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Two-Photon exchange correction from the *ab-initio* No-Core Shell Model

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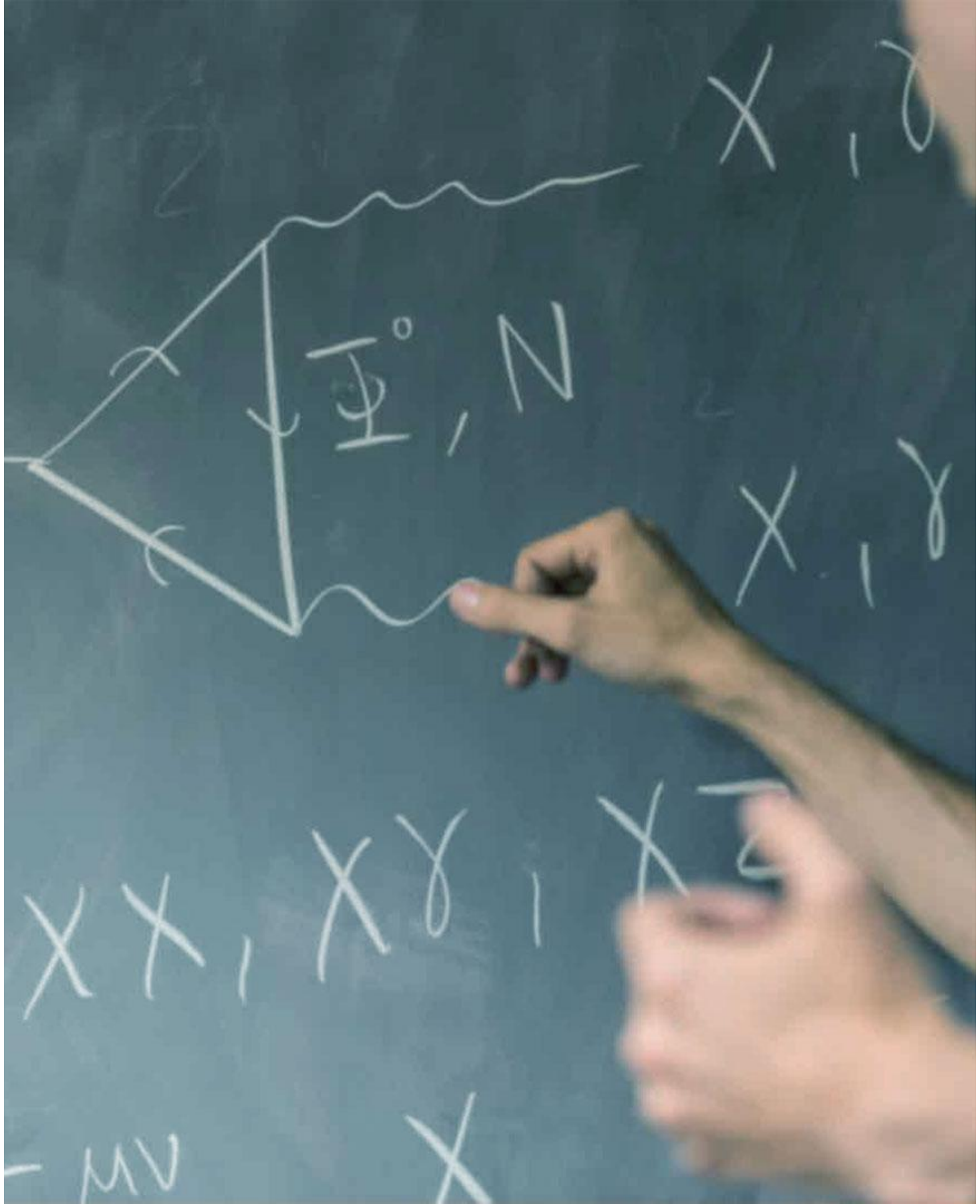
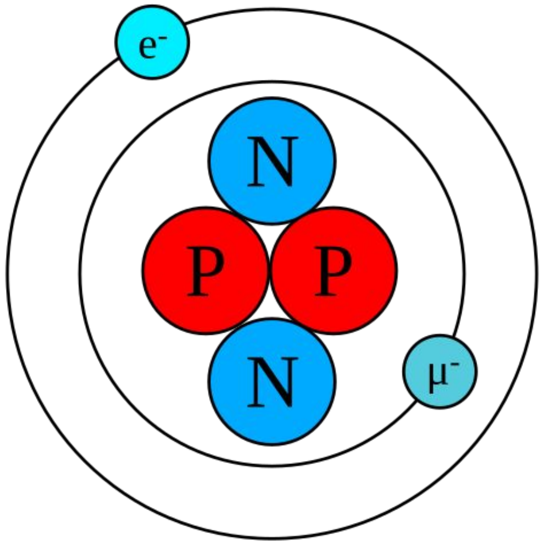


