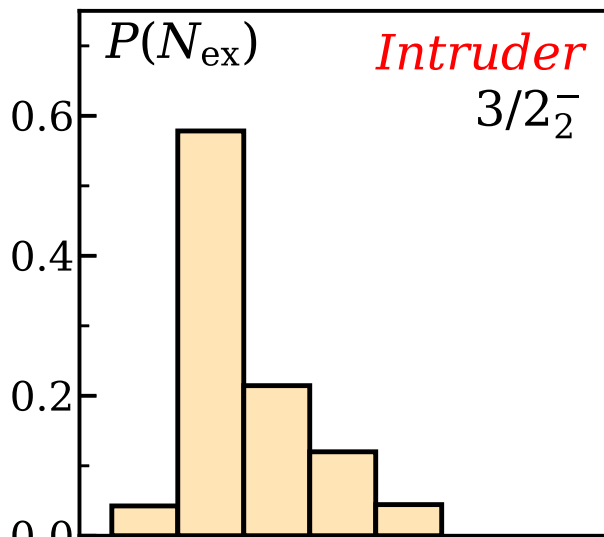


Group-theoretical Decomposition

Symplectic $\text{Sp}(3,\mathbb{R})$

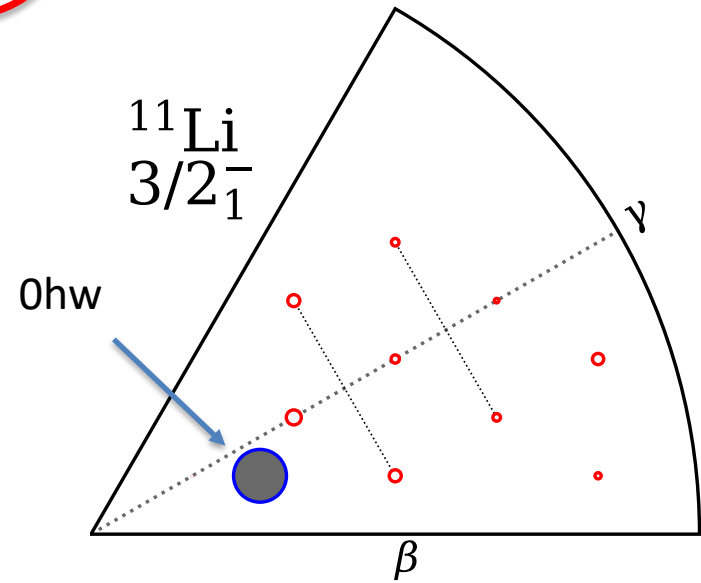
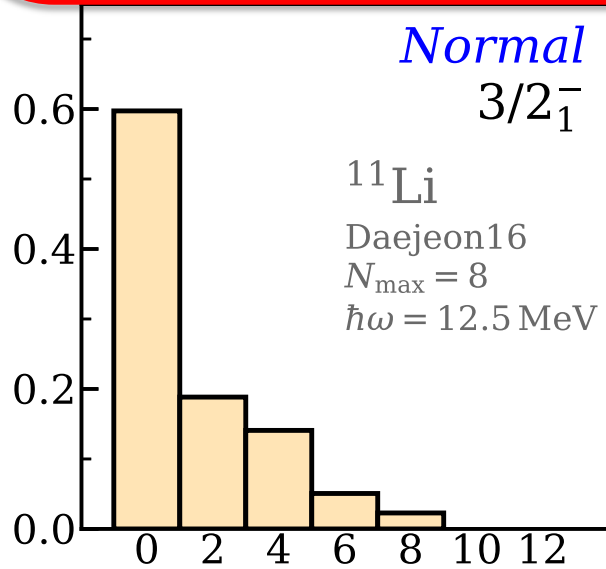
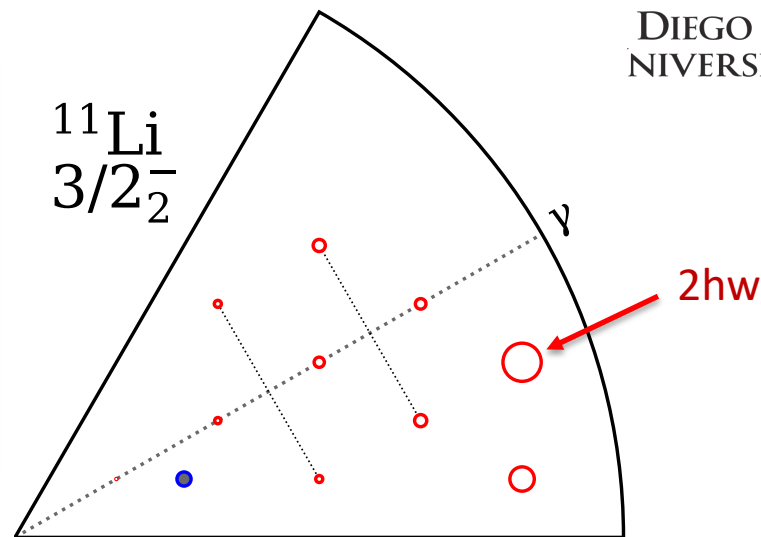
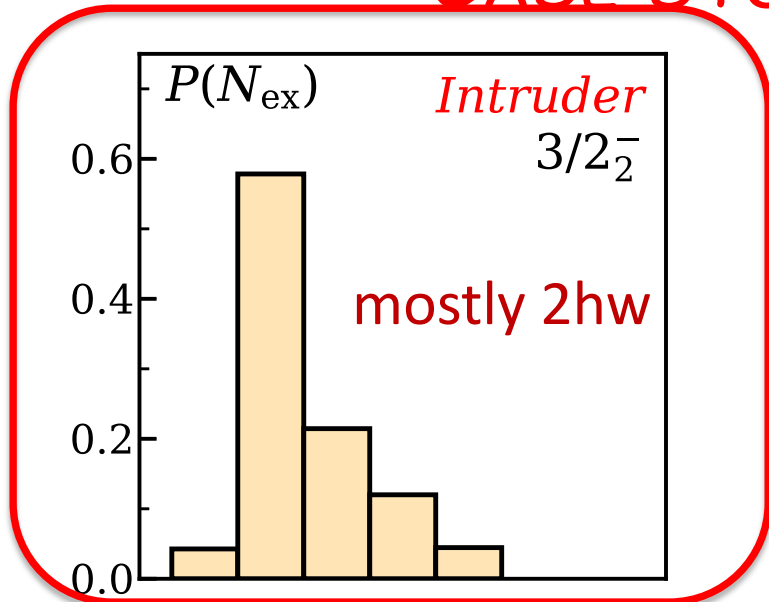


CASE STUDY: ^{11}Li





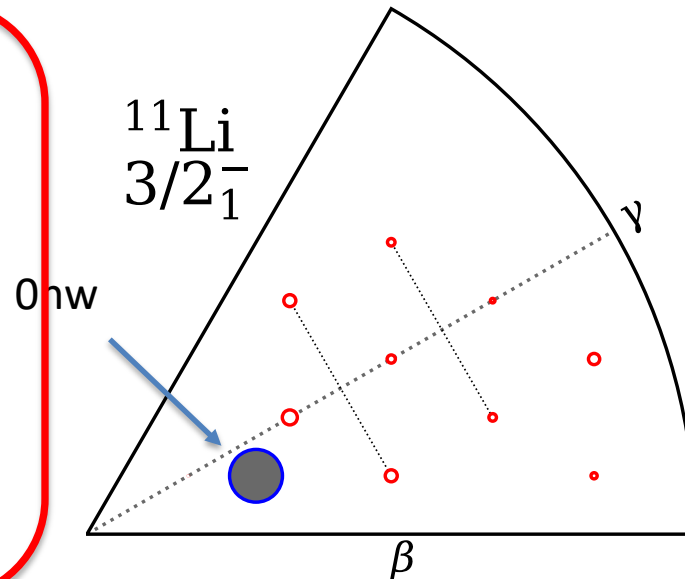
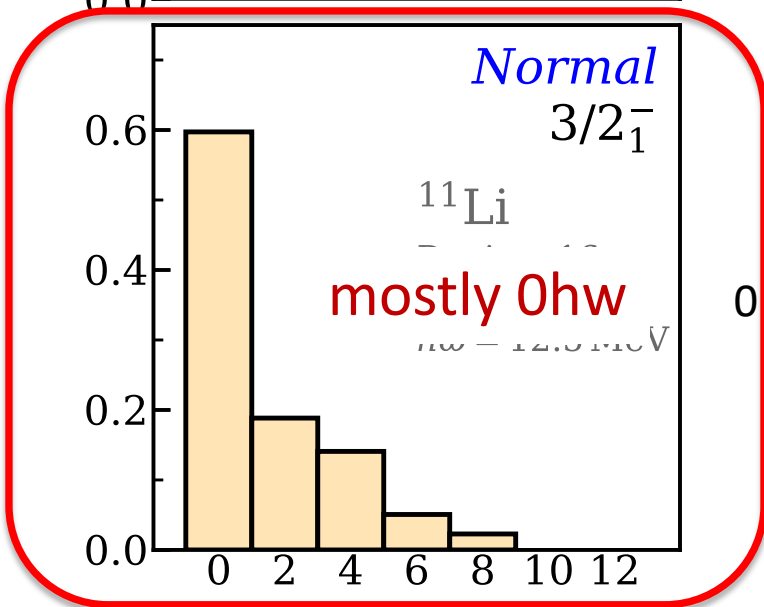
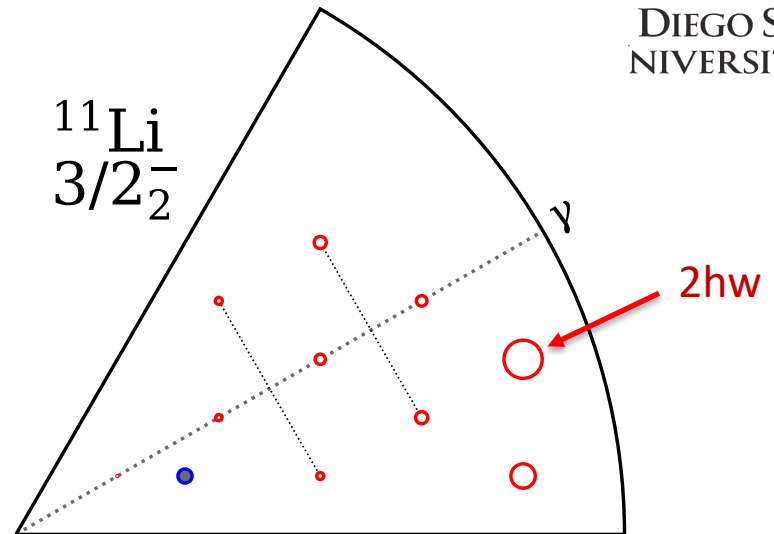
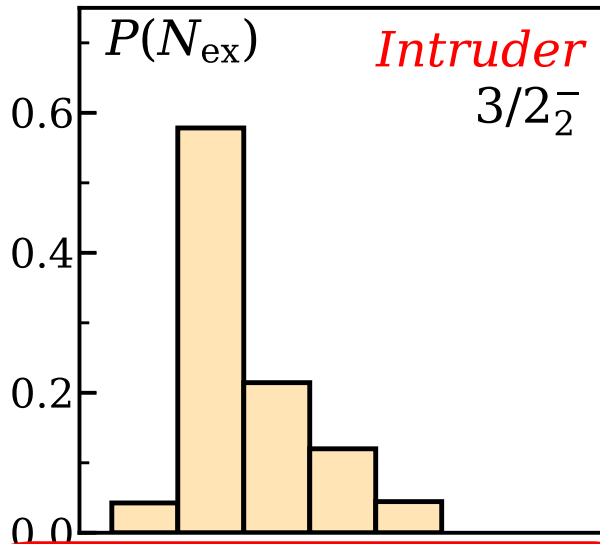
CASE STUDY: ^{11}Li



