No-Core Shell Model for Strength Functions in Even-A Helium

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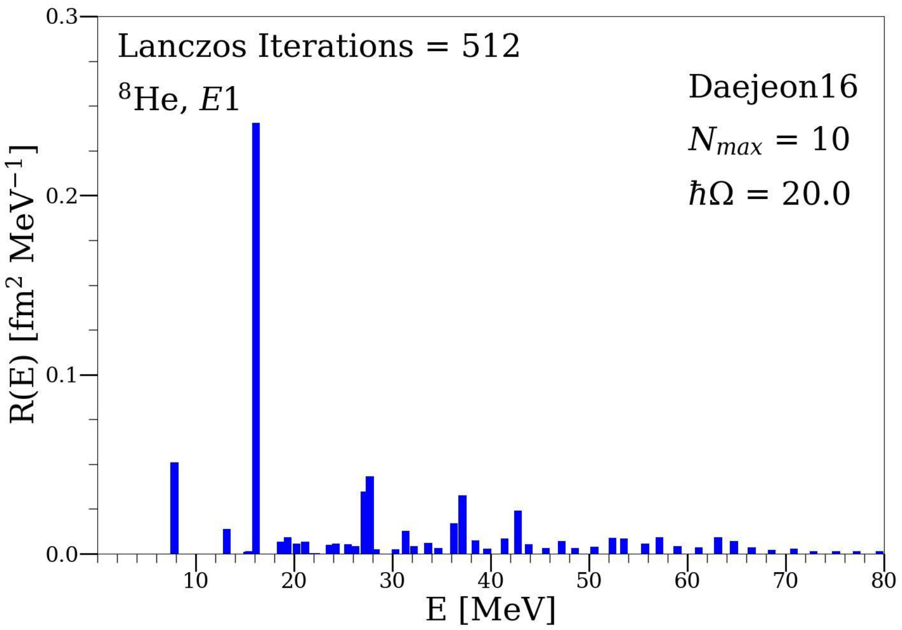
Strength Functions

- Strength functions characterize transition probability as a function of excitation energy

$$R(E) = \sum_f |\langle \Psi_f | \hat{O} | \Psi_0 \rangle|^2 \delta(E_f - E_0 - E)$$

- Requires computation of transition matrix element between the initial and each possible final state



Lanczos Method for Strength Functions

- Lanczos algorithm can be used to avoid needing to calculate each individual overlap

- Proposed by Whitehead (1980) and widely used in calculation of strength functions

- With the no-core shell model, has been used to calculate electromagnetic strength functions for even oxygen nuclei (C. Stumpf et. al, 2017)

$$\hat{H}\left|v_i\right\rangle = \beta_{i+1}\left|v_{i+1}\right\rangle + \alpha_i\left|v_i\right\rangle + \beta_i\left|v_{i-1}\right\rangle$$

After n iterations, writing $\hat{H}$ in the $\{|v_i\rangle\}$ basis:

$$\hat{H}_{n\times n} = \begin{pmatrix} \alpha_1 & \beta_2 & 0 & \dots & 0 \\ \beta_2 & \alpha_2 & \beta_3 & \ddots & \vdots \\ 0 & \beta_3 & \alpha_3 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \beta_n \\ 0 & \dots & 0 & \beta_n & \alpha_n \end{pmatrix}$$

$$\hat{H}_{n\times n} = \mathrm{proj}_{\mathcal{K}_n}(\hat{H}) \quad (\mathcal{K}_n = \mathrm{span}(\left|v_1\right\rangle, \hat{H}\left|v_1\right\rangle, \hat{H}^2\left|v_1\right\rangle, \dots \hat{H}^{n-1}\left|v_1\right\rangle))$$

Lanczos Method for Strength Functions

1. Diagonalize the Hamiltonian and get initial state:

$$\hat{H}\,|i\rangle = E_i\,|i\rangle$$

2. Apply transition operator to initial state to get pivot:

$$|v_1\rangle = \hat{O}\,|i\rangle$$

3. Decompose pivot as energy eigenstates to get overlaps:

$$|f\rangle = \sum_i c_i(f)\,|v_i\rangle$$

$$|\,\langle f|\hat{O}|i\rangle\,|^2 = |c_1(f)|^2$$

4. Get strength function from overlaps:

$$R(E) = \sum_f |\,\langle \Psi_f|\hat{O}|\Psi_0\rangle\,|^2 \delta(E_f - E_0 - E)$$