The study of muonic atoms & Nuclear charge radii



- Smaller Bohr radius in muonic atoms makes low-lying states more sensitive to the **nuclear charge radius.**

- Nuclear charge radii is used in precision tests, e.g the extraction of V_{ud} [1]

$$m_\mu \sim 200 m_e$$

$$r_\mu \sim \frac{1}{200} r_e$$

[1] M. Gorchtein. Phys. Rev. C, 113:L011301, 2026.

Measuring nuclear charge radii

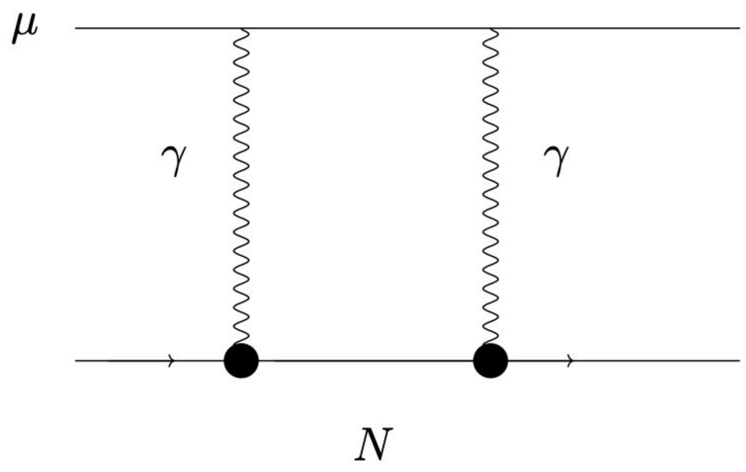
$$E = E_D + \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{FNS}} r_C^2 + \delta_{\mathrm{TPE}}$$

Energy difference measured

Nuclear charge radii

Two photon exchange correction (Not well known ⚠)

- Requires precise input from nuclear theory on δ_{TPE} [2,3]

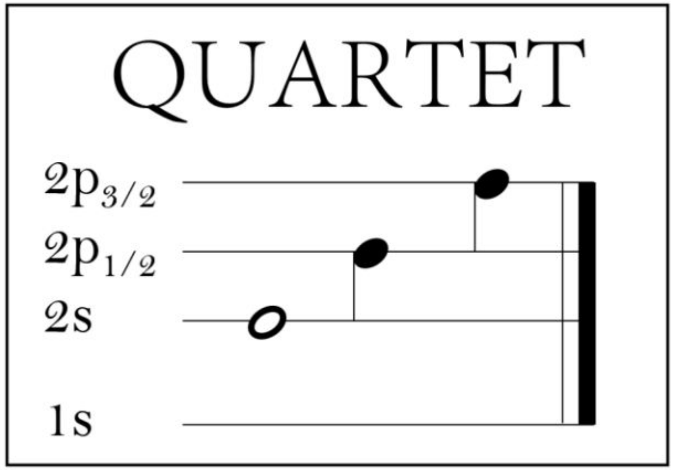


[2] O. Hernandez, C.Ji, S. Bacca, N. Barnea. Phys. Rev. C, 100(6):064315, 2019.

[3] Ben Ohayon et al. MDPI Physics, 6(1):206–215, 2024.

Measuring nuclear charge radii

Ben Ohayon et al. MDPI Physics, 6(1):206–215, 2024.
Daniel Unger et al. J. Low Temp. Phys., 216(1-2):344–351, 2024.