CASE STUDY: ^{11}Li



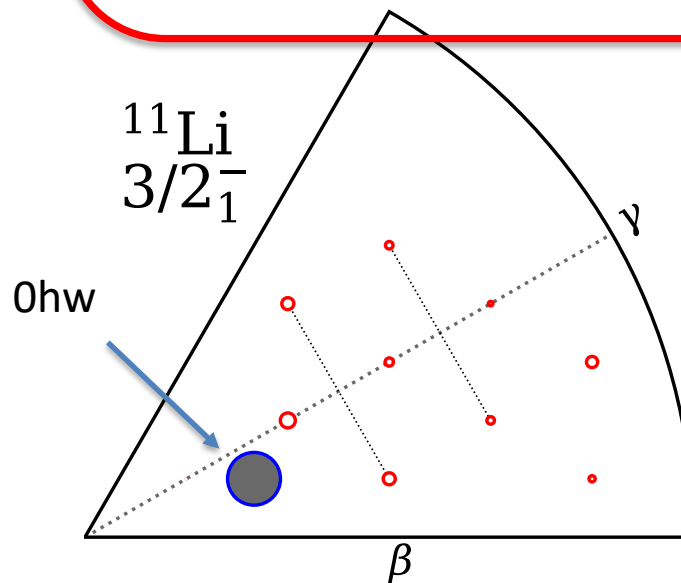
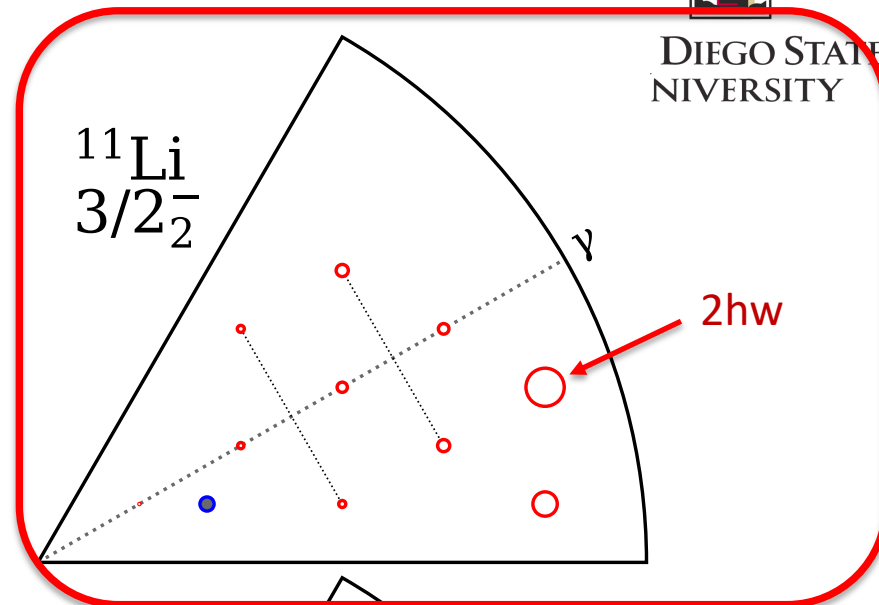
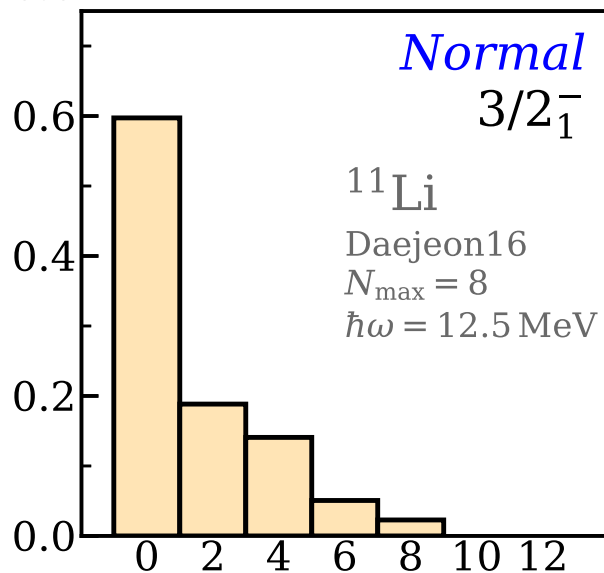
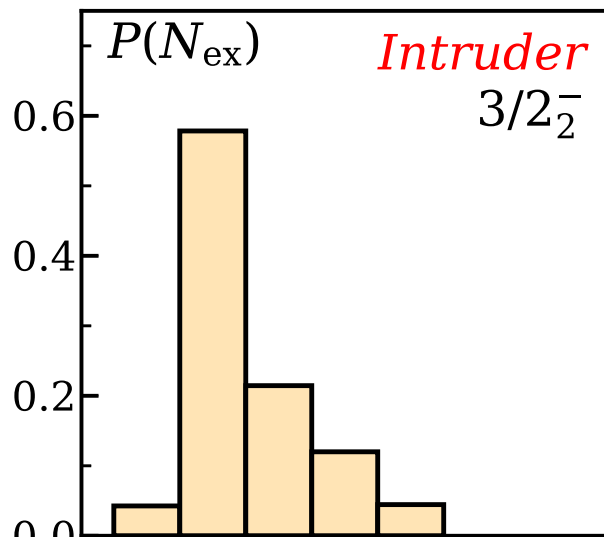
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CASE STUDY: ^{11}Li



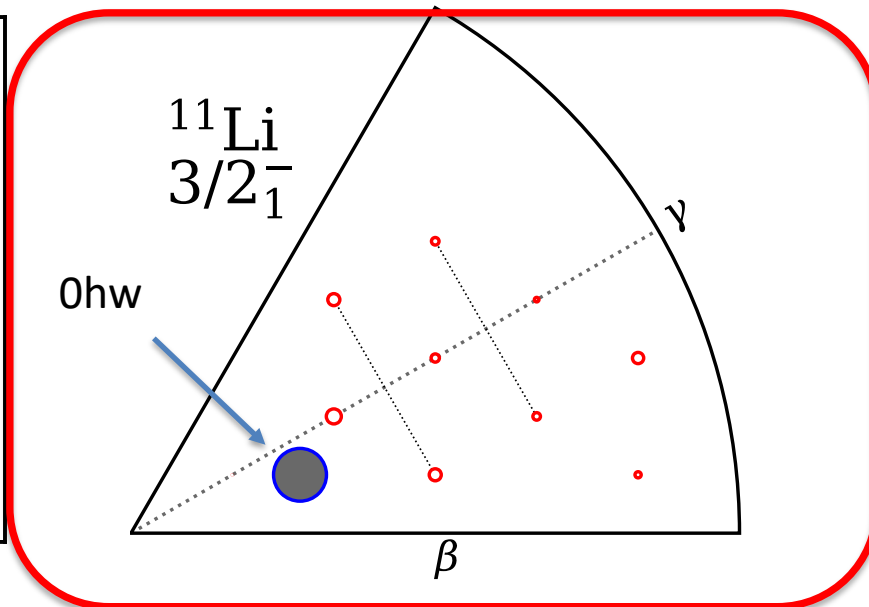
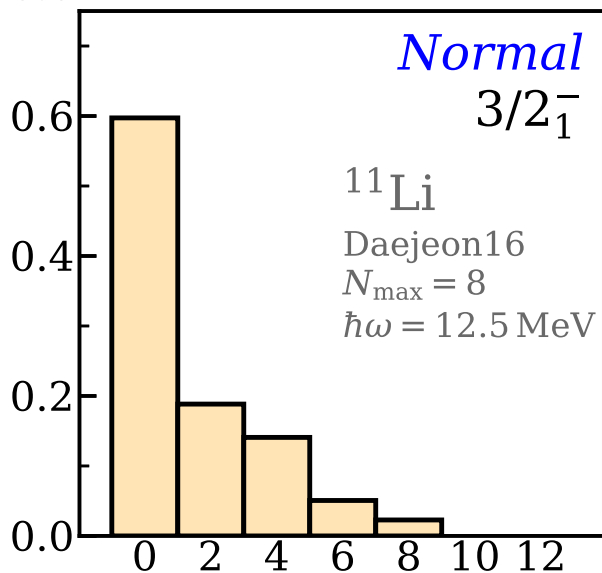
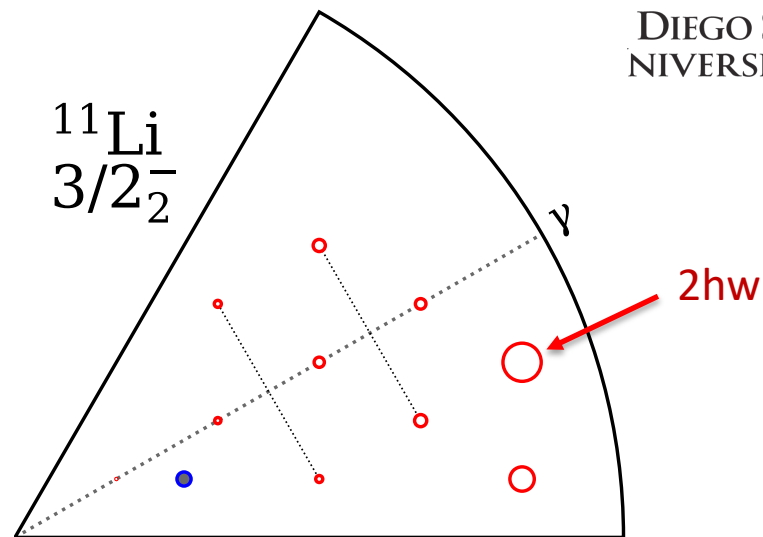
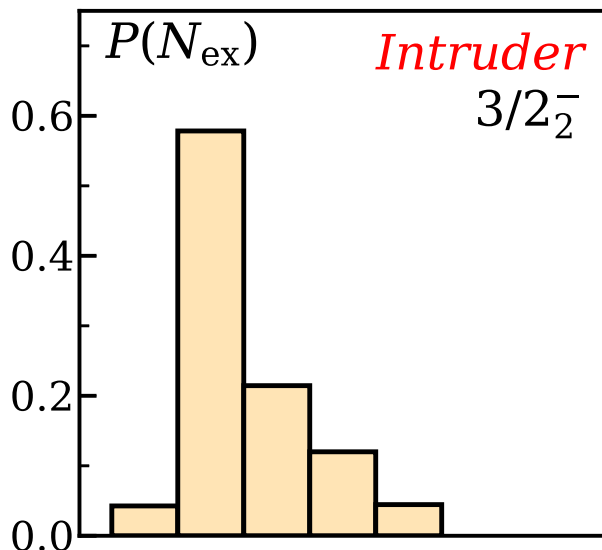
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CASE STUDY: ^{11}Li



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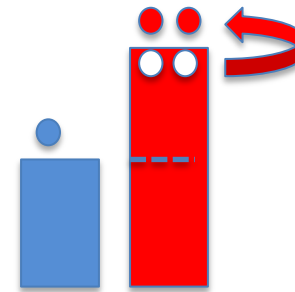
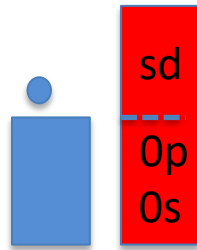


CASE STUDY: ^{29}F



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^{29}F is an analog of ^{11}Li



One proton outside a
filled shell
+ filled neutron shell

One proton outside a
filled shell
+ neutron 2p-2h

“island of inversion”