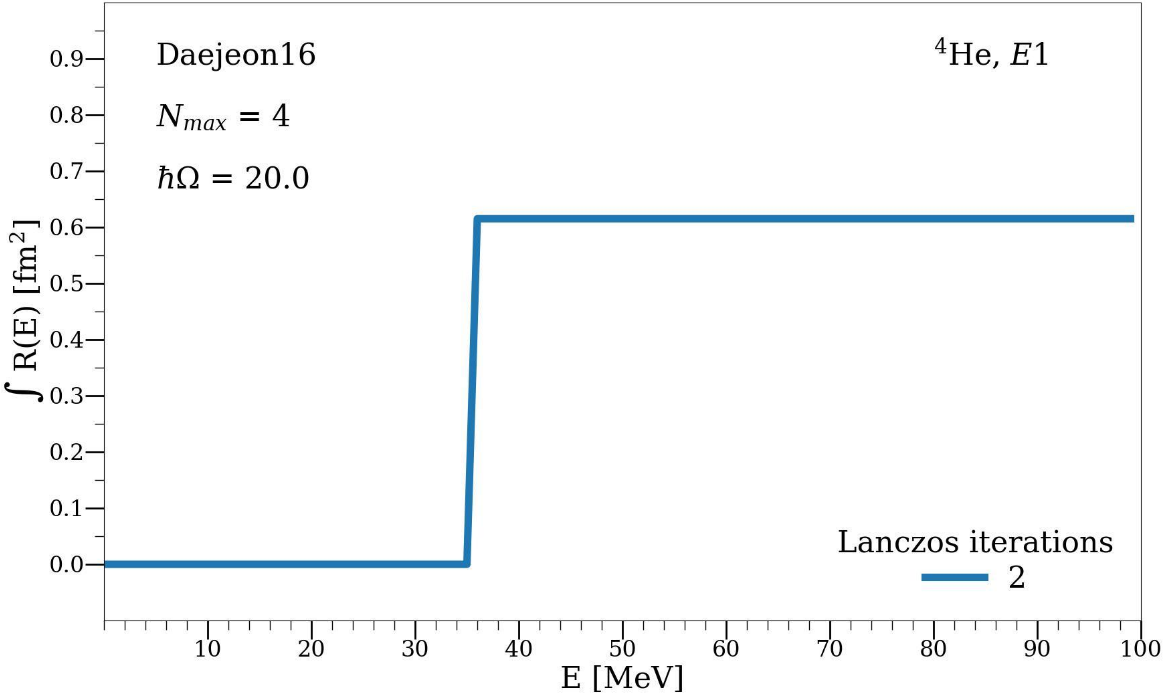
Direct Method Comparison for ^{4}He $E1$ Strength Function

- $E1$ strength functions for ^{4}He calculated explicitly in no-core shell model using Daejeon16 interaction (Yin et al., 2024)