QUARTET: Determine radii via x-ray spectroscopy of light muonic atoms. Targets include:

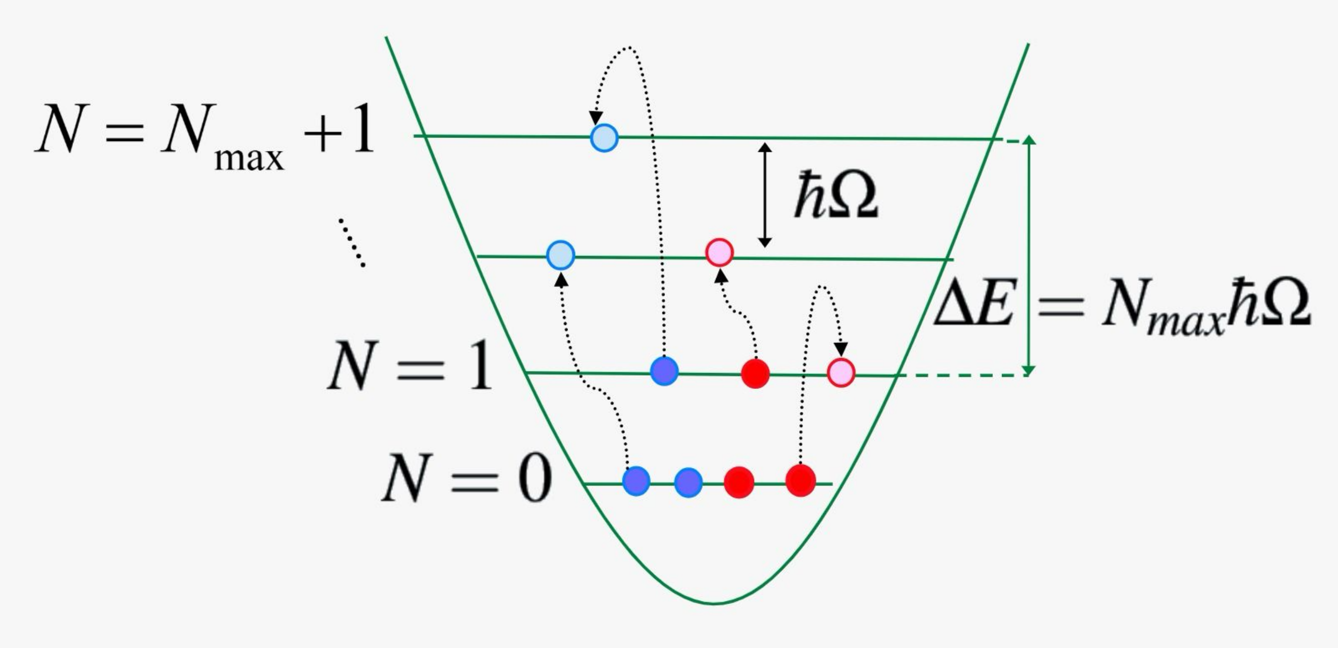
- C
- Li
- Be
- **B**

En … n … ction …)

- Re

The *ab-initio* NCSM

- Nucleons are active degrees of freedom
- NN & $3N$ interactions derived from χEFT
- Nuclear eigenstates expanded in HO basis, parameterized by $\mathbf{\Omega}$, truncated by N_{max}



$$\left|\Psi_A^{J\pi}\right\rangle = \sum_{N=0}^{N_{\max}} \sum_{\alpha} c_{N\alpha}^{J\pi} \left|\Phi_{N\alpha}^{J\pi}\right\rangle$$

Calculating δ_{TPE} [2]

- Elastic and inelastic terms

$$\delta_{\mathrm{TPE}} = \delta_{\mathrm{pol}} + \delta_{\mathrm{el}}$$

- Multipole expansion of the electromagnetic current:

$$\delta_{\mathrm{pol}} = \underbrace{\Delta_{\mathrm{L}}}_{\text{Longitudinal}} + \underbrace{\Delta_{\mathrm{T,E}} + \Delta_{\mathrm{T,M}}}_{\text{Transverse}}$$

Longitudinal

Transverse

[2] O. Hernandez, C.Ji, S. Bacca, N. Barnea. Phys. Rev. C, 100(6):064315, 2019.

Calculating δ_{TPE} [2]

- Generally, each Δ_i is given by

$$\Delta = -8(Z\alpha)^2|\phi_\mu(0)|^2 \int \mathrm{d}q \int \mathrm{d}\omega K(\omega) \sum_i \left| \langle \Psi_i | \hat{O}_{\mathcal{J}}(q) | \Psi_0 \rangle \right|^2 \delta(\omega_i - \omega)$$

Computationally infeasible ⚠

Lanczos method [4]

[2] O. Hernandez, C.Ji, S. Bacca, N. Barnea. Phys. Rev. C, 100(6):064315, 2019.