CASE STUDY: ^{29}F

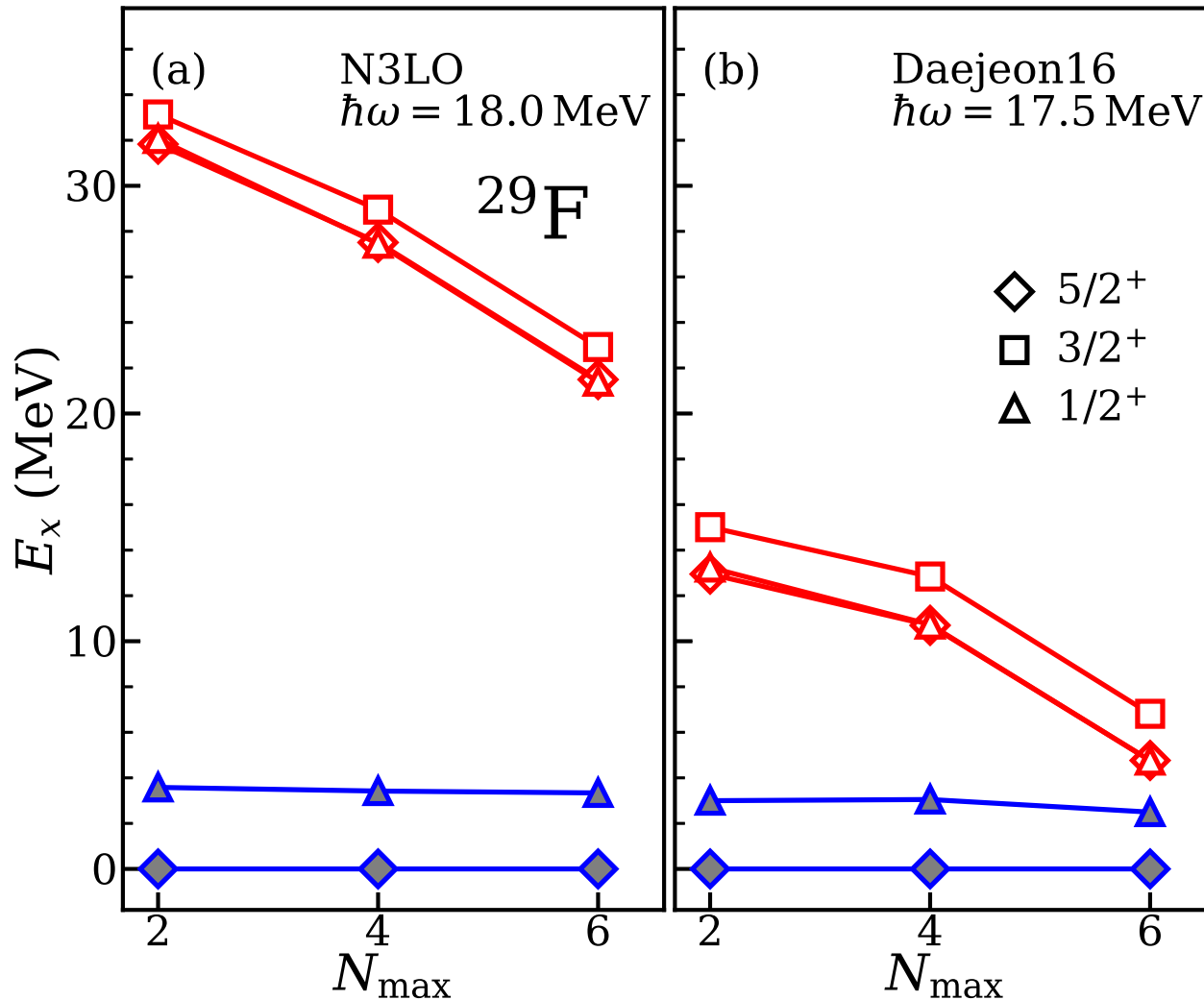


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^{29}F is an analog of ^{11}Li

"intruder"

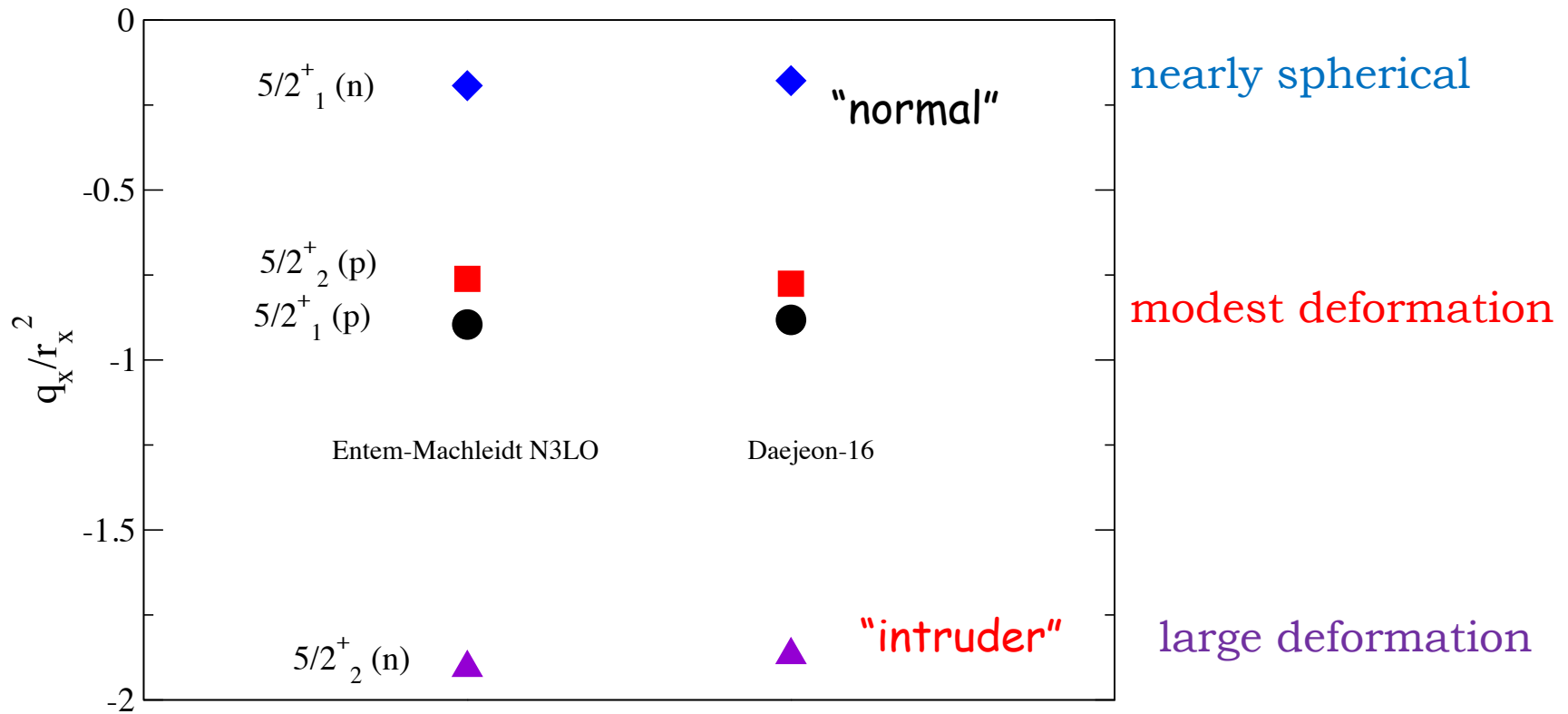
"normal"



CASE STUDY: ^{29}F



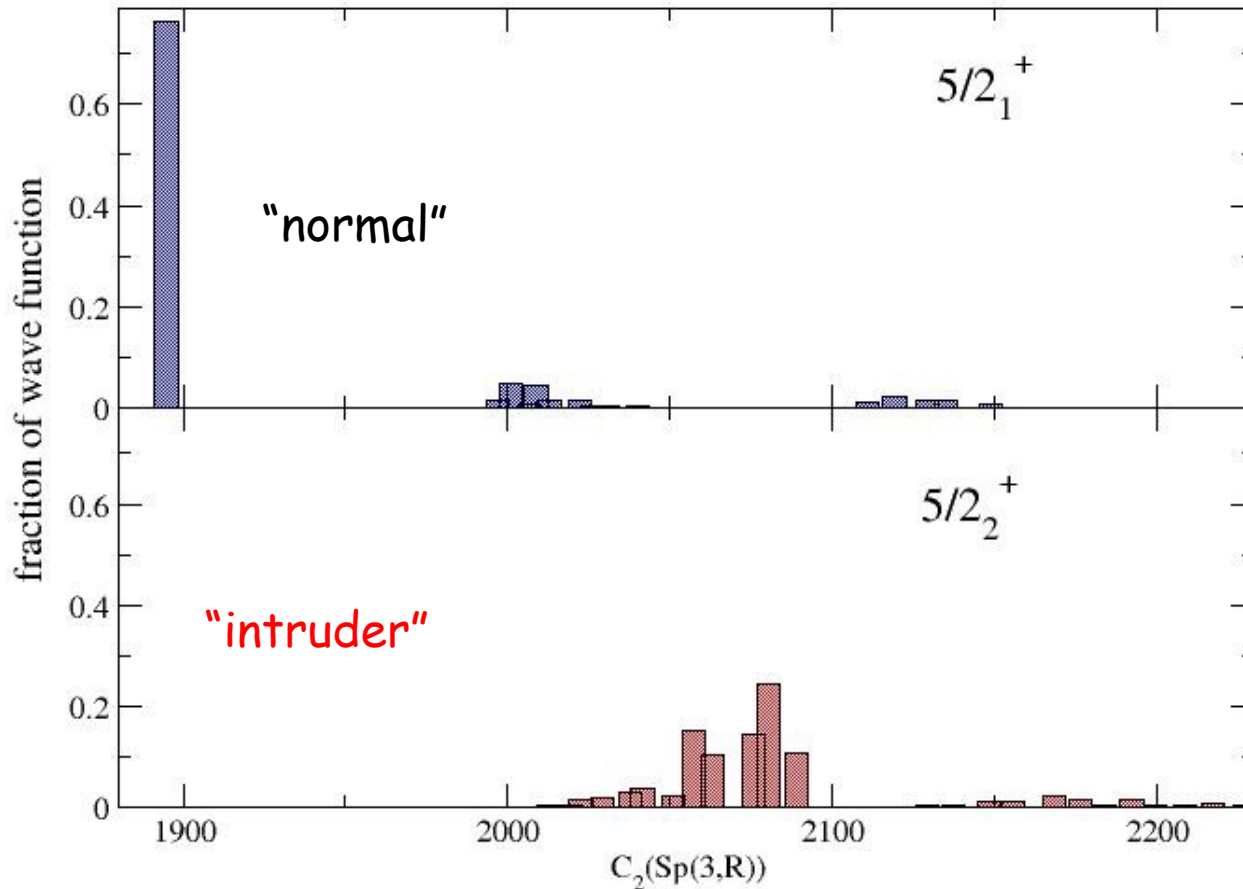
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CASE STUDY: ^{29}F



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$N_{\text{max}} = 4$ (natural orbitals)