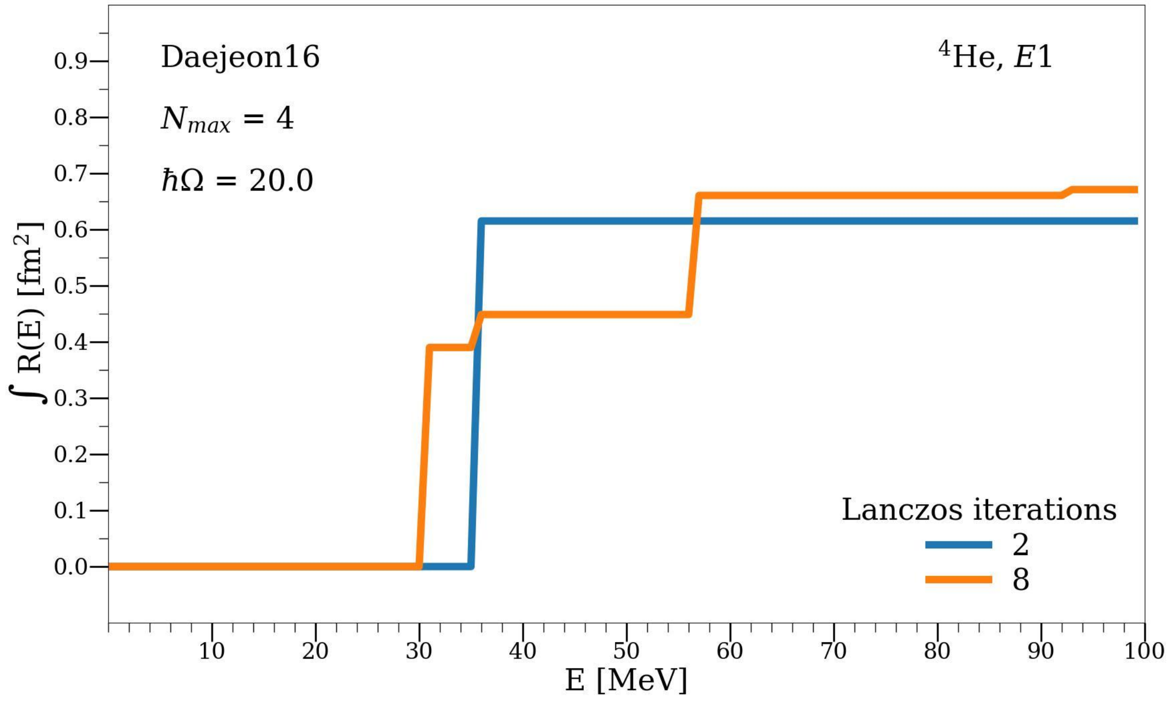
- Comparison shown with Lanczos method using same interaction and truncation parameters

- Plot of integrated strength functions shown

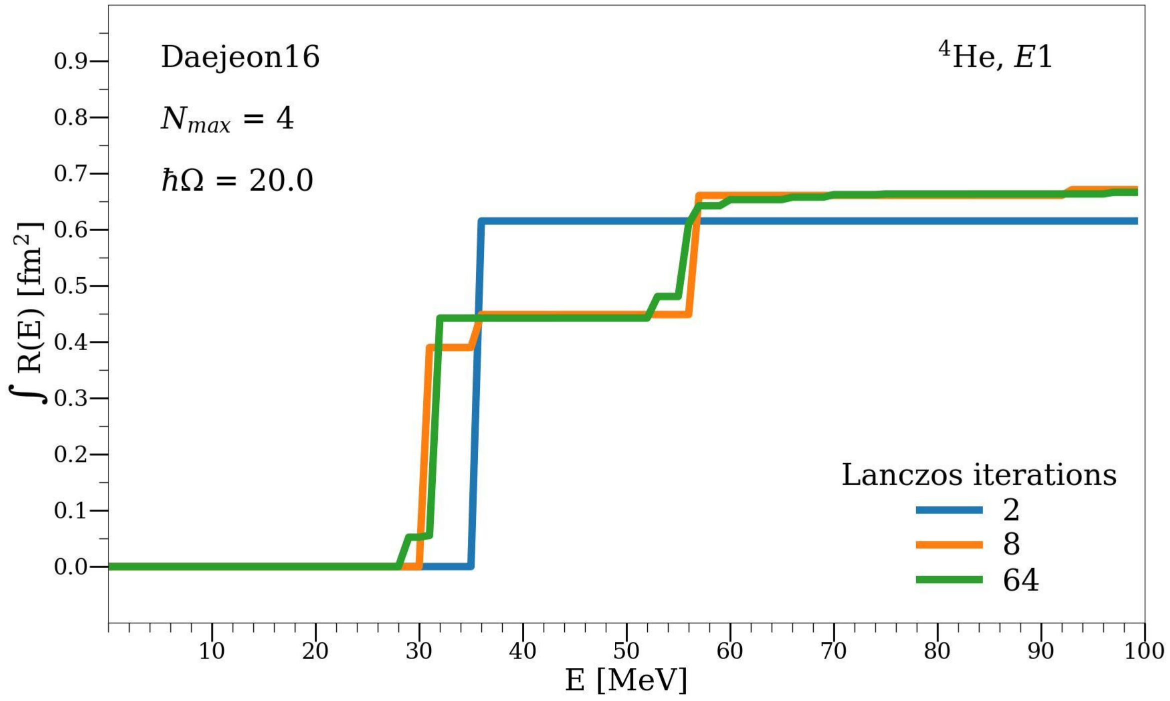
Direct Method Comparison for ^{4}He $E1$ Strength Function