[4] N. Dinur, C. Ji, S. Bacca, N. Barnea. Phys. Rev. C, 89(6):064317, 2014.

Calculating δ_{TPE} [2]

- Lanczos Method[4]: Put H in tridiagonal form via $\hat{\mathrm{H}}$ and a pivot

$$|\phi_0\rangle = \hat{O}_{\mathcal{J}} |\Psi_0\rangle \,/\sqrt{\langle\Psi_0|\hat{O}_{\mathcal{J}}\hat{O}^{\dagger}_{\mathcal{J}}|\Psi_0\rangle}$$

- Diagonalize $\hat{\mathrm{H}}$:

$$\hat{H} = QDQ^{\dagger}$$

- Then:

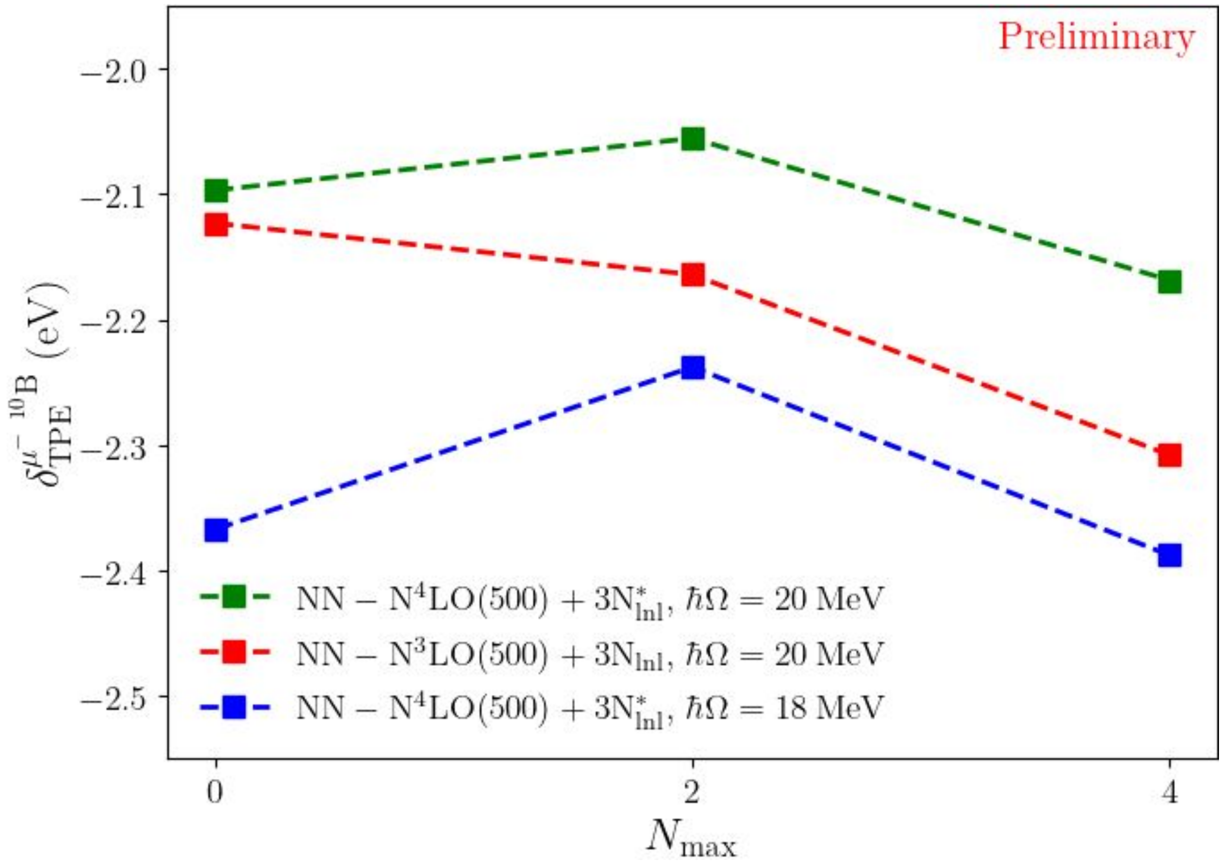
$$\int \mathrm{d}\omega \sum_i \left| \langle\Psi_i|\hat{O}_{\mathcal{J}}(q)|\Psi_0\rangle \right|^2 \delta(\omega_i - \omega) = \langle\Psi_0|\hat{O}_{\mathcal{J}} O^{\dagger}_{\mathcal{J}}|\Psi_0\rangle \sum_{\alpha} |Q_{\alpha 0}|^2$$