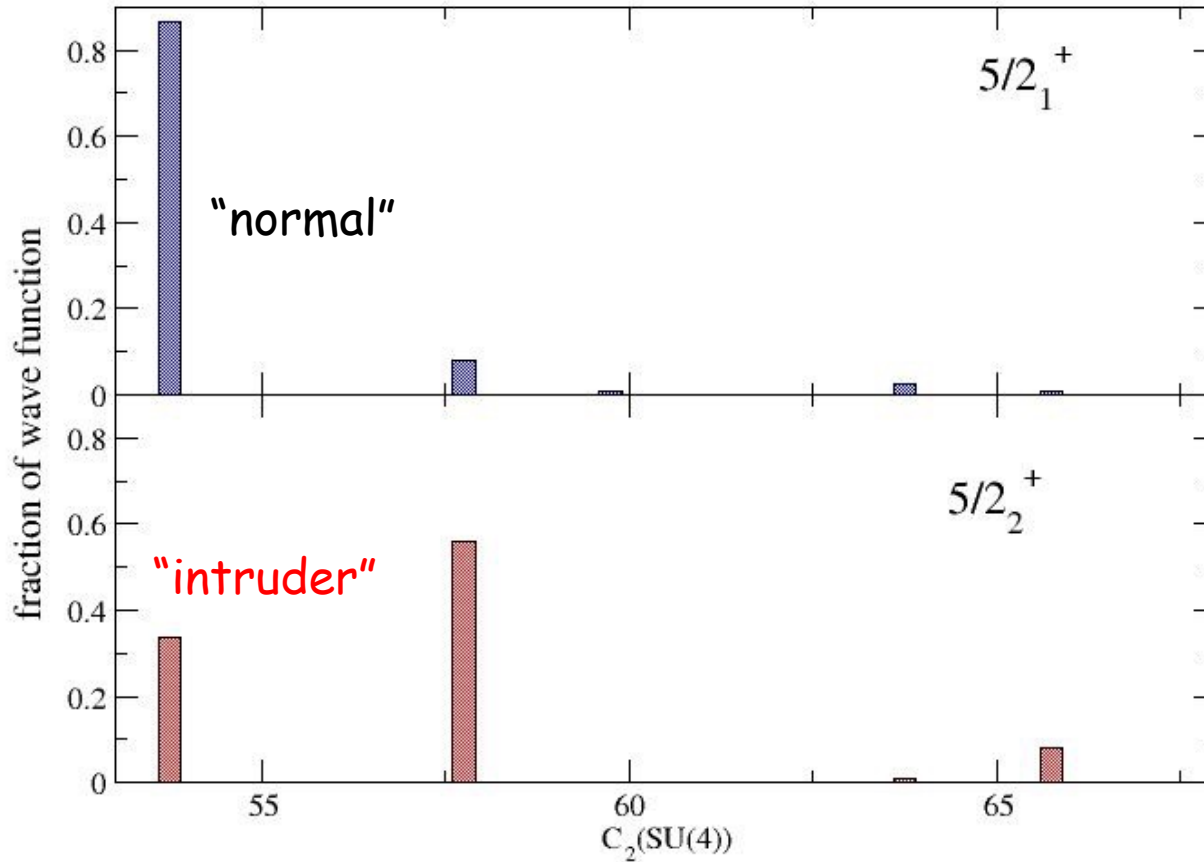
Group-
theoretical
Decomposition

Symplectic
 $\text{Sp}(3,\mathbb{R})$

CASE STUDY: ^{29}F



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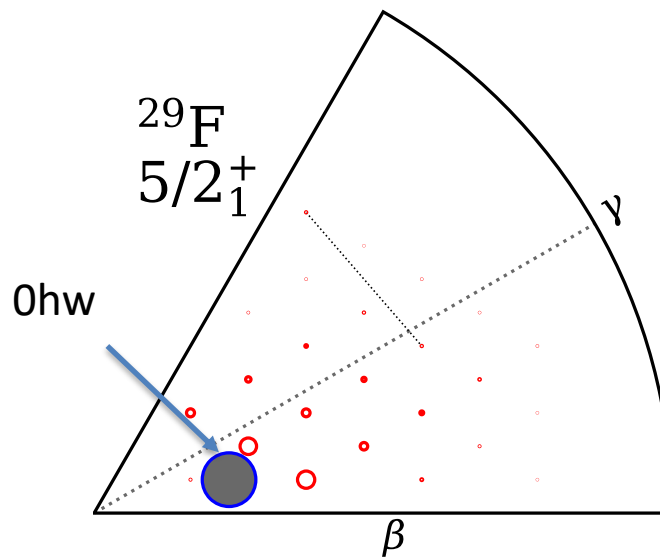
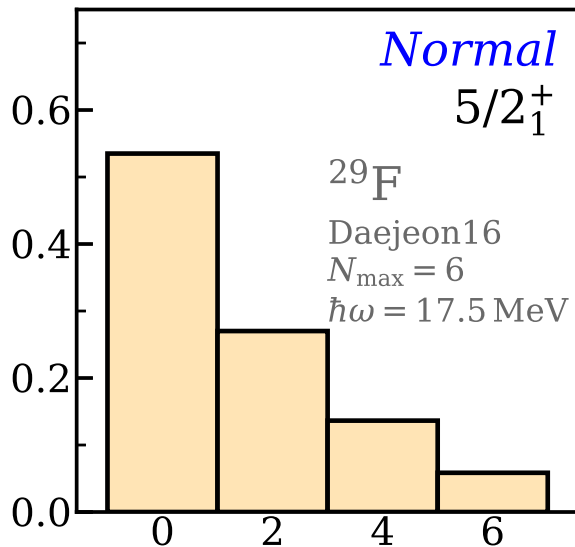
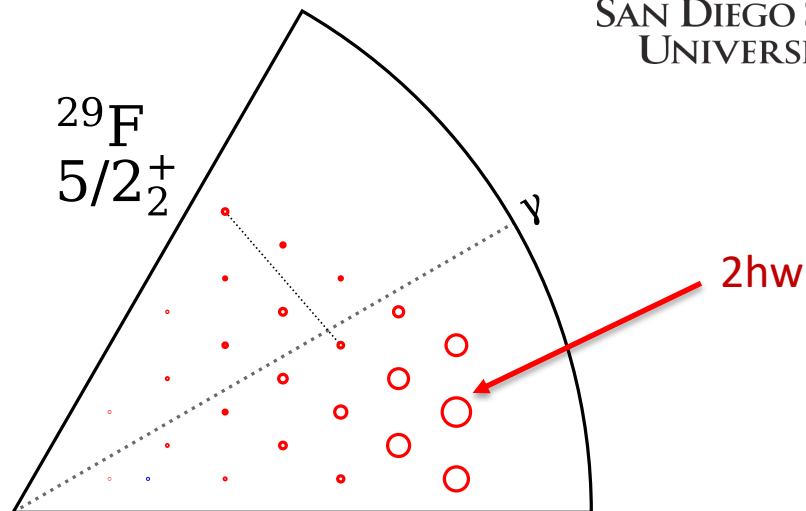
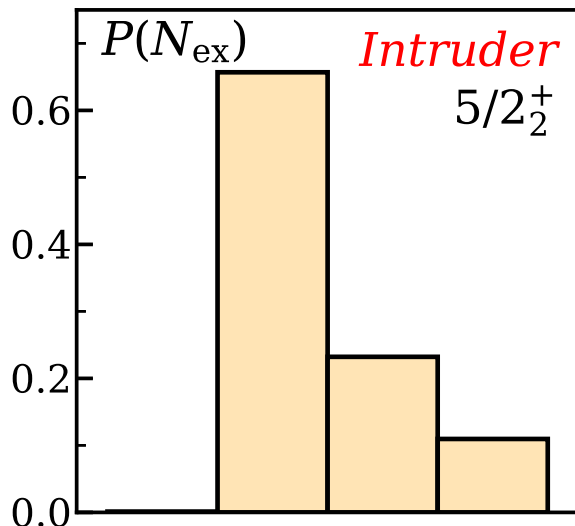
Group-
theoretical
decomposition

SU(4)

$N_{\text{max}} = 4$, natural orbitals



CASE STUDY: ^{29}F



CASE STUDIES: ^{11}Li , ^{29}F

What do we learn?



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CASE STUDIES: ^{11}Li , ^{29}F



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What do we learn?



Convergence of intruder states is especially slow!

For these cases, the intruders appear to be deformed (stronger signal in neutron quadrupole moment)

CASE STUDIES: ^{11}Li , ^{29}F



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Admittedly, our
calculations are not
fully converged....

CASE STUDIES: ^{11}Li , ^{29}F



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Admittedly, our
calculations are not
fully converged....



CASE STUDIES: ^{11}Li , ^{29}F



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I suggest ^{11}Li , ^{29}F as case studies for other methods (coupled cluster, IM-SRG, symmetry adapted, lattice, etc.).

The neutron quadrupole moment is a particularly useful **calculational** signal of deformation (unfortunately not for experiment....)



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The N=14 Island: ^{20}C ?

(preliminary results)



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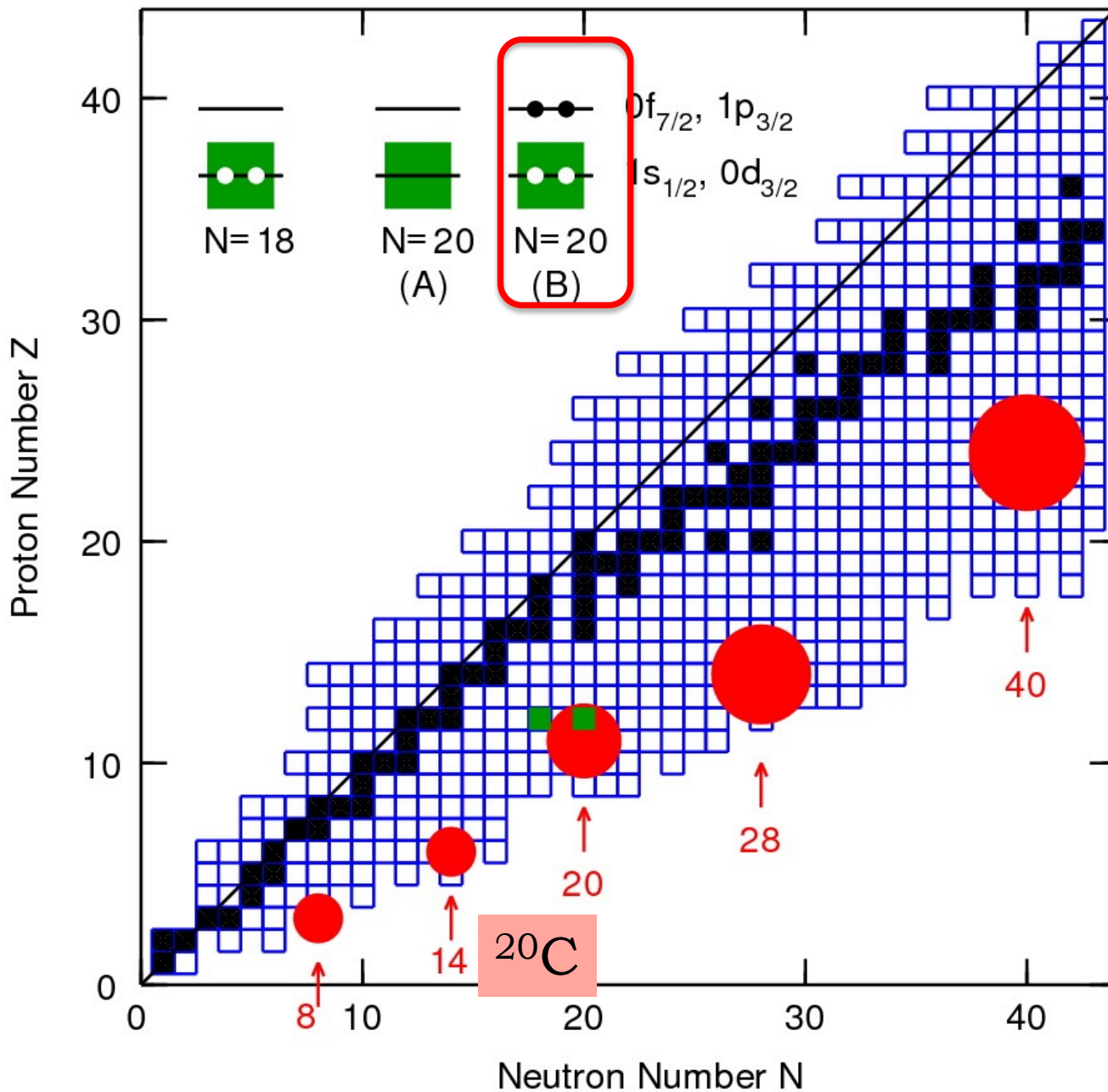


Figure:
Alex Brown





The N=14 Island: ^{20}C ?

(preliminary results)

N=14 semi-magic = closure of $0d_{5/2}$ shell.

Note that filled $0d_{5/2}$ shell does not have $L = 0$, but is admixture of many values of L (and also many $SU(3)$ irreps, and thus admixture of many shape \rightarrow interpretation may be less clean than N=8, 20.



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The N=14 Island: ^{20}C ?

(preliminary results)

I computed ^{20}C and nearby nuclides in NCSM
with Daejeon 16 and only at $N_{\text{max}} = 4$, $hw = 17.5$.



The N=14 Island: ^{20}C ?

(preliminary results)

I computed ^{20}C and nearby nuclides in NCSM with Daejeon 16 and only at $N_{\text{max}} = 4$, $hw = 17.5$.