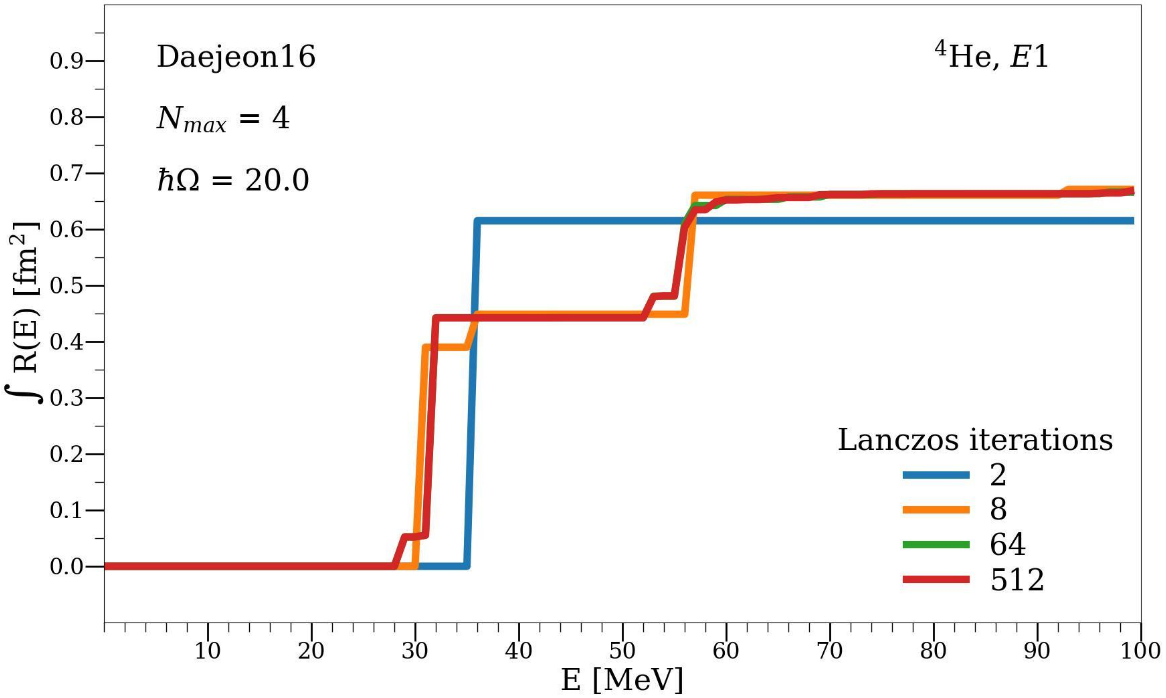
Direct Method Comparison for ^{4}He $E1$ Strength Function



