Competitive Fierz constraints with electron capture

DUMENIL Victor & HAYEN Leendert

PAINT2026 Workshop @ TRIUMF

February 26th 2026

LPC CAEN

cnrs NUCLÉAIRE & PARTICULES

Contents

CKM matrix

CKM matrix

Cabibbo-Kobayashi-Maskawa matrix relates weak and mass eigenstates

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_w = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_m$$

CKM matrix

Cabibbo-Kobayashi-Maskawa matrix relates weak and mass eigenstates

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_w = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_m$$

The unitarity requires that

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Nuclear β-decay, meson decay (π, K), $|\mathrm{V}_{\mathrm{ub}}|^2 \sim 10^{-5}$

CKM matrix

Cabibbo-Kobayashi-Maskawa matrix relates weak and mass eigenstates

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_w = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_m$$

The unitarity requires that

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Nuclear β-decay, meson decay (π, K), $|V_{ub}|^2 \sim 10^{-5}$

$$\Delta^{CKM} - 1 = -0.00166(69)$$

Δ^{CKM} : exotic physics (new particle or **exotic currents**)

Exotic currents

The **Effective Field Theory:** BSM physics at scale $\Lambda_{BSM} \gg LHC$ (~14 TeV)

Exotic currents

The **Effective Field Theory:** BSM physics at scale $\Lambda_{BSM} \gg LHC$ (~14 TeV)

$$\mathcal{L}_{eff} = -\frac{G_F \tilde{V}_{ud}}{\sqrt{2}} \Big\{ \bar{e}\gamma_\mu \nu_L \cdot \bar{u}\gamma^\mu [C_V - (C_A - 2\epsilon_R)\gamma^5] d + \epsilon_S \bar{e}\nu_L \cdot \bar{u}d$$

$$- \epsilon_P \bar{e}\nu_L \cdot \bar{u}\gamma^5 d + \epsilon_T \bar{e}\sigma_{\mu\nu}\nu_L \cdot \bar{u}\sigma^{\mu\nu}(1-\gamma^5)d \Big\}$$

Exotic currents

The **Effective Field Theory:** BSM physics at scale $\Lambda_{BSM} \gg LHC$ (~14 TeV)

$$\mathcal{L}_{eff} = -\frac{G_F \tilde{V}_{ud}}{\sqrt{2}} \Big\{ \bar{e}\gamma_\mu \nu_L \cdot \bar{u}\gamma^\mu [C_V - (C_A - 2\epsilon_R)\gamma^5] d + \epsilon_S \bar{e}\nu_L \cdot \bar{u}d$$

$$- \epsilon_P \bar{e}\nu_L \cdot \bar{u}\gamma^5 d + \epsilon_T \bar{e}\sigma_{\mu\nu}\nu_L \cdot \bar{u}\sigma^{\mu\nu}(1-\gamma^5)d \Big\}$$