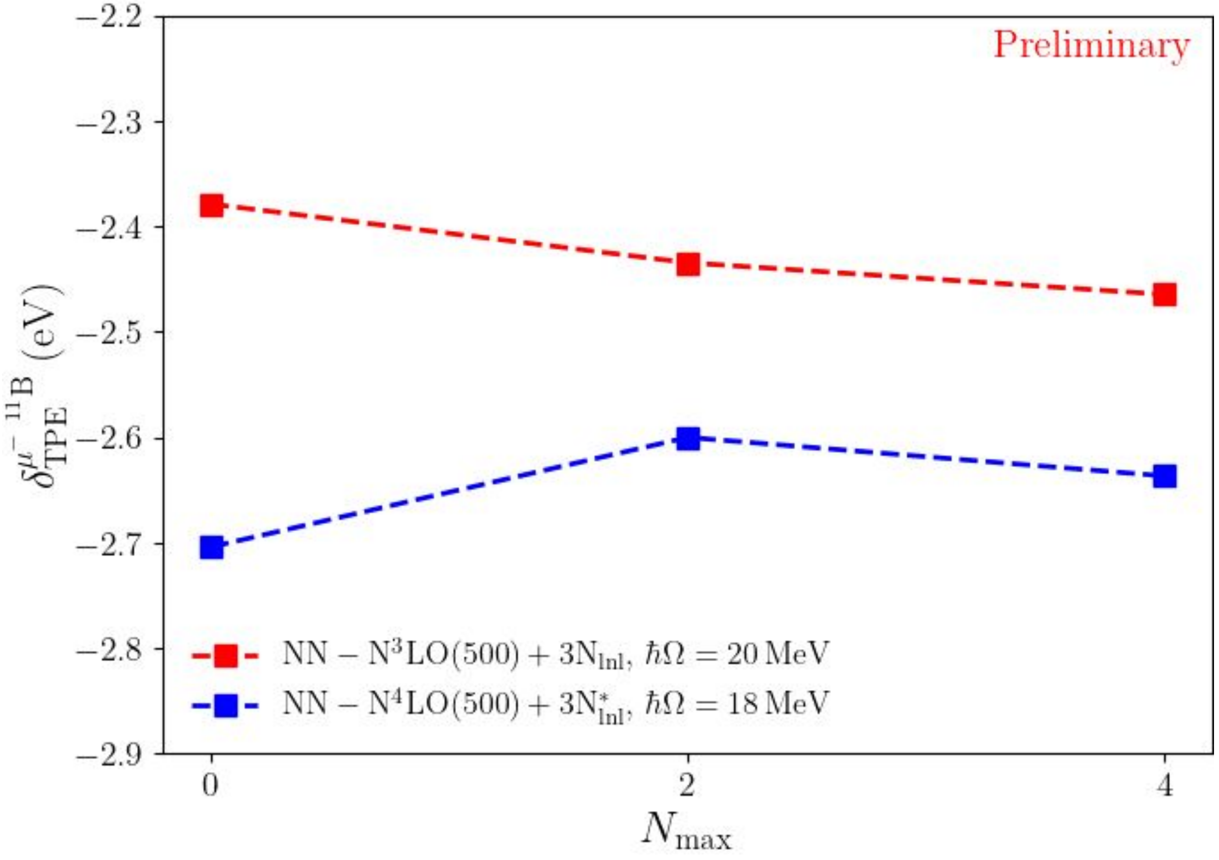
No explicit dependence on intermediate states

[2] O. Hernandez, C.Ji, S. Bacca, N. Barnea. Phys. Rev. C, 100(6):064315, 2019.

[4] N. Dinur, C. Ji, S. Bacca, N. Barnea. Phys. Rev. C, 89(6):064317, 2014.

δ_{TPE} for ^{10}B and ^{11}B





Outlook

- Calculate at higher N_{max} and different Ω to check convergence
- Verify contributions from angular momentum projections of intermediate states
- Perform calculations on Be

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Thank you
Merci

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