



# CORRELATED MASS MODEL FOR CALCIUM ISOTOPES

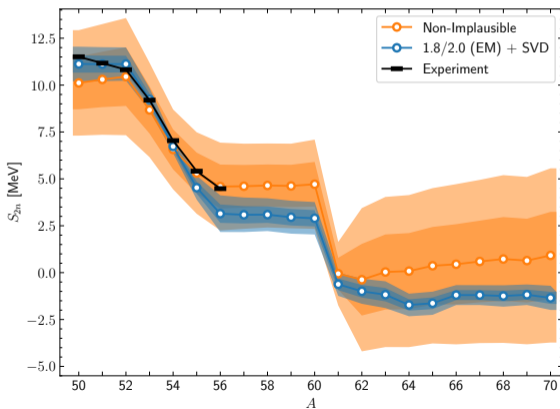
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# Introduction & Motivation

- Goal: Statistical Model for  $S_{2n}$  of Ca isotopes
- Distribution of interactions to understand uncertainties and correlations
- 34 Non-implausible (NI) [Hu et al. NP \(2022\)](#)
- 101 1.8/2.0 (EM) + SVD  
[Hebeler et al. PRC \(2011\)](#), [Plies et al. arXiv:2509.24671 \(2025\)](#)
- VS-IMSRG(2) and  $e_{\max} = 12$
- In our model:
  - Include additional uncertainties
  - Condition on experimental data





# Error model

- Assume **independent normally distributed errors**

$$\mathcal{N}(\mu, \Sigma) = \mathcal{N}(\mu_0, \Sigma_0) + \sum_i \mathcal{N}(\Delta\mu_i, \Sigma_i)$$

- Systematic shift (Fit for NI from Ca isotopes) [Stroberg et al. PRL \(2021\)](#)

$$\delta S_{2n}(A) = S_{2n}^{\text{th}}(A) - S_{2n}^{\text{exp}}(A) = aS_{2n}^{\text{th}}(A) + b$$

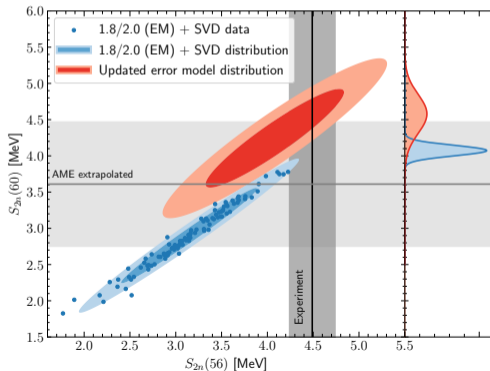
- Include **experimental data** by taking conditional probability

$$p(S_{2n} | S_{2n,\text{exp.}}) = \frac{p(S_{2n}, S_{2n,\text{exp.}})}{p(S_{2n,\text{exp.}})}$$



# Error model example

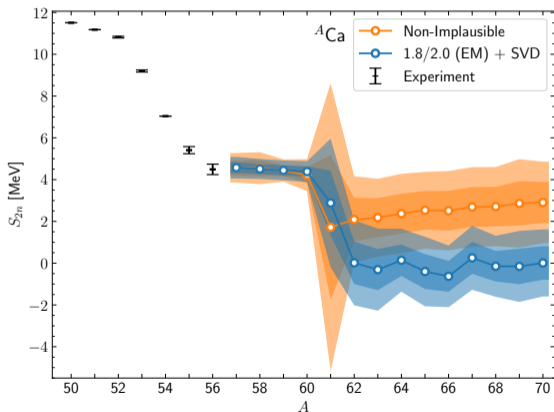
- Two-dimensional distribution for  $^{56}\text{Ca}$  and  $^{60}\text{Ca}$
- Error model gives **updated prediction** accounting for various sources of uncertainty
- Good agreement with AME 2020  
[Meng, et al. CPC \(2021\)](#)
- Additional uncertainties:
  - Model-space uncertainty ( $e_{\max}$ )
  - Many-body truncations
- Details see poster





# Results for Ca isotopic chain

- Experimental data for even  $S_{2n}$   $^{50-56}\text{Ca}$  → **predictions** for neutron-rich Ca isotopes
- Neutron dripline** between  $^{62}\text{Ca}$  and  $^{70}\text{Ca}$
- Largest uncertainty from:
  - interaction distribution (Non-implausible)
  - interaction systematics (1.8/2.0 (EM) + SVD)
- Limited by number of calculations per nucleus