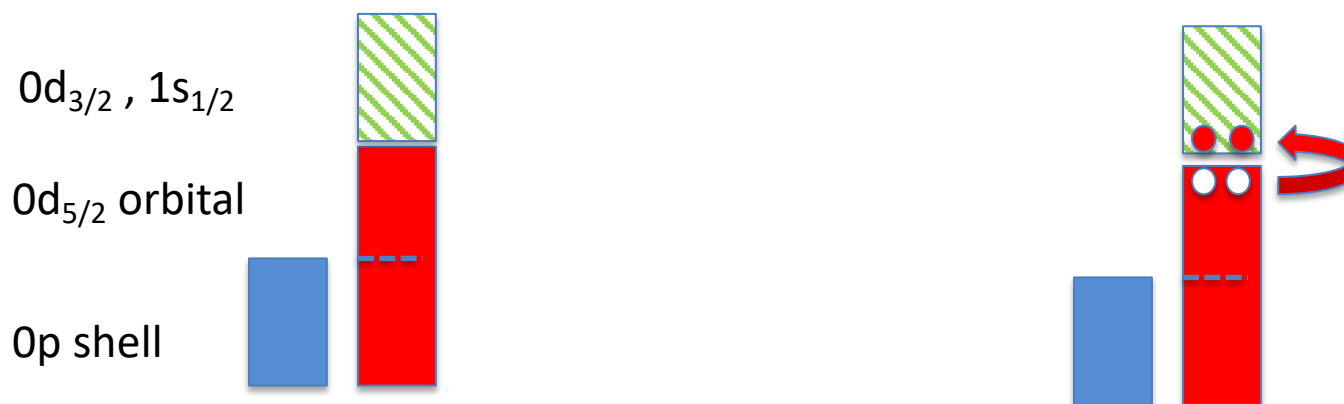
Remember, we're looking for:

- particle excitation out of the filled orbital ($0d_{5/2}$)
- Lower state more deformed than excited state



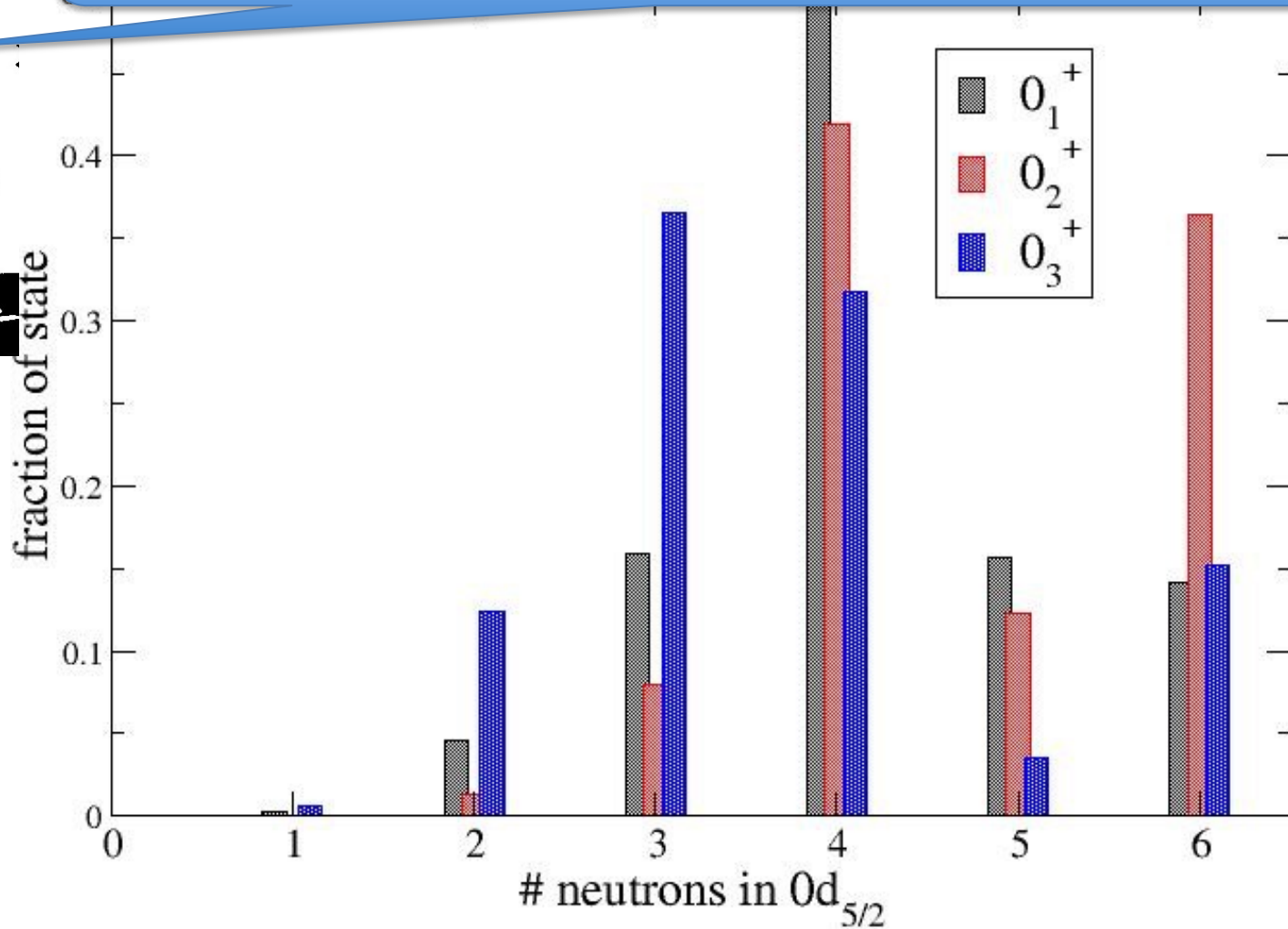


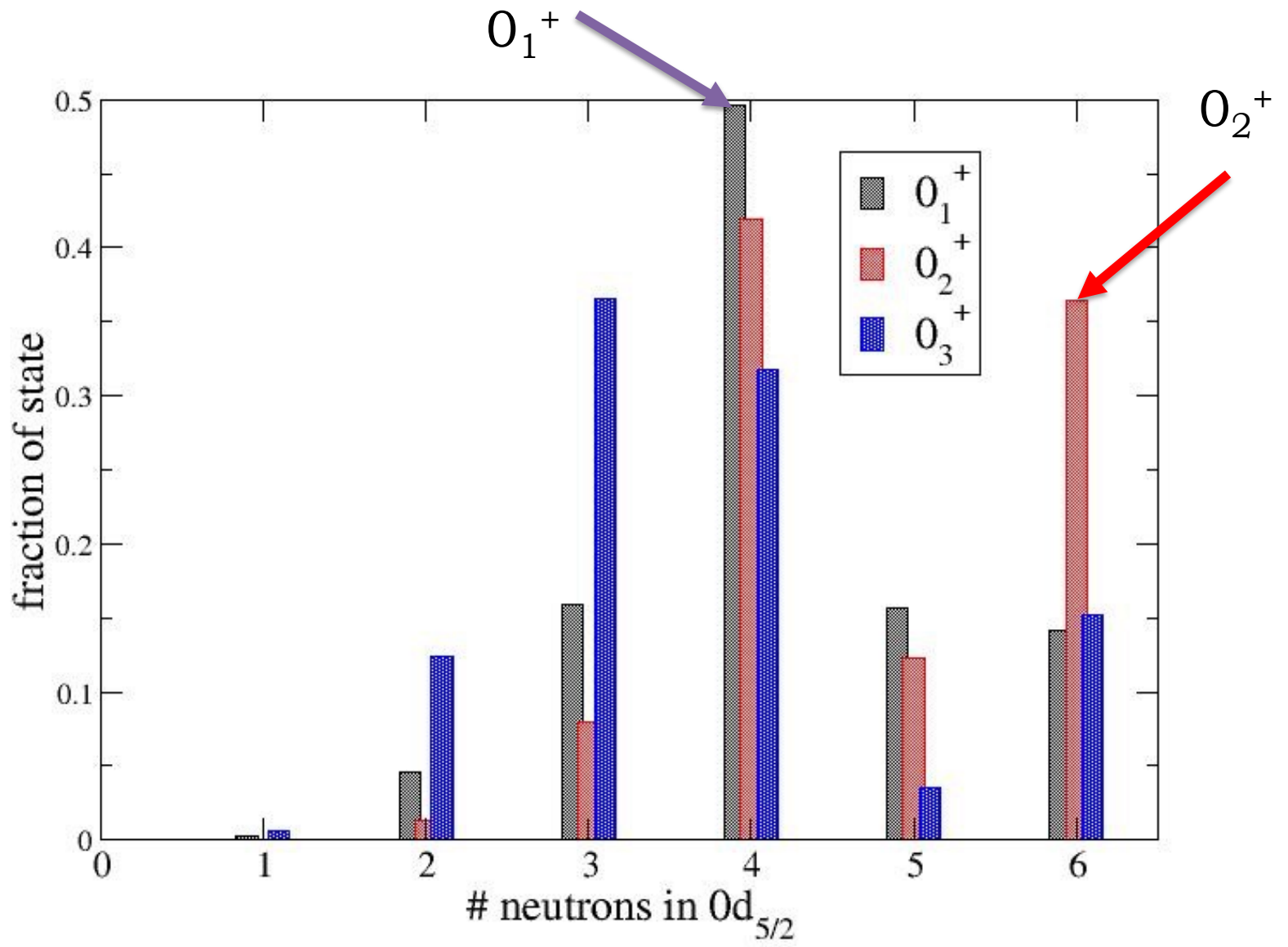
Particle excitation out of $0d_{5/2}$ /s



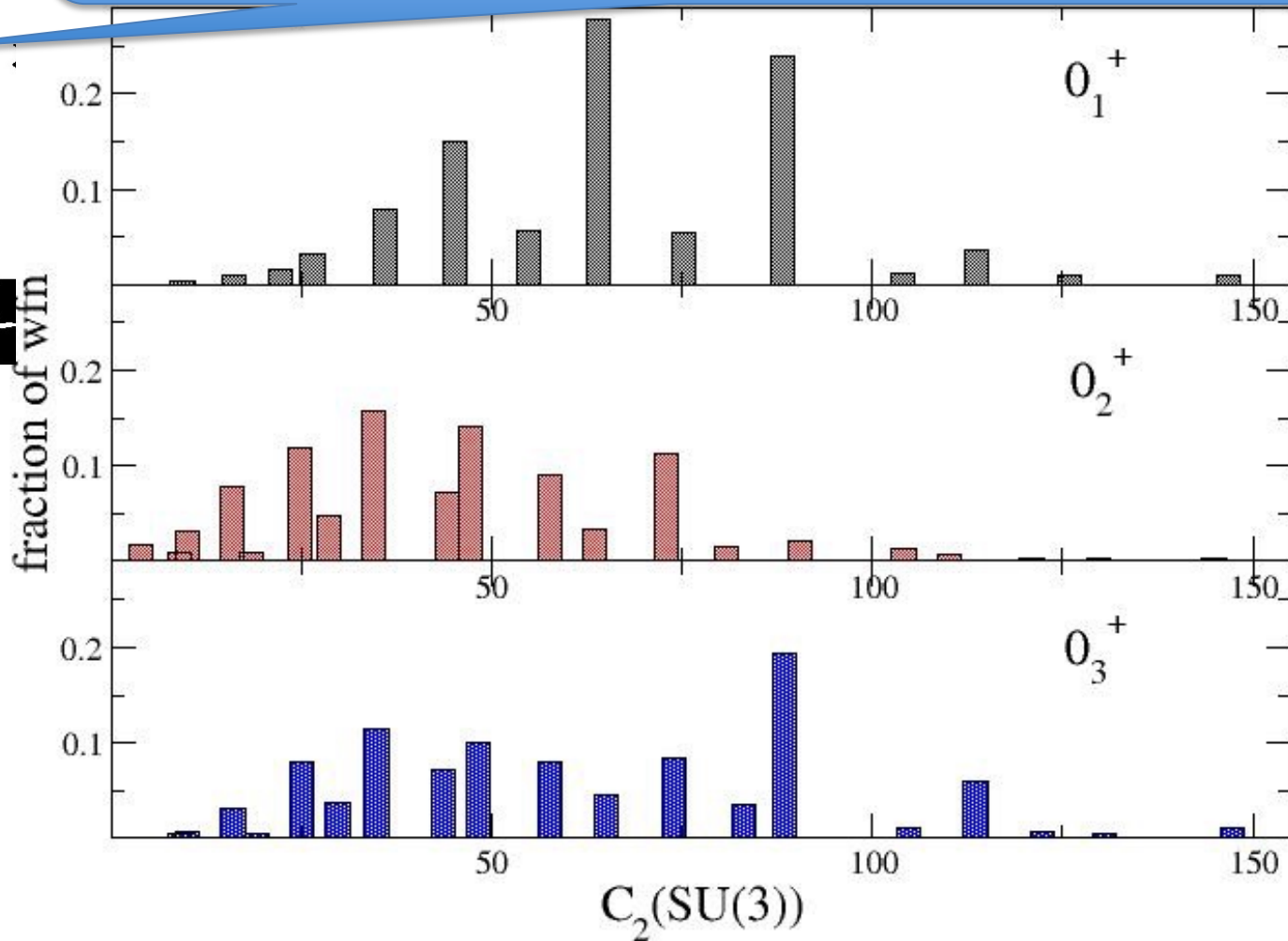
Naïve expectation: 6 neutrons in $0d_{5/2}$ orbital