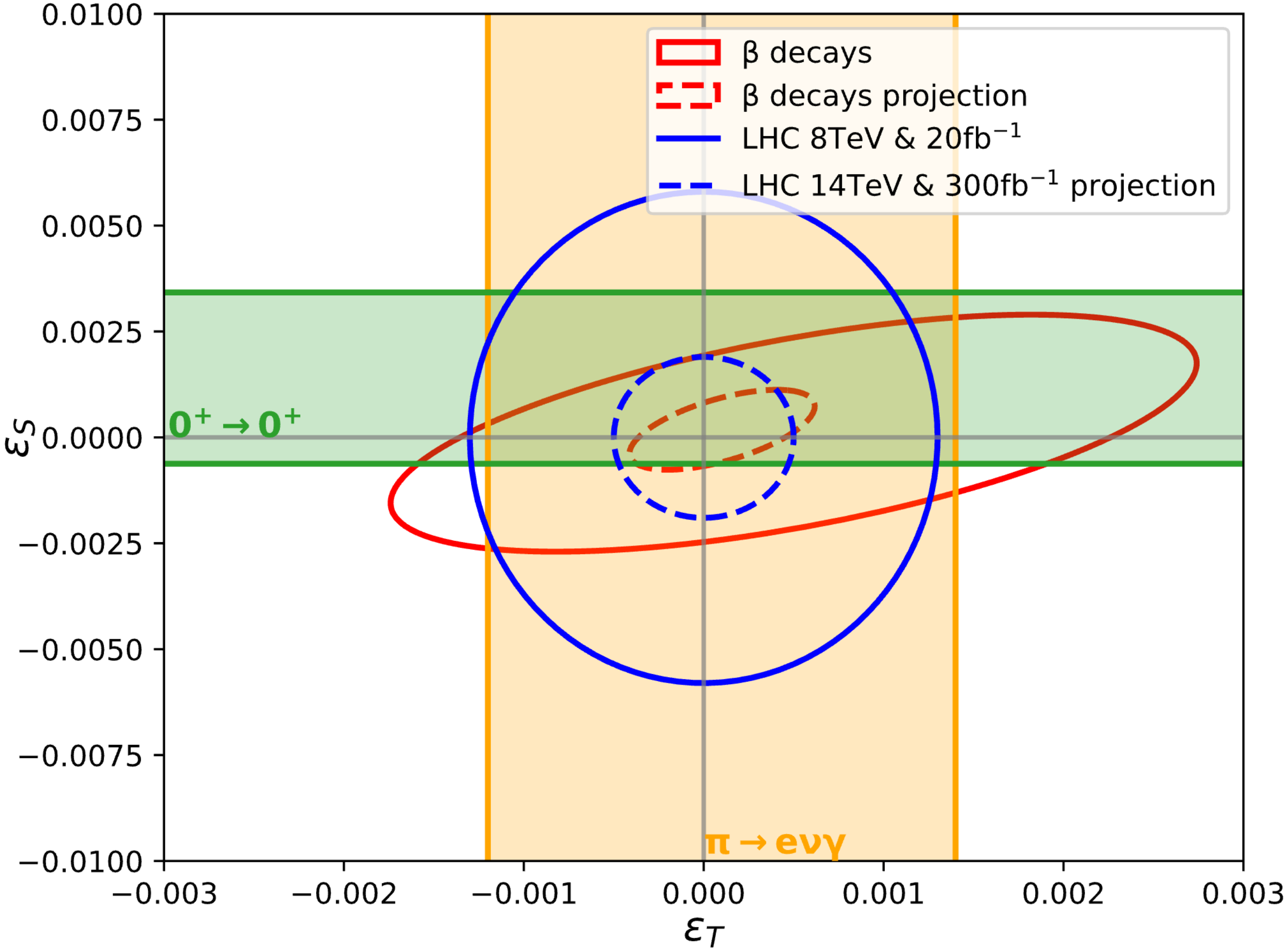
V – A structure in Sandard Model

$\epsilon_i \propto (M_W/\Lambda_{BSM})^2 \rightarrow \epsilon_i \leq 10^{-4}$

$\rightarrow \Lambda_{BSM} \geq$ 15 TeV (natural couplings)

Exotic current constraints

→ Nuclear β-decay is **competitive** with the LHC in the search for exotic currents



A.Falkowski et al, 2021
M.Gonzalez-Alonso et al, 2019

Fierz interference term

Fierz interference term

- bSTILED : Precision measurement of β-spectrum shape

$$\frac{dN}{dE} = \frac{dN_{SM}}{dE} \times \left(1 \pm b_F \frac{m_e}{E}\right)$$

Fierz interference term

- bSTILED : Precision measurement of β-spectrum shape

$$\frac{dN}{dE} = \frac{dN_{SM}}{dE} \times \left(1 \pm b_F \frac{m_e}{E}\right)$$

→ Interference between the Standard Model & Beyond Standard Model

$$b_F = \pm \frac{2\gamma}{1 + |\rho|^2} \mathrm{Re}\left(\frac{\epsilon_S g_S}{g_V} + |\rho|^2 \frac{8\epsilon_T g_T}{-2g_A}\right)$$

Fierz interference term

- bSTILED : Precision measurement of β-spectrum shape

$$\frac{dN}{dE} = \frac{dN_{SM}}{dE} \times \left(1 \pm b_F \frac{m_e}{E}\right)$$

→ Interference between the Standard Model & Beyond Standard Model

$$b_F = \pm \frac{2\gamma}{1+|\rho|^2} \mathrm{Re}\left(\frac{\epsilon_S g_S}{g_V} + |\rho|^2 \frac{8\epsilon_T g_T}{-2g_A} \right)$$

→ Linear dependence on exotic scalar & tensor currents

Fierz interference term

- bSTILED : Precision measurement of β-spectrum shape

$$\frac{dN}{dE} = \frac{dN_{SM}}{dE} \times \left(1 \pm b_F \frac{m_e}{E}\right)$$

→ Interference between the Standard Model & Beyond Standard Model

$$b_F = \pm \frac{2\gamma}{1 + |\rho|^2} \mathrm{Re}\left(\frac{\epsilon_S g_S}{g_V} + |\rho|^2 \frac{8\epsilon_T g_T}{-2g_A}\right)$$

→ **0 in SM**

→ Linear dependence on exotic scalar & tensor currents

Fierz interference term

- bSTILED : Precision measurement of β-spectrum shape

$$\frac{dN}{dE} = \frac{dN_{SM}}{dE} \times \left(1 \pm b_F \frac{m_e}{E}\right)$$

→ Interference between the Standard Model & Beyond Standard Model

$$b_F = \pm \frac{2\gamma}{1 + |\rho|^2} \mathrm{Re}\left(\frac{\epsilon_S g_S}{g_V} + |\rho|^2 \frac{8 \epsilon_T g_T}{-2 g_A} \right)$$

→ **0 in SM**

→ Linear dependence on exotic scalar & tensor currents

- **SALER/ASGARD** : Measurement of **EC/β+ ratio**

How to obtain $\mathbf{b_F}$?