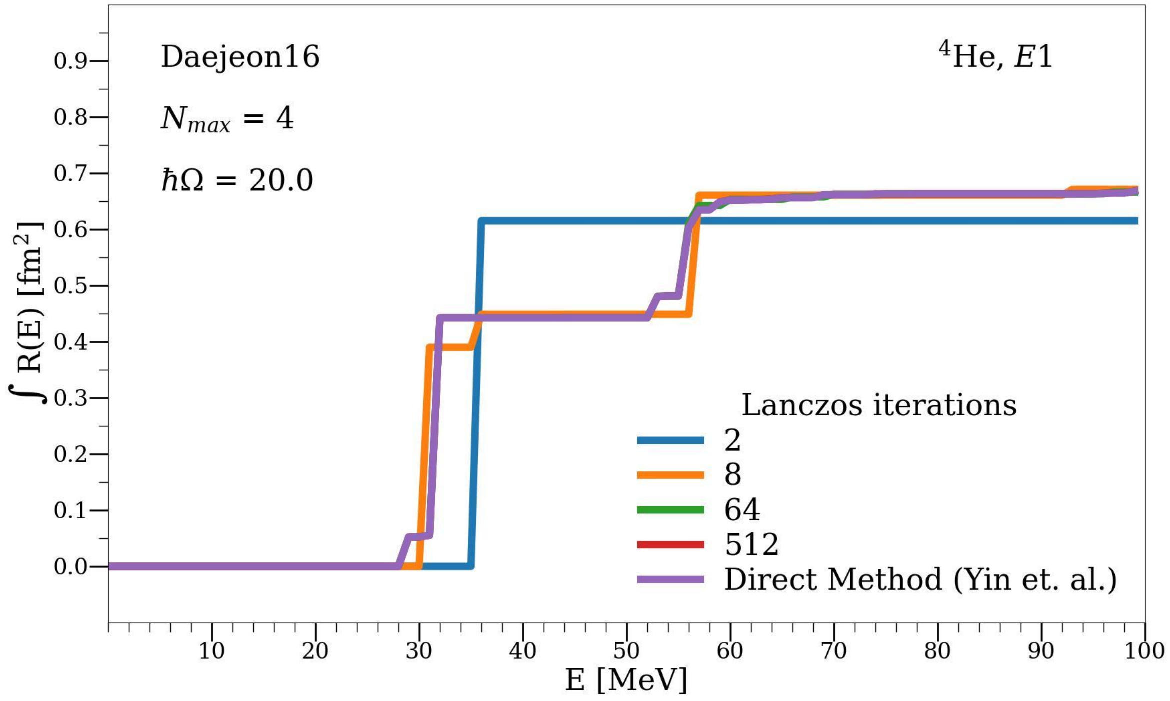
Direct Method Comparison for ^{4}He $E1$ Strength Function