# Summary & Outlook


- Statistical error model based on ab initio calculations
- Various sources of uncertainty
- Predictions for  $S_{2n}$  of neutron rich Ca isotopes
- Need efficient emulators for IMSRG calculations
  - Validate underlying assumptions (independence of uncertainties)
  - Application to heavy nuclei
- Propagation of uncertainties to astrophysical applications see, e.g., [Kuske et al. arXiv:2509.19131 \(2025\)](#)




## Correlated mass model for calcium isotopes

**Introduction & Motivation**


- Goal: Statistical model of  $S_{2n}$  for neutron rich Ca nuclei
- 34 Non-implausible (NI) interactions at  $\hbar\omega = 14$  MeV [3]
- 107 1.8/2.0 (IM) + 303 interactions at  $\hbar\omega = 12$  MeV [2,4]
- Starting point to IMSRG(2),  $\hbar\omega = 12$  MeV,  $\hbar\omega_{cut} = 24$  calculations
- reaction of valence space for  $0 \leq \nu \leq 10$
- reaction only valence space for  $0 \leq \nu \leq 7$
- 10-body uncertainty
- 303 only parametric uncertainty  $\rightarrow$  fully understood
- Build model that includes additional uncertainties:
  - Many-body expansion
  - Model space parameters
  - Systematic shifts
  - Condition model on data from experiment



**Statistical Model**

- Model sources of uncertainty as independent normal distributions
- Systematic shift using model form [5]
 
$$\delta S_{2n} = \sum_{i=1}^N \frac{\partial S_{2n}}{\partial \mu_i} \mu_i = \sum_{i=1}^N \frac{\partial S_{2n}}{\partial \mu_i} \mu_i + \sum_{i=1}^N \frac{\partial S_{2n}}{\partial \mu_i} \sigma_i$$
- Systematic uncertainty model on 8-body expansion with  $\mu_{8b} = \mu_{8b} - \delta \mu_{8b}$  [5]
- Arbitrary systematic shift for 16 Ca isotopes
- 1.8/2.0 (IM) + 303 interaction taken from [4]
- Condition model on experimental data to reduce prediction
 
$$\mu_{8b} = \mu_{8b} - \frac{\partial S_{2n}(\mu_{8b}) - S_{2n}^{exp}}{\partial \mu_{8b}}$$
- Include experimental uncertainty when conditioning
 

**Model space & Many-body uncertainties**

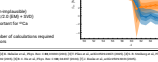
- Conditioning with respect to  $\mu_{8b}$  to mitigate model space uncertainty [5]
- Calculations using IMSRG(2) for many-body uncertainty [4]
- Additional many-body uncertainty from size of IMSRG(2) correction ( $\hbar\omega_{cut}$ )
- Subsampling down to 13 interaction to reduce computational cost
- For  $\nu = 0$  ensure mean field occupation of  $p_{1/2}$  orbit for stable extrapolation
- High-charge calculations of  $\delta S_{2n}$  orbit for 1.8/2.0 (IM) + 303
 

**$^{44}\text{Ca}$  calculations**

- For  $\nu = 0$  no filling of either  $p_{3/2}$  or  $p_{1/2}$  possible
- Investigate sensitivity by testing  $^{44}\text{Ca}$
- Both model space and many-body effects play a role
- Table shows location of experimental  $^{44}\text{Ca}$  ground state of  $^{44}\text{Ca}$ 

$^{44}\text{Ca}$ state	$\hbar\omega$	$\hbar\omega_{cut}$	IM	NI
16 1/2 <sup>+</sup> - $\mu_{8b} = 12$	12	24	0	0
16 1/2 <sup>+</sup> - $\mu_{8b} = 14$	14	28	0	0
16 1/2 <sup>+</sup> - $\mu_{8b} = 14$	14	4	0	0

**Results & Discussion**

- Final ab-initio based mass model for  $\nu = 0 - 10$  isotopes
- Result in correlated distribution for Ca isotopes  $\nu = 17 - 21$
- Conditioning on experimental data 2020 data
- Experimental data gives strong constraint up to  $^{44}\text{Ca}$
- Median splits between  $^{44}\text{Ca}$  and  $^{42}\text{Ca}$
- Largest uncertainty from interaction distribution (Non-implausible) interaction operators 2.0/2.0 (IM) + 303
- Many-body uncertainty important for  $^{42}\text{Ca}$
- change in valence space
- Application involving number of calculations required per nucleus  $\rightarrow$  need emulation


**Outlook**

- Investigate independence assumption to uncertainties
- Extension to more theory models requires efficient emulators
- Feasibility to propagate uncertainties to astrophysical applications [5]
- Extension to more observations
- Include additional sources of valence uncertainty
- $\rightarrow$   $\mu_{8b}$
- NI/2B approximation
- Continuum contributions