Decompose the $J=0$ states by number of neutrons in $0d_{5/2}$ orbital

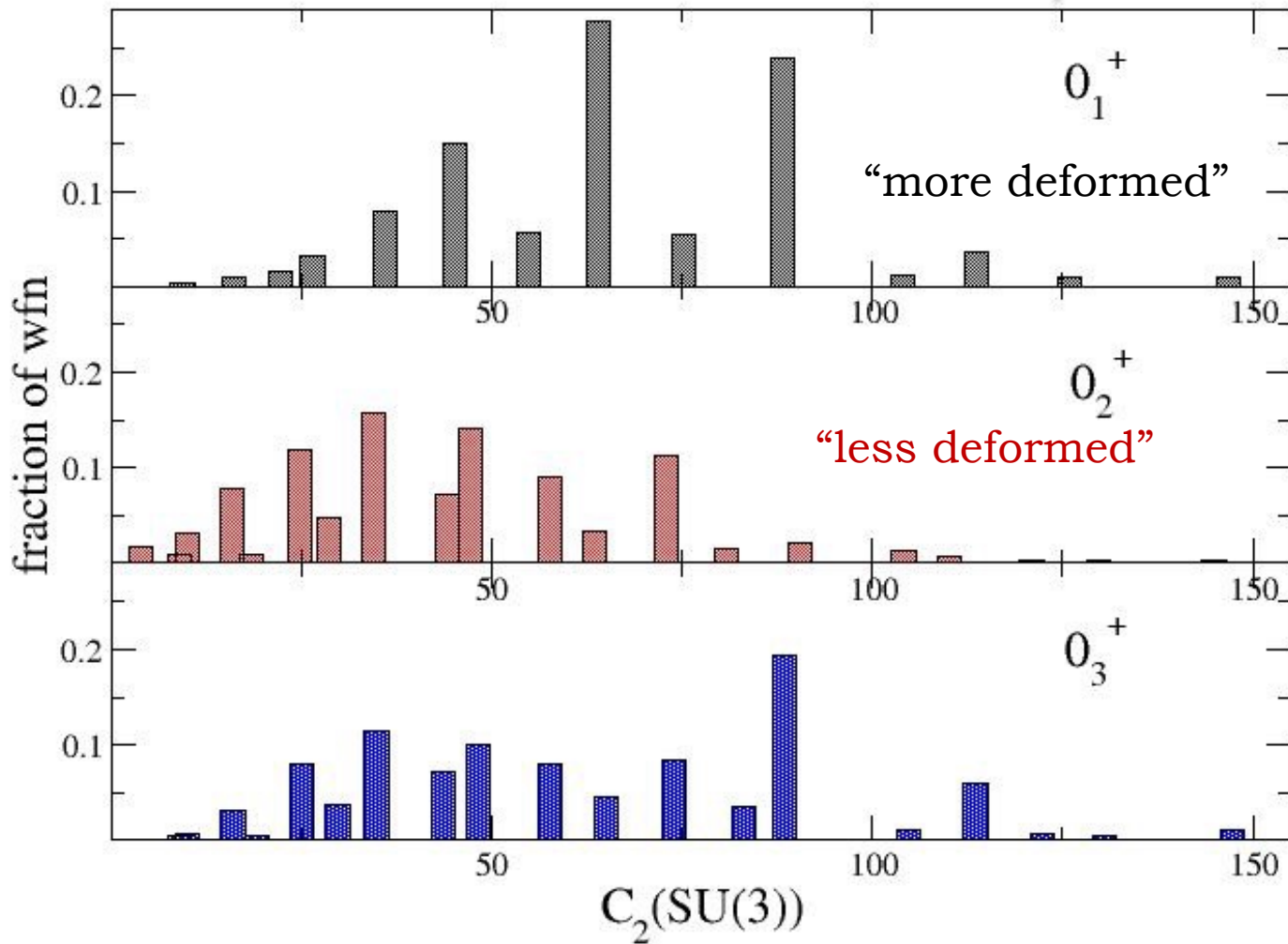




We can also decompose using $SU(3)$



more deformed





The N=14 Island: ^{20}C ?

The initial results are not as clean as for ^{11}Li and ^{29}F .

The shape distributions are more complicated, unsurprising given that the $0d_{5/2}$ is only half of spin-orbit partners



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The N=14 Island: ^{20}C ?

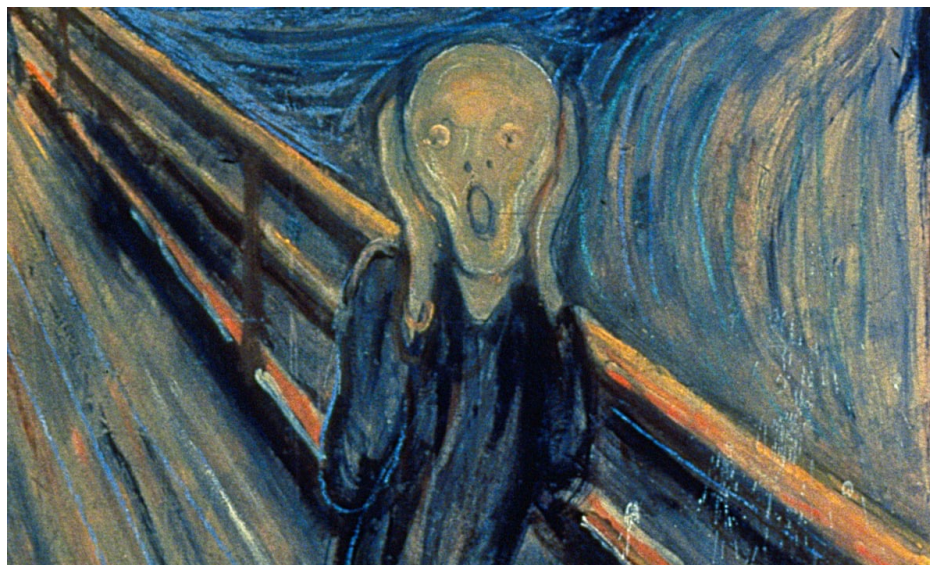
Of course the calculations are not converged...



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The N=14 Island: 20°C ?

Of course the calculations are not converged...





The $N=14$ Island: ^{20}C ?

Next steps:

- Higher N_{max}
- Other interactions (e.g., EM N3LO)
- Compute effective single particle energies



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The hunt for shape coexistence and intruders in the NCSM:

NCSM Hartree-Fock

preliminary results



NCSM Hartree-Fock

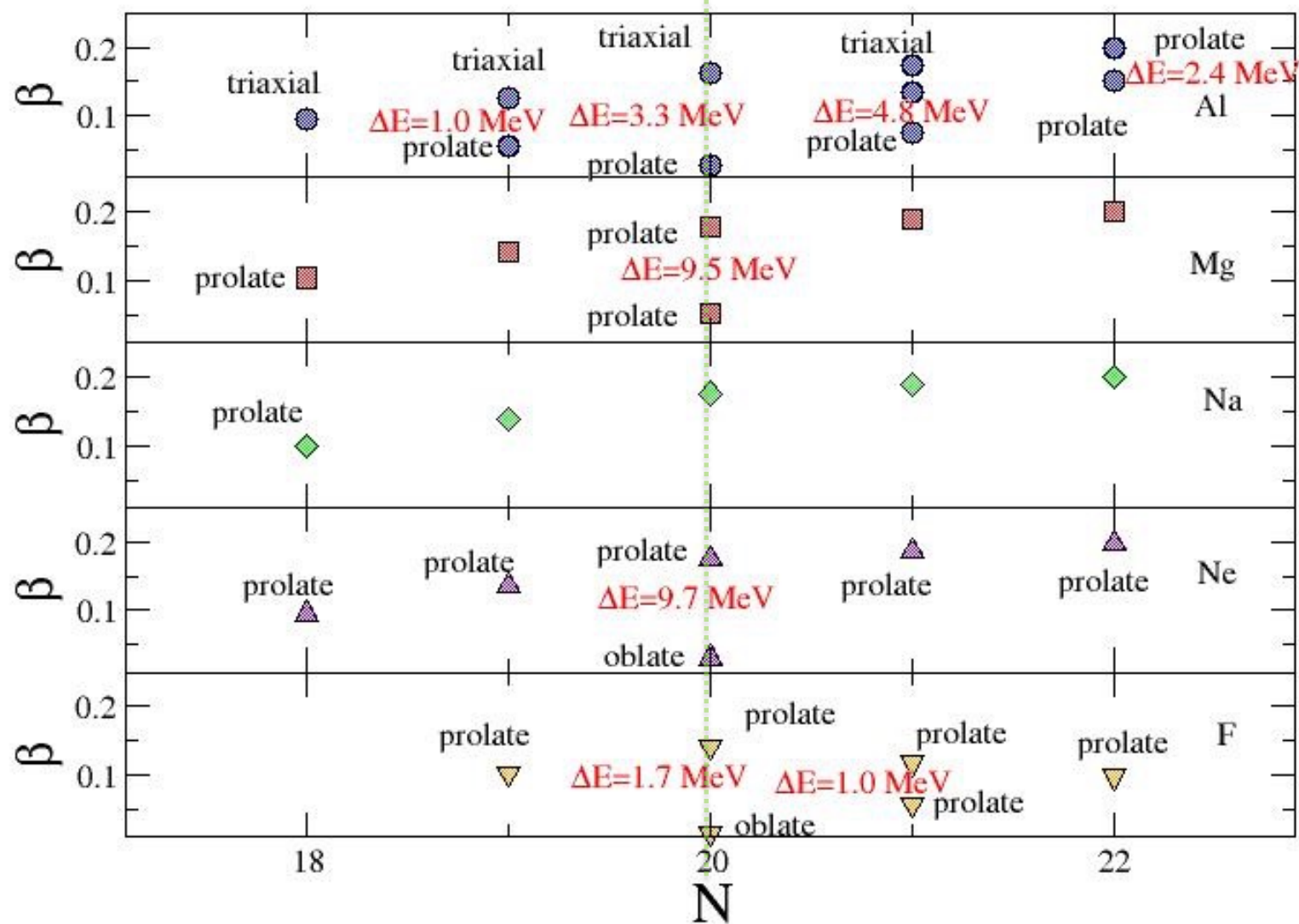
- * Construct unrestricted (no axial symmetry) Slater determinant in NCSM space and minimize $\langle H \rangle$ via gradient descent
see e.g., Lauber, Frye, CWJ, J. Phys. G. **48**, 095107 (2021)
- * Daejeon16 in 7-10 major shells, hw \sim 15-20 MeV (results not very sensitive to details of space).
- * Added H_{cm} to reduce spuriousity to a few percent.
- * Random initial states allow to find multiple minima.