Direct Method Comparison for ^{4}He $E1$ Strength Function



Direct Method Comparison for ^{4}He $E1$ Strength Function

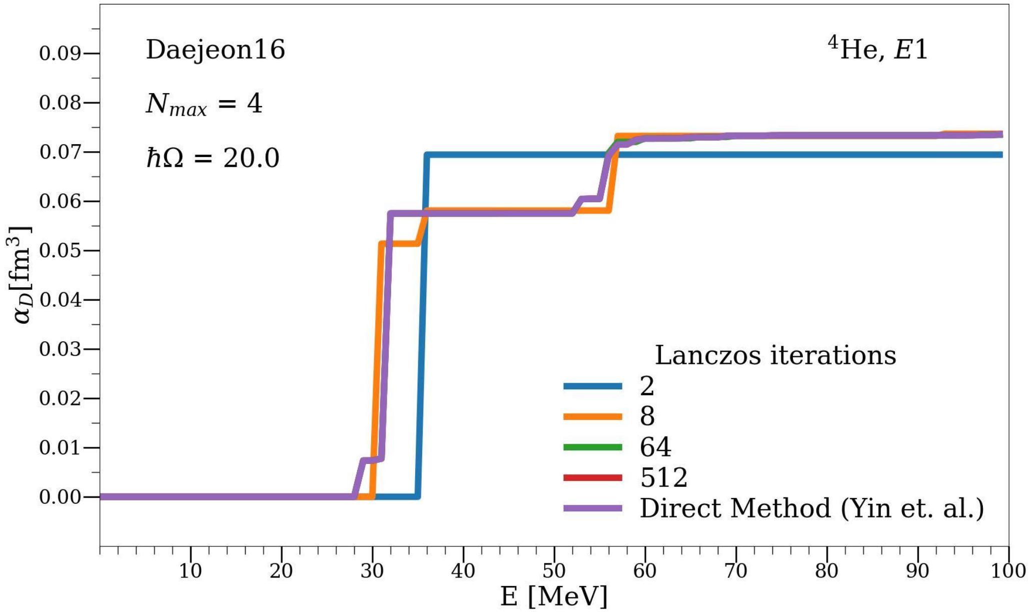


^{4}He Dipole Polarizability

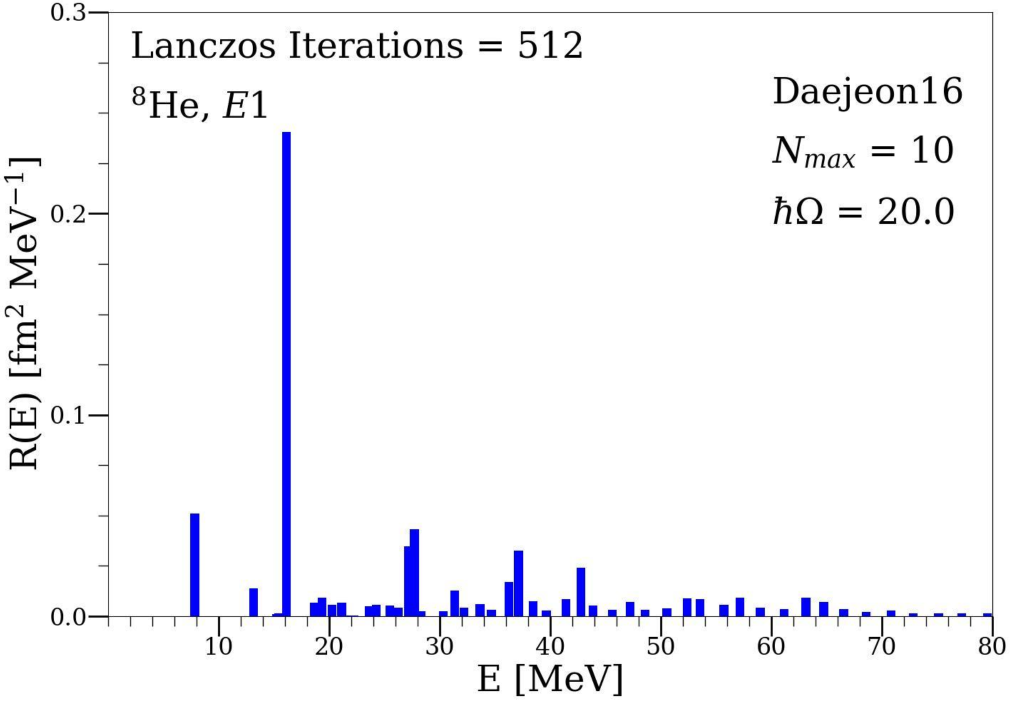
- Often useful to look at the moments of strength functions