How to obtain b_F ?

EC/β+ : **opposite sign** b_F & probes **same nuclear matrix elements**

$$\frac{\lambda_{EC}}{\lambda_{\beta^+}} = \frac{f_{EC}}{f_{\beta^+}}(1+\delta_{NS}) \cdot \frac{1+b_F\frac{m_e}{E_{EC}}}{1-b_F\left\langle\frac{m_e}{E_e}\right\rangle}$$

How to obtain b_F ?

EC/β+ : **opposite sign** b_F & probes **same nuclear matrix elements**

$$\frac{\lambda_{EC}}{\lambda_{\beta^+}} = \frac{f_{EC}}{f_{\beta^+}}(1 + \delta_{NS}) \cdot \frac{1 + b_F \frac{m_e}{E_{EC}}}{1 - b_F \left\langle \frac{m_e}{E_e} \right\rangle}$$

Nuclear Structure effects

How to obtain b_F ?

EC/β+ : **opposite sign** b_F & probes **same nuclear matrix elements**

$$\frac{\lambda_{EC}}{\lambda_{\beta^+}} = \frac{f_{EC}}{f_{\beta^+}}(1 + \delta_{NS}) \cdot \frac{1 + b_F \frac{m_e}{E_{EC}}}{1 - b_F \left\langle \frac{m_e}{E_e} \right\rangle}$$

Nuclear Structure effects

→ **extremely sensitive** to new physics

We can probe new physic at tens of TeV in **new way**

How to obtain b_F ?

EC/β+ : **opposite sign** b_F & probes **same nuclear matrix elements**

$$\frac{\lambda_{EC}}{\lambda_{\beta^+}} = \frac{f_{EC}}{f_{\beta^+}}(1+\delta_{NS}) \cdot \frac{1+b_F \frac{m_e}{E_{EC}}}{1-b_F \left\langle \frac{m_e}{E_e} \right\rangle}$$

Nuclear Structure effects

→ **extremely sensitive** to new physics

We can probe new physic at tens of TeV in **new way**

How to access **nuclear structures** (and δ_{NS}) ?

*) L.Hayen et al., Rev. Mod. Phys. 90, 015008 (2018)
**) W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

Branching ratio calculation

$$\frac{\lambda_{EC}}{\lambda_{\beta^+}} = \frac{f_{EC}}{f_{\beta^+}}(1+\delta_{NS}) \cdot \frac{1+b_F \frac{m_e}{E_{EC}}}{1-b_F \left\langle \frac{m_e}{E_e} \right\rangle}$$

$$\lambda_{\beta^+} \propto \int_{m_e}^{E_0} dE\, pE(E_0-E)^2 F(Z,E) C(Z,E) K(Z,E)$$

Shape factor
Phase space
Fermi function
Other corrections (from thecobs/BSG *)

$$\lambda_{EC} = \sum_x n_x C_x f_x = n_x C_x \times \left(\frac{\pi}{2} q_x^2 \beta_x^2 B_x\right)$$

Relative occupation number
Shape factor
Neutrino momentum
Amplitude of wave function **
Overlap and exchange corrections **

Shape factor (nuclear structures information) → H.Behrens & W.Bühring formalism

H.Behrens & W.Bühring formalism

H.Behrens & W.Bühring formalism

Nuclear β-decay involves complex **many-body interactions**

H.Behrens & W.Bühring formalism

Nuclear β-decay involves complex **many-body interactions**

BUT angular momentum conservation → multipole decomposition

The many-body physics are encoded into the form factors

H.Behrens & W.Bühring formalism

Nuclear β-decay involves complex **many-body interactions**

BUT angular momentum conservation → multipole decomposition

The many-body physics are encoded into the form factors

$$\left\langle f \middle| H_0^{\text{hadr}} L_{\text{lep}}^0 \middle| i \right\rangle \propto \sum_{L,M} j_L(qR)\, Y_M^L(\hat{q})\, F_L(q^2)$$

H.Behrens & W.Bühring formalism

Nuclear β-decay involves complex **many-body interactions**

BUT angular momentum conservation → multipole decomposition

The many-body physics are encoded into the form factors

$$\left\langle f \middle| H_0^{\text{hadr}} L_{\text{lep}}^0 \middle| i \right\rangle \propto \sum_{L,M} j_L(qR)\, Y_M^L(\hat{q})\, F_L(q^2)$$

→ « old school » model independent with $\mathrm{q} = \mathrm{p_f} - \mathrm{p_i}$
→ Separates the Lepton & Hadronic calculation
→ Processes **all types of transitions**