NCSM Hartree-Fock

First stop: around $N = 20$







Results for $N = 20$

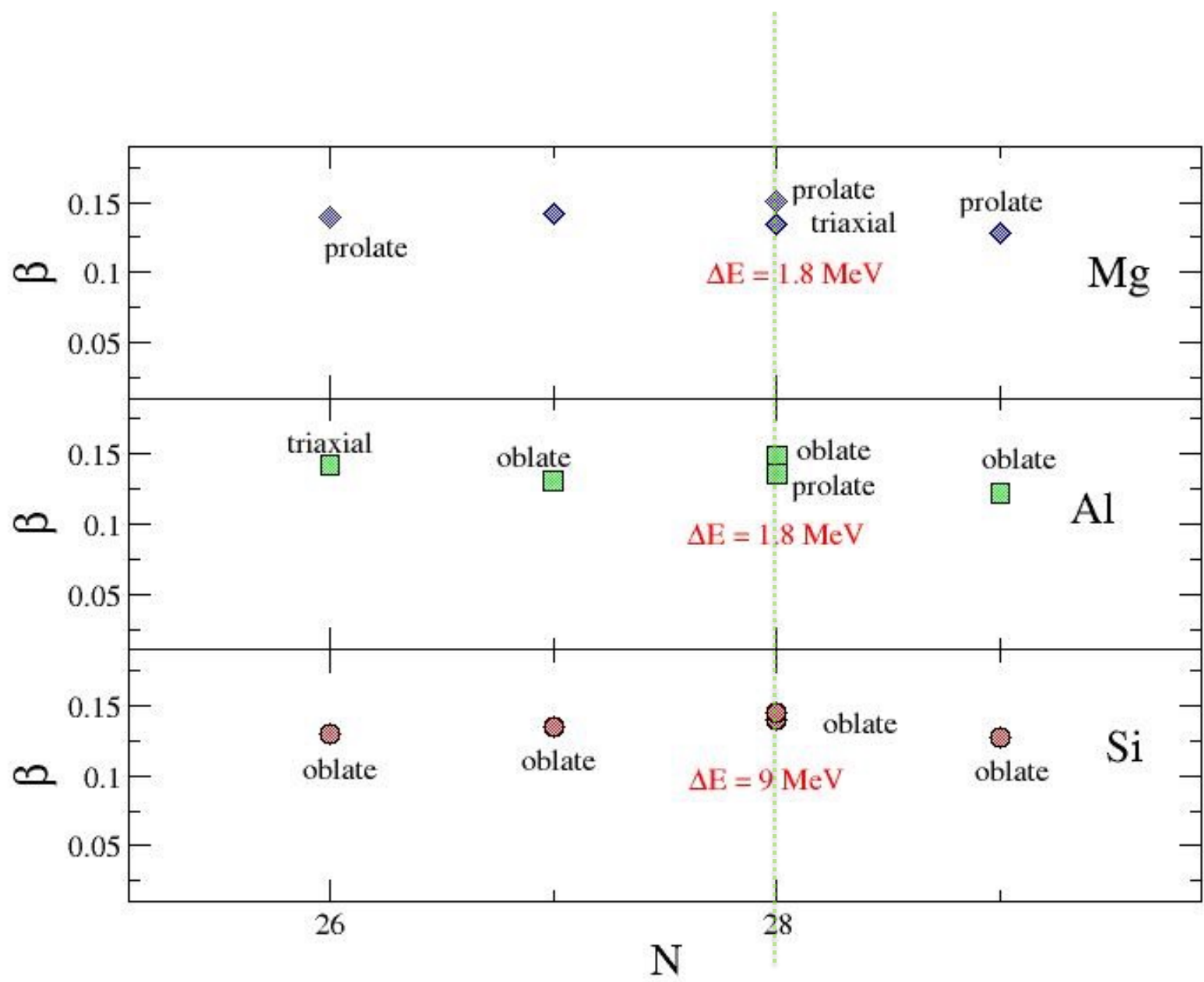
- Clear shape coexistence at and near $N=20$ (except for Na isotopes)
- * More deformed state is lower in energy, from 1-9 MeV, typically prolate
- * Al tends to triaxiality and multiple minima (?)

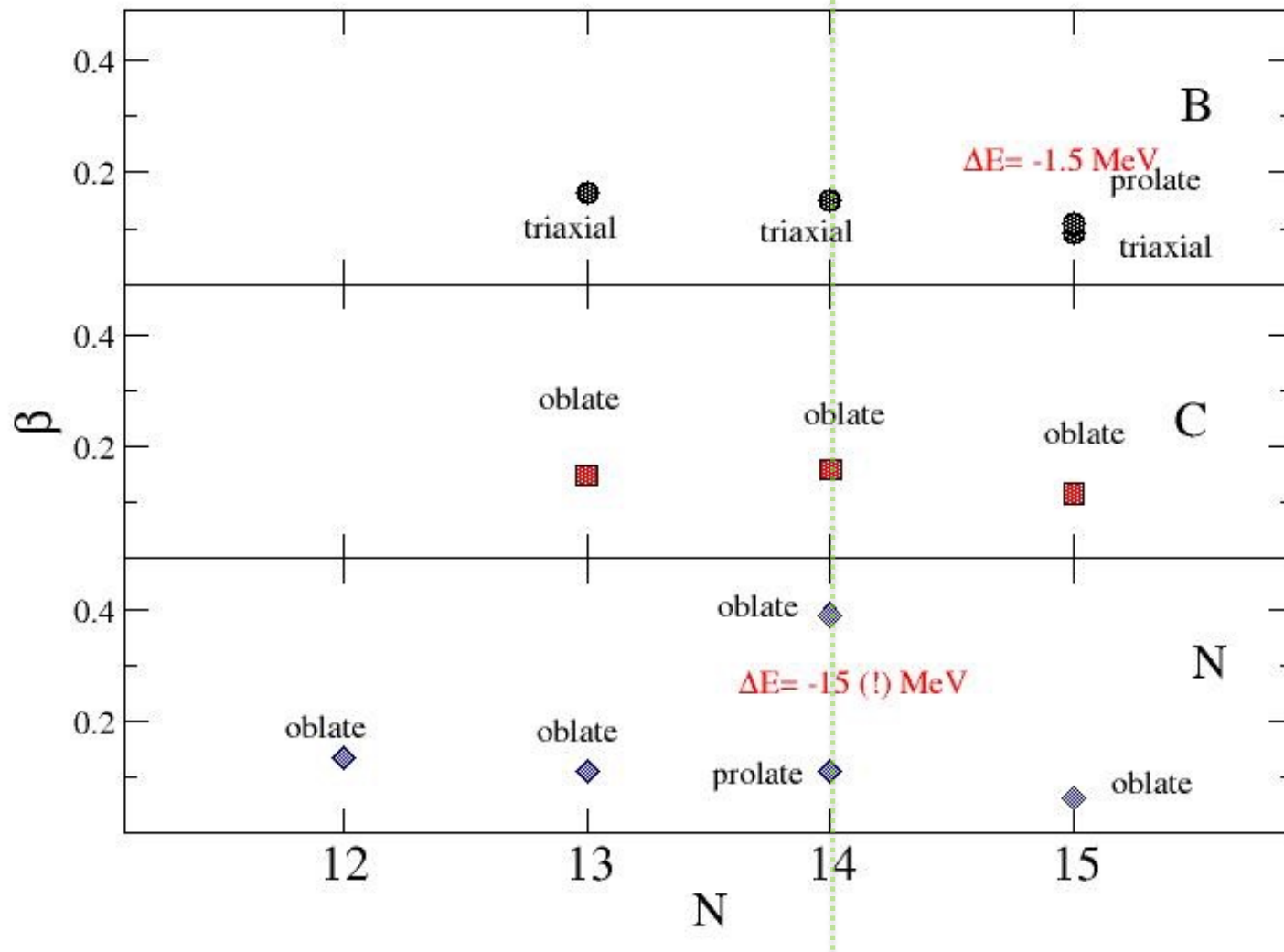


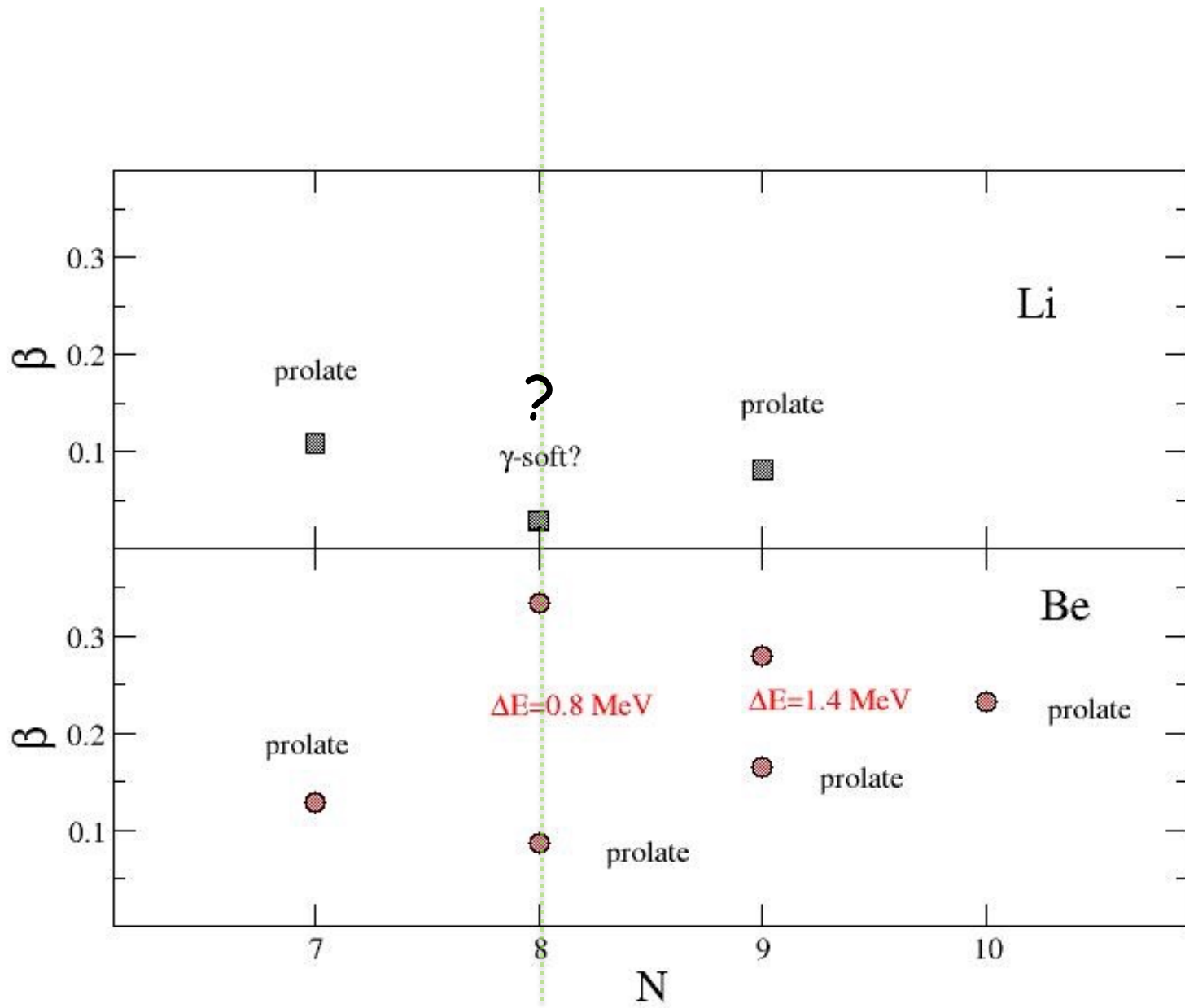
NCSM Hartree-Fock

Next: around $N = 28$











Next steps:

- Try other interactions (e.g. E&M N3LO)
- Map out potential energy surface (GCM)
- Compute effective single-particle energies
- 3-body forces? Continuum?

Summary



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The no-core configuration-interaction **shell model** remains useful.

But 'intruder' states are very challenging! They are highly deformed and require large model spaces

Mean-field and beyond *may* be helpful in exploring



Summary



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The no-core configuration-interaction **shell model** remains useful.

But 'intruder' states are very challenging! They are highly deformed and require large model spaces

Other methods (NLEFT, symmetry-adapted) may be necessary. We suggest $^{11}\text{Li}/^{29}\text{F}$ as useful test cases





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Extra slides
for extraordinary audiences





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M-scheme dimensions are huge—into the
tens of billions!
How can we possibly 'understand' them?



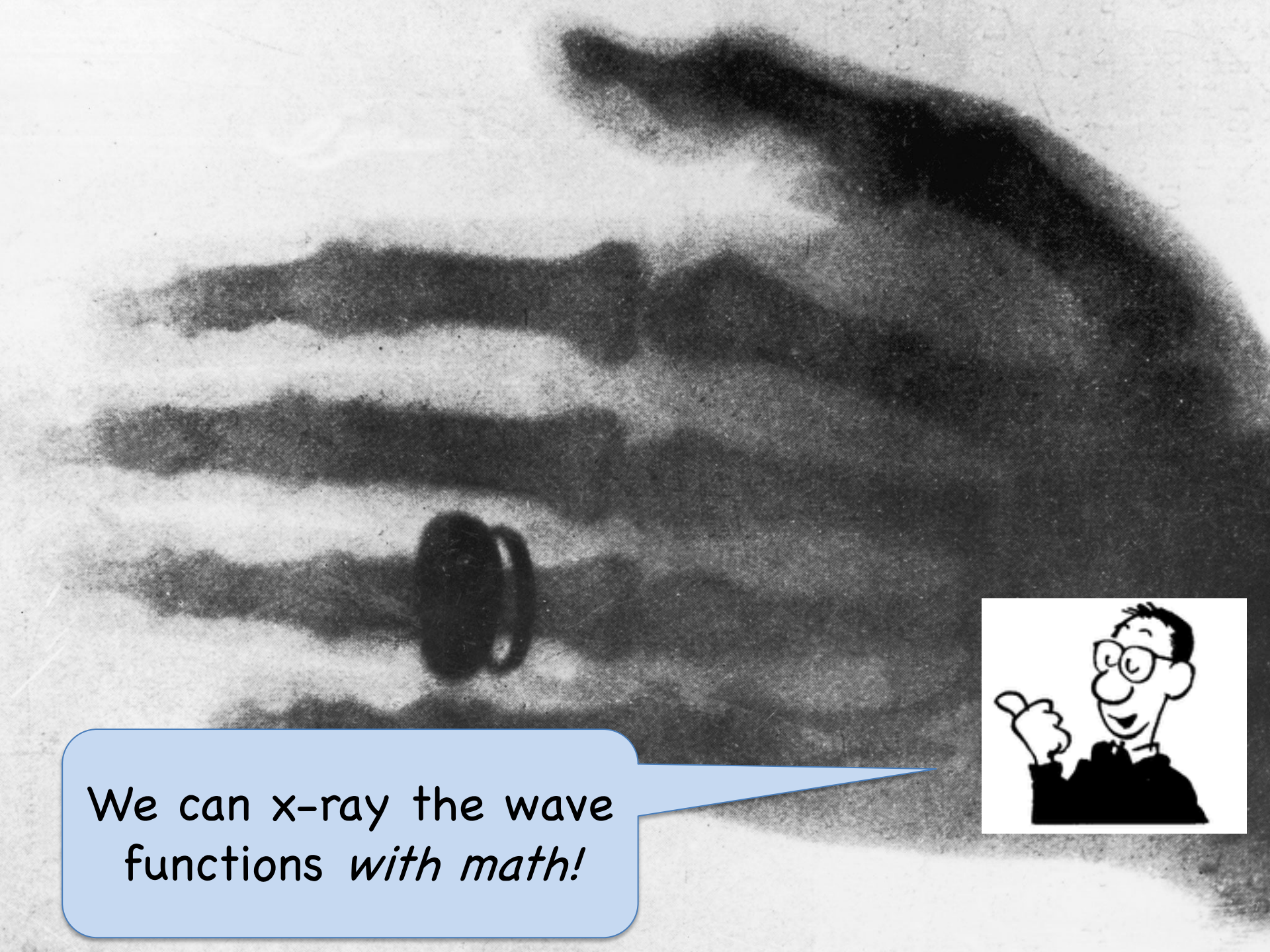


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M-scheme dimensions are huge—into the
tens of billions!
How can we possibly 'understand' them?



Richard Hamming:
*The purpose of computing
is **insight**, not numbers.*



We can x-ray the wave functions *with math!*





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Use eigenvalues
of Casimir operators to label
subspaces (“irreps”)





Casimir

$$\hat{C} |z, \alpha\rangle = z |z, \alpha\rangle$$


z (eigenvalue)
labels the
subspace

α indexes all the
states in the subspace
(same value of z)





Casimir

 $\hat{C} |z, \alpha\rangle = z |z, \alpha\rangle$

The best known Casimir is \mathbf{J}^2 ,
which has eigenvalues $j(j+1)$





Casimir

$$\hat{C} |z, \alpha\rangle = z |z, \alpha\rangle$$

Another is Elliott's representation
of an $SU(3)$ Casimir:


$$\hat{C}_{SU(3)} = \vec{Q} \cdot \vec{Q} - \frac{1}{4} \vec{L}^2$$

For this 2-body $SU(3)$ Casimir,
the eigenvalue $z = \lambda^2 + \lambda\mu + \mu^2 + 3(\lambda + \mu)$,
where λ, μ label the irreps





Casimir

 $\hat{C} |z, \alpha\rangle = z |z, \alpha\rangle$

If the Casimir(s) commute(s)
with the Hamiltonian, $[\hat{H}, \hat{C}] = 0$
then the Hamiltonian is block-diagonal
in the *irreps* (irreducible representation)

This is known as *dynamical symmetry*





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A key idea: A Casimir can be used
to divide up a Hilbert space into subspaces,
labeled by eigenvalues

*even if the Casimir does not commute with
the Hamiltonian*





Casimir

$$\hat{C} |z, \alpha\rangle = z |z, \alpha\rangle$$

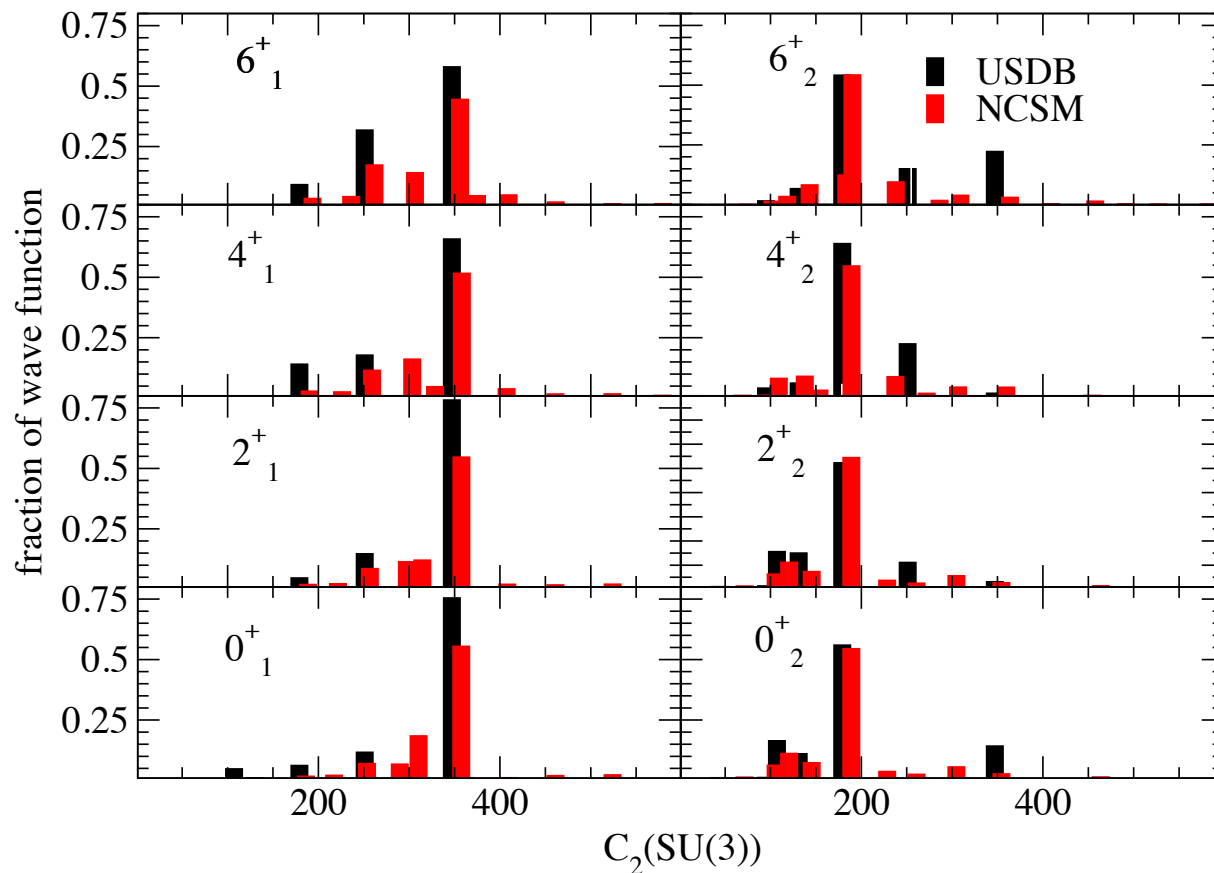
For some wavefunction $|\Psi\rangle$, we define
the *fraction of the wavefunction in an irrep*

$$F(z) = \sum_{\alpha} \left| \langle z, \alpha | \Psi \rangle \right|^2$$





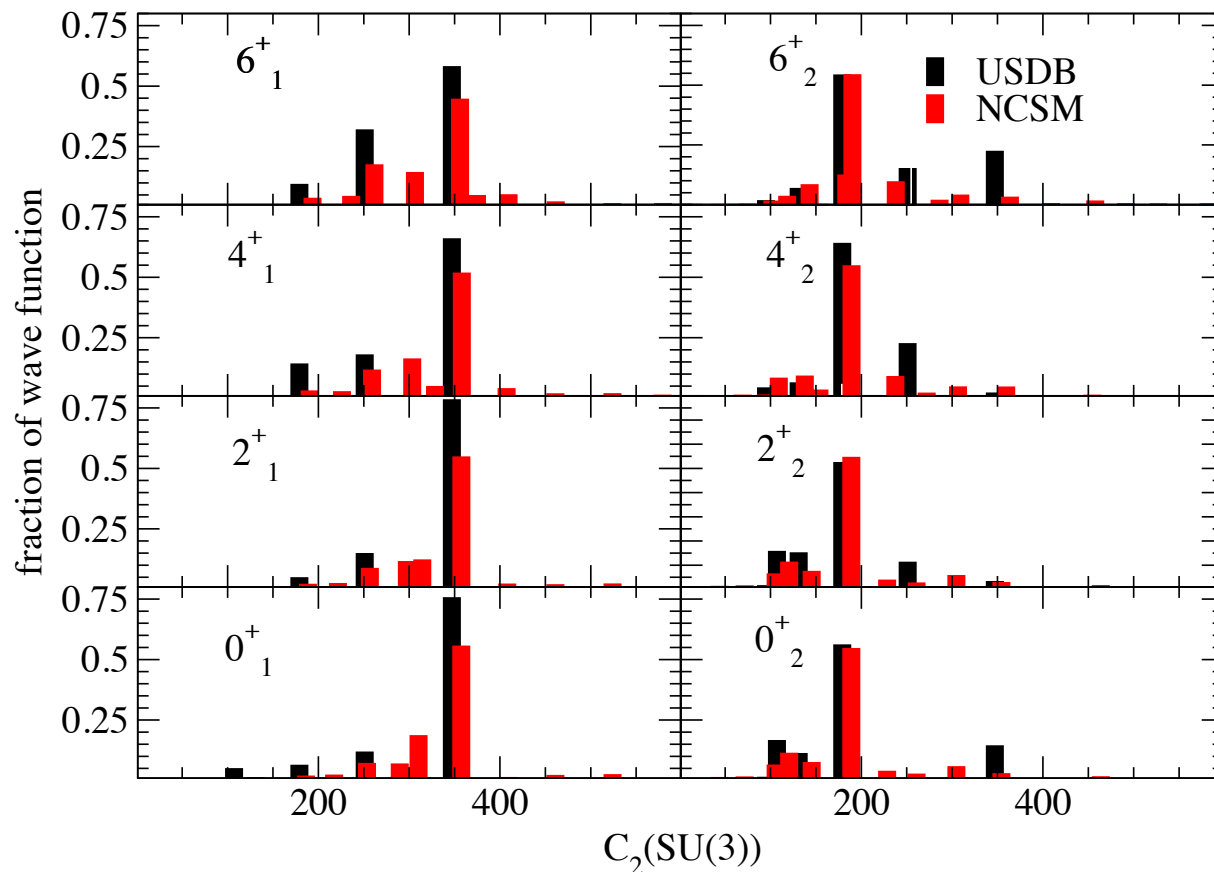
^{20}Ne



This can be done efficiently using a variant of the Lanczos algorithm:
CWJ, PRC **91**, 034313 (2015)



^{20}Ne



By looking at the group-theoretical decomposition, we can even show that the valence-space empirical and *ab initio* multi-shell wave functions have similar structure!



Particle excitation out of $0d_{5/2}$ /s



First, compute average neutron occupation
of $0d_{5/2}$