$$m_n = \int_{E_0}^{\infty} R(E)(E - E_0)^n \, \mathrm{d}E$$

$$\alpha_D = \frac{8\pi}{9} m_{-1}$$



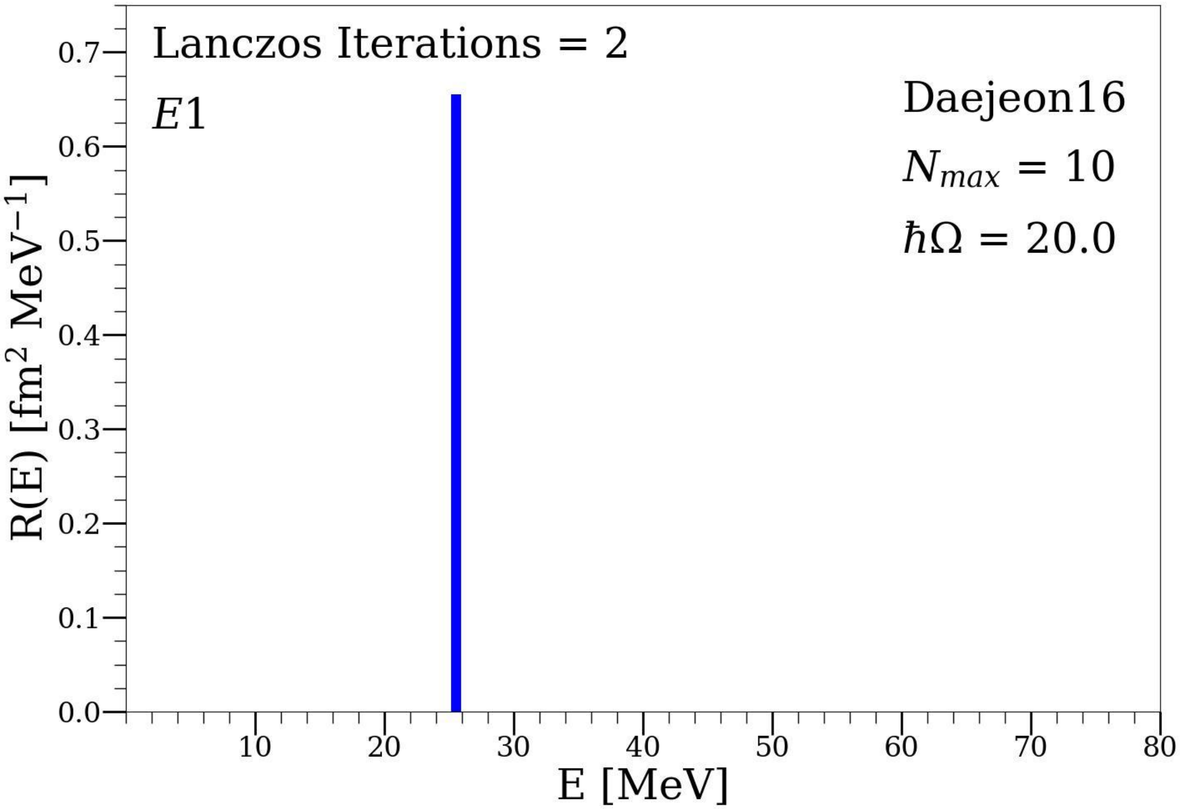
Outlook: Electric Dipole Strength Functions in ^{8}He



- Calculations were performed using Daejeon16 and NNLOopt interactions
- Compare with coupled-cluster results (F. Bonaiti et al., 2022)
- Look at energy-weighted sum rule (m_1) and dipole polarizability.

Extra Slides

Convergence of Strength Function with Lanczos Iterations



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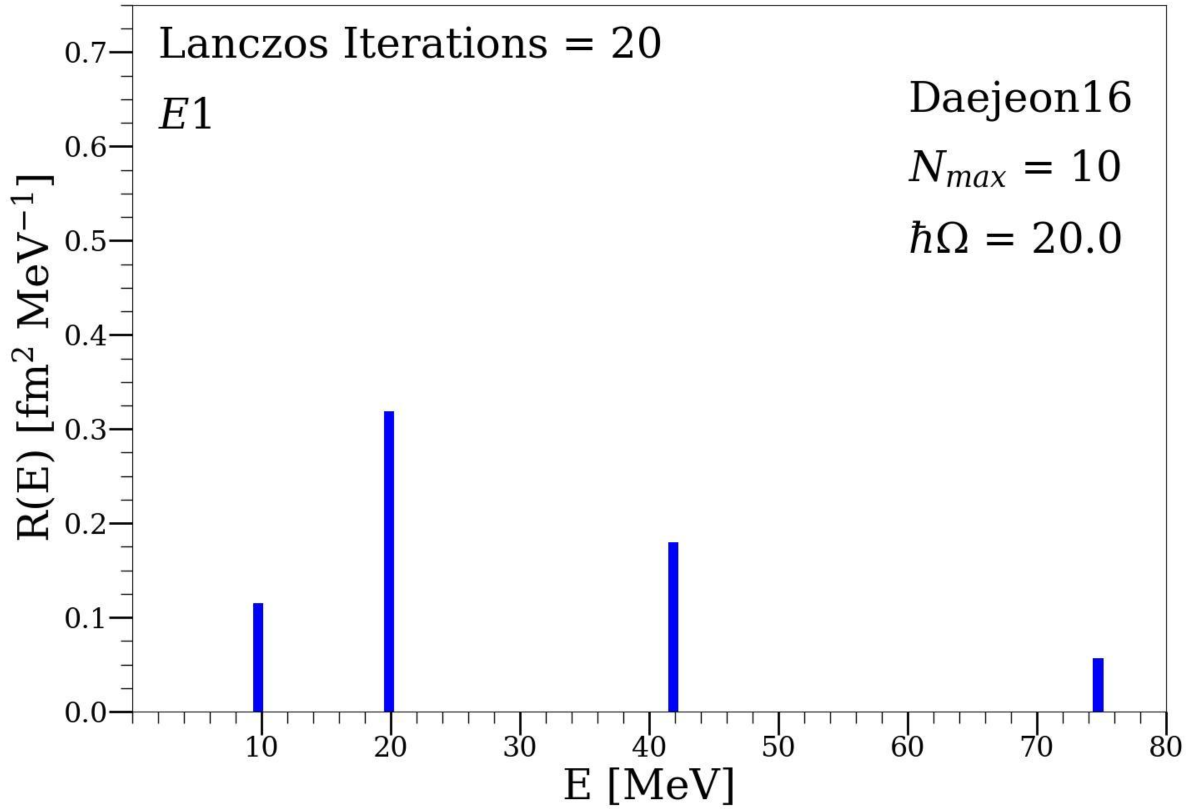