Calculation of b_F

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

2 datasets : the most *recent* (2019)[1] and *old* one (~1970)[2]

$$b_F = \frac{Br^{\text{exp}} / Br^{\text{theo}}(1 + \delta_{NS}) - 1}{\frac{m_e}{E_{\text{EC}}} + \left\langle \frac{m_e}{E_e} \right\rangle}$$

Calculation of b_F

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

2 datasets : the most *recent* (2019)[1] and *old* one (~1970)[2]

$$b_F = \frac{Br^{\text{exp}} / Br^{\text{theo}}(1 + \delta_{NS}) - 1}{\frac{m_e}{E_{\text{EC}}} + \left\langle \frac{m_e}{E_e} \right\rangle}$$

We break down δ_{NS} into <u>two parts</u>

$$\delta_{NS} = \delta_{NS}^{exp} + \delta_{NS}^{theo}$$

Calculation of b_F

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

2 datasets : the most *recent* (2019)[1] and *old* one (~1970)[2]

$$b_F = \frac{Br^{\mathrm{exp}}/Br^{\mathrm{theo}}(1+\delta_{NS})-1}{\frac{m_e}{E_{\mathrm{EC}}}+\left\langle \frac{m_e}{E_e}\right\rangle}$$

We break down δ_{NS} into two parts

$$\delta_{NS} = \delta_{NS}^{exp} + \delta_{NS}^{theo}$$

$\delta_{NS}^{\mathrm{exp}}$ is estimated with the **experimental values** of predominant form factors

$$^{V}F^{0}_{000}(M_F),{}^{A}F^{0}_{101}(M_{GT}),{}^{V}F^{0}_{111}(\mathrm{WM})$$

Calculation of b_F

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

2 datasets : the most *recent* (2019)[1] and *old* one (~1970)[2]

$$b_F = \frac{Br^{\mathrm{exp}}/Br^{\mathrm{theo}}(1+\delta_{NS})-1}{\frac{m_e}{E_{\mathrm{EC}}}+\left\langle \frac{m_e}{E_e}\right\rangle}$$

We break down δ_{NS} into two parts

$$\delta_{NS} = \delta_{NS}^{exp} + \delta_{NS}^{theo}$$

$\delta_{\mathrm{NS}}^{\mathrm{exp}}$ is estimated with the **experimental values** of predominant form factors

$${}^{V}F_{000}^{0}(M_F), {}^{A}F_{101}^{0}(M_{GT}), {}^{V}F_{111}^{0}(\mathrm{WM})$$

$\delta_{\mathrm{NS}}^{\mathrm{theo}}$ is calculated numerically with **shell model input** (OBTDs)

Calculation of b_F

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

2 datasets : the most *recent* (2019)[1] and *old* one (~1970)[2]

$$b_F = \frac{Br^{\text{exp}}/Br^{\text{theo}}(1+\delta_{NS}) - 1}{\frac{m_e}{E_{EC}} + \left\langle \frac{m_e}{E_e} \right\rangle}$$

We break down δ_{NS} into <u>two parts</u>

$$\delta_{NS} = \delta_{NS}^{exp} + \delta_{NS}^{theo}$$

δ_{NS}^{exp} is estimated with the **experimental values** of predominant form factors

$${}^{V}F^{0}_{000}(M_F), {}^{A}F^{0}_{101}(M_{GT}), {}^{V}F^{0}_{111}(\text{WM})$$

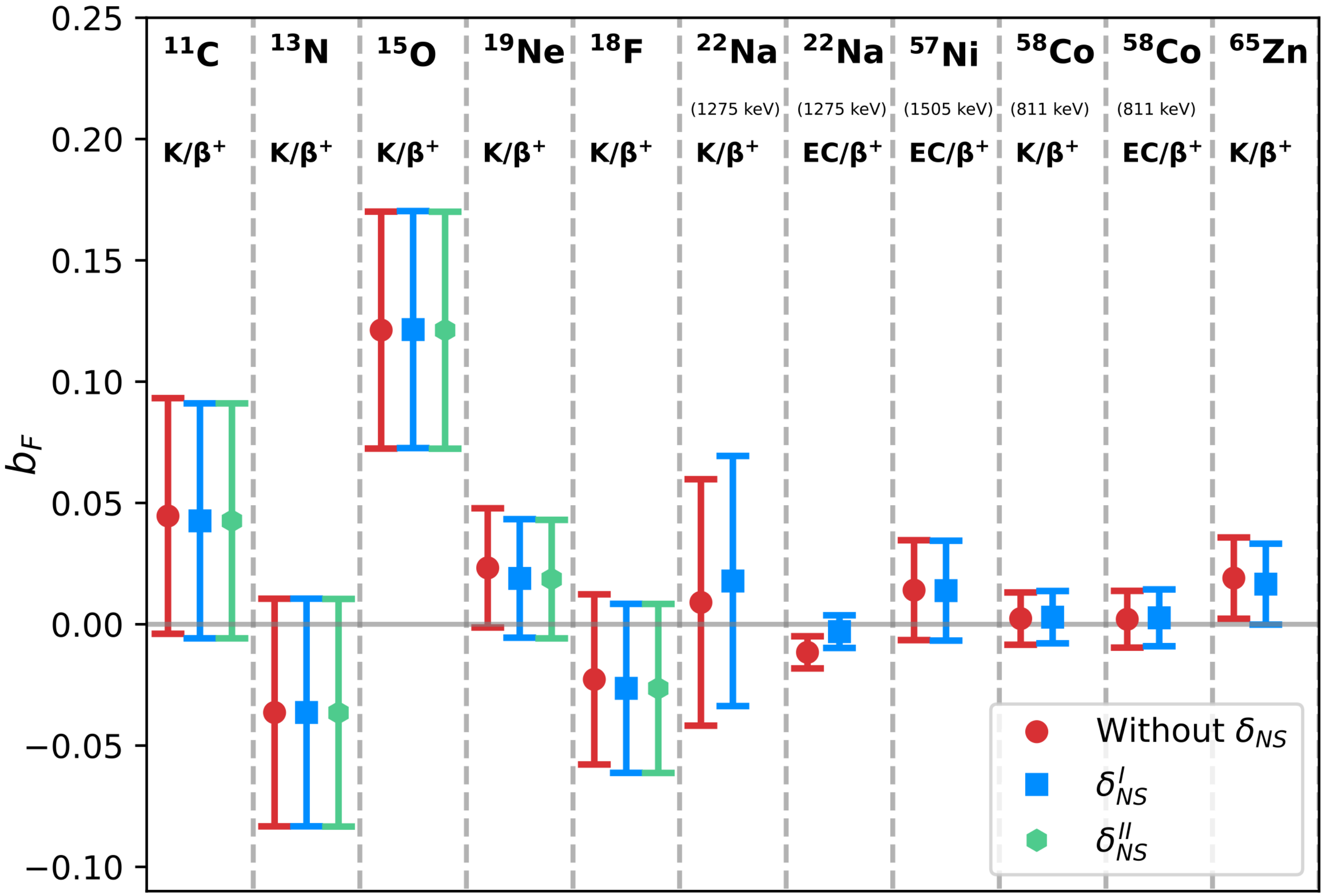
$\delta_{NS}^{\text{theo}}$ is calculated numerically with **shell model input** (OBTDs) → part of uncertainties

Result for $\mathbf{b_F}$

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

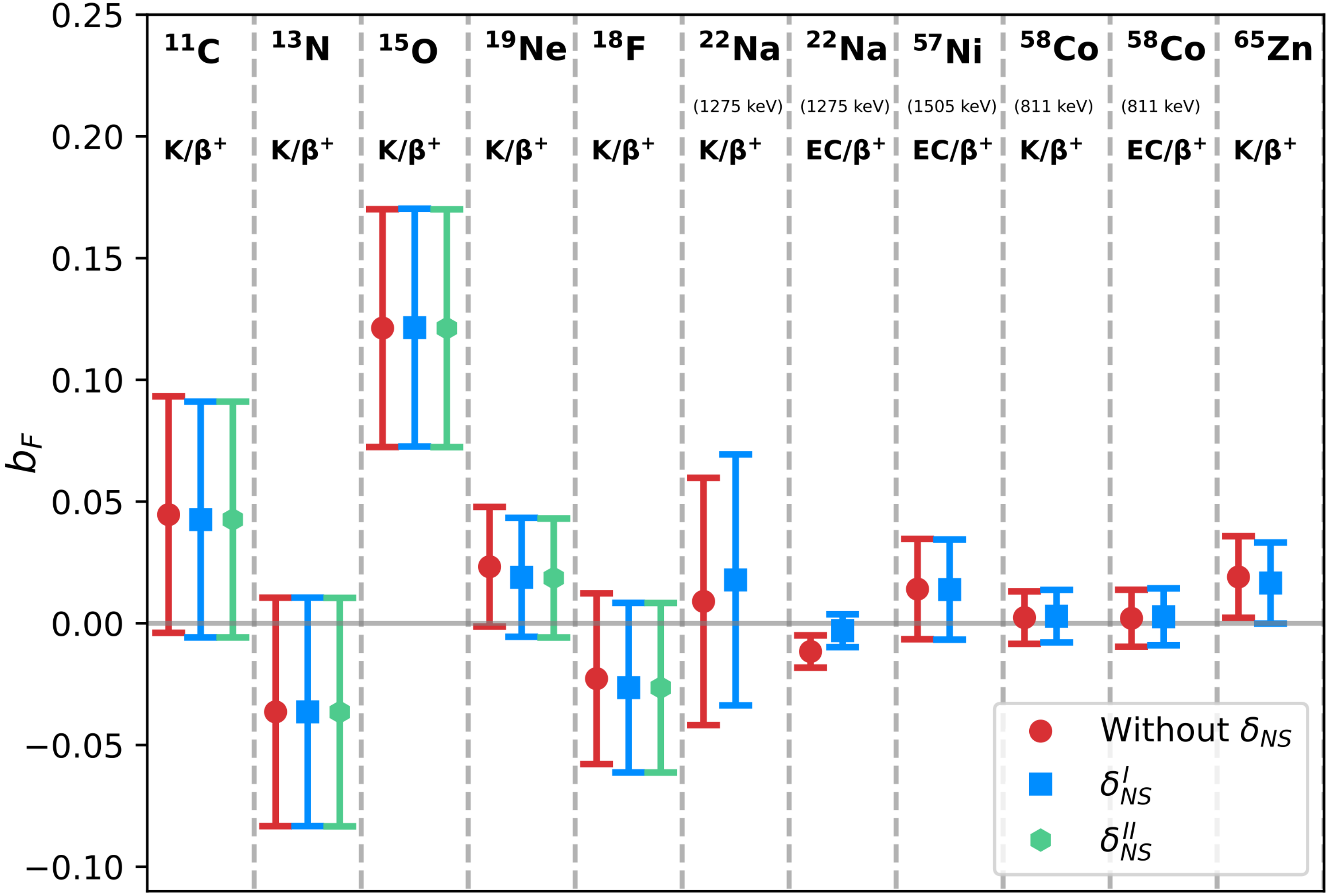
Result for b_F

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)



Result for b_F

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)



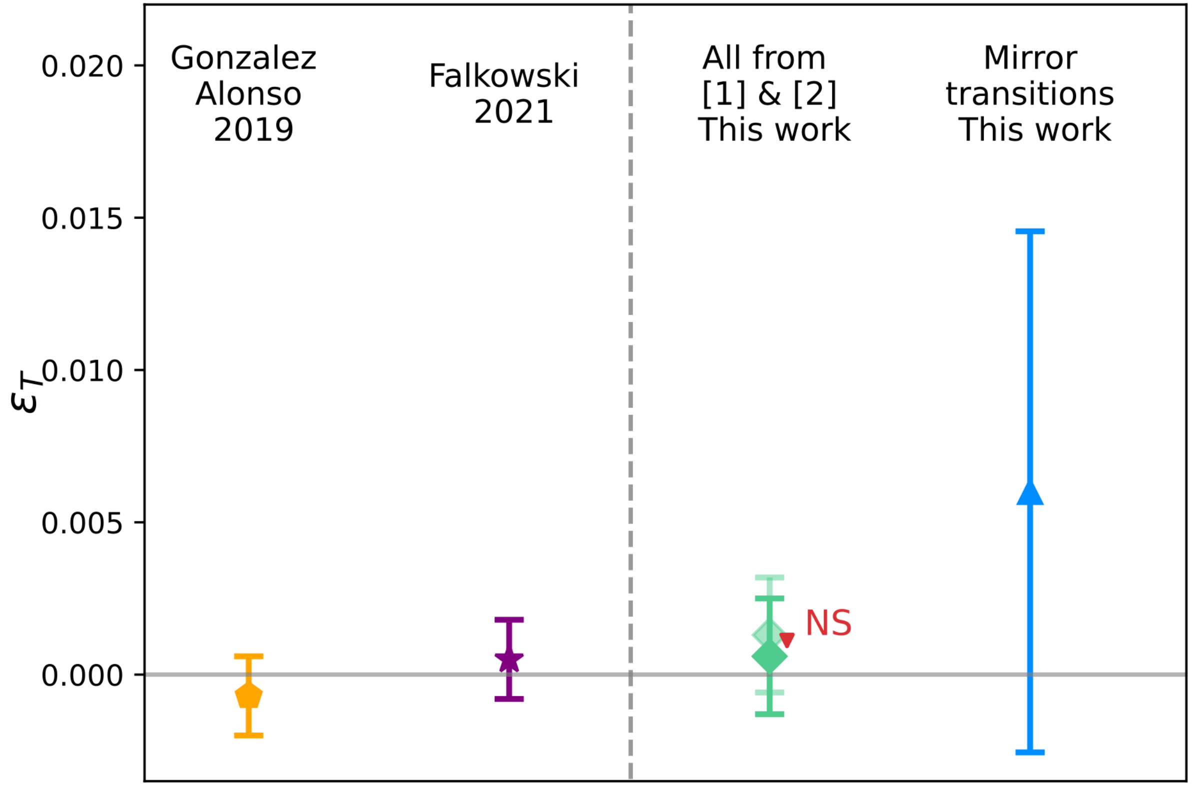
→ **First time** that b_F extraction is done with this *new way* with **old data**

Result for ε_T constraints

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

Result for ε_T constraints

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

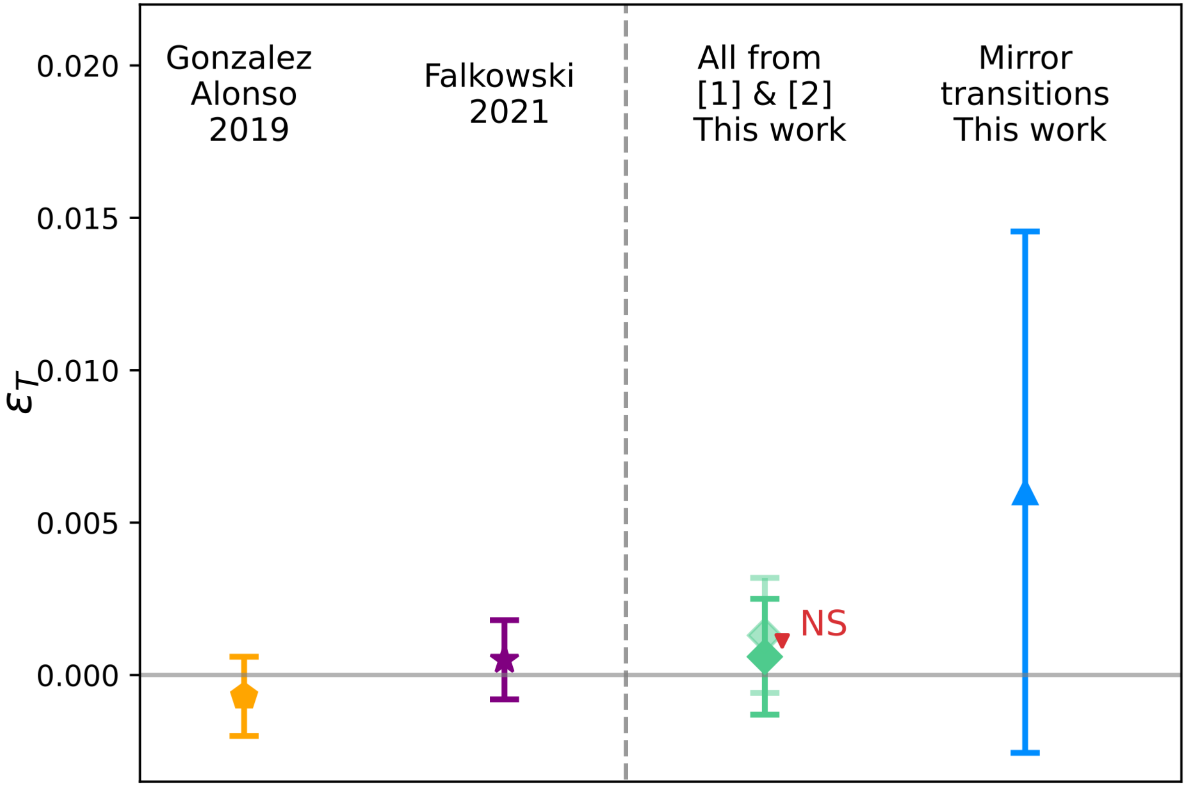


$$\varepsilon_T^{All} = 0.0006(15)_{exp}(12)_{atom}(1)_{NS} \text{ (90\%CL)}$$

$$\varepsilon_T^{MT} = 0.0060(80)_{exp}(28)_{atom}(6)_{NS} \text{ (90\%CL)}$$

Result for ε_T constraints

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)



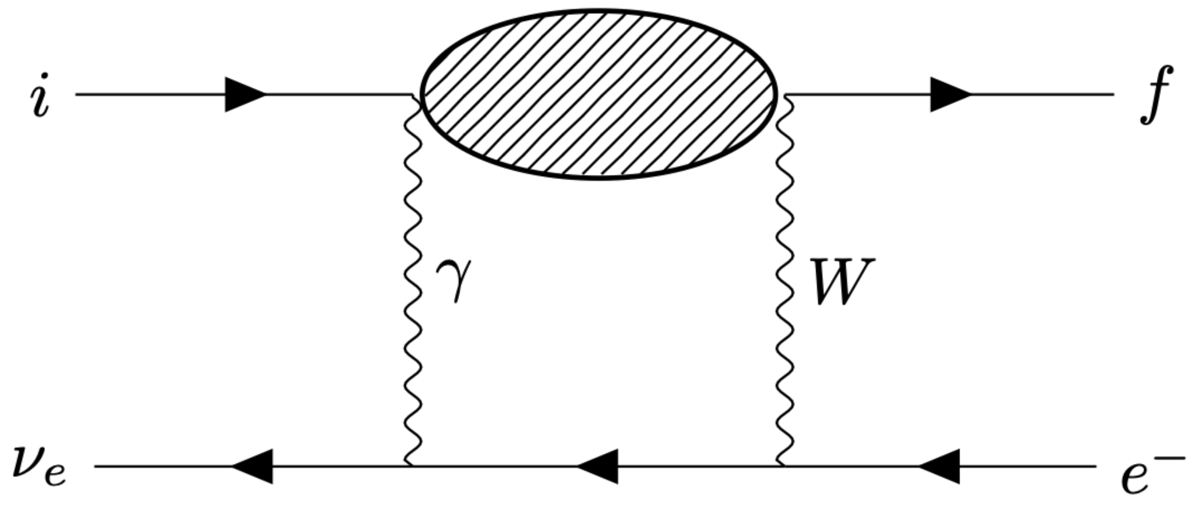
$$\varepsilon_T^{All} = 0.0006(15)_{exp}(12)_{atom}(1)_{NS}\ (90\%CL)$$