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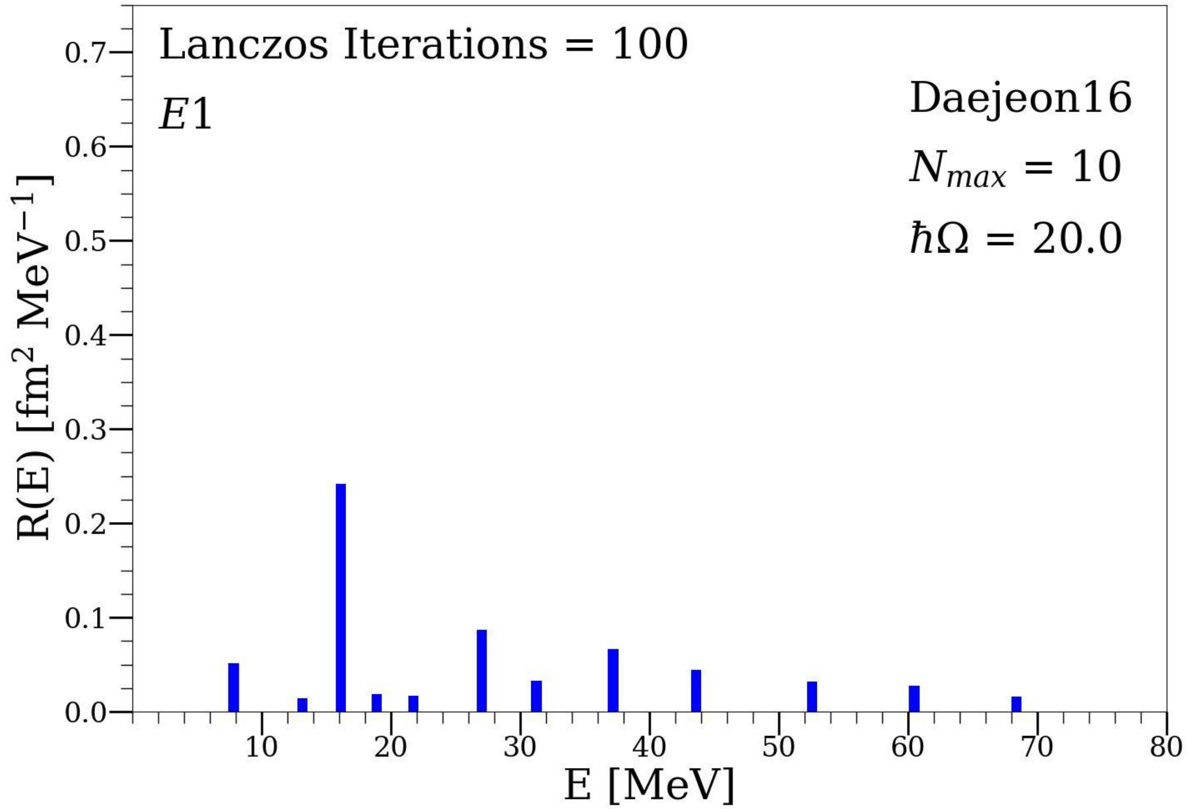

Convergence of Strength Function with Lanczos Iterations