$$\varepsilon_T^{MT} = 0.0060(80)_{exp}(28)_{atom}(6)_{NS}\ (90\%CL)$$

→ Nuclear structure uncertaintes **do not play** significant role in constraints

… and *ab initio* ?

… and *ab initio* ?



Calculate the δ_{NS} correction from γW-box diagram with NCCI @ UND

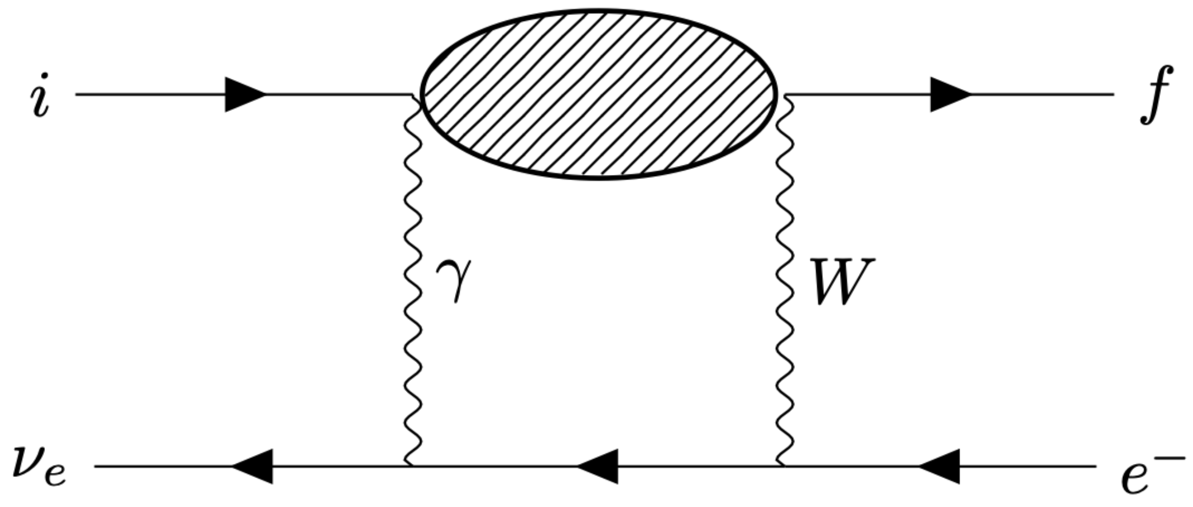
→ **Dispersive relation** for all transitions

→ Walecka multipole decomposition ⇒ Behrens & Bühring

→ **Mirror transitions → improvement**

$$\delta\mathcal{M}^{\text{nucl}}_{\text{inner}} = \left\{\frac{\alpha}{4\pi}\left[3\ln\frac{M_Z}{m_p} + \ln\frac{M_Z}{M_W} + \tilde{a}_g\right] + \frac{1}{2}\delta^{\text{QED}}_{\text{HO}} + \Box^{n}_{\gamma W}\right\}\mathcal{M}_0 + \left\{\Box^{\text{nucl}}_{\gamma W}(E_e) - \Box^{n}_{\gamma W}\right\}\mathcal{M}_0$$

… and *ab initio* ?



Calculate the δ_{NS} correction from γW-box diagram with NCCI @ UND

→ **Dispersive relation** for all transitions

→ Walecka multipole decomposition ⇒ Behrens & Bühring

→ **Mirror transitions → improvement**

$$\delta\mathcal{M}^{\text{nucl}}_{\text{inner}} = \left\{\frac{\alpha}{4\pi}\left[3\ln\frac{M_Z}{m_p} + \ln\frac{M_Z}{M_W} + \tilde{a}_g\right] + \frac{1}{2}\delta^{\text{QED}}_{\text{HO}} + \square^{n}_{\gamma W}\right\}\mathcal{M}_0 + \left\{\square^{\text{nucl}}_{\gamma W}(E_e) - \square^{n}_{\gamma W}\right\}\mathcal{M}_0$$

nuclear-structure-independent inner RC

nuclear-structure-dependent inner RC

Summary

Summary

→ Large code infrastructure for **high precision** calculations

→ Competitive constraints for $\mathbf{b_F}$ & $\boldsymbol{\varepsilon_T}$ with *old data* in a **new way**

→ **Nuclear structure** uncertainties **do not play** significant role in **exotic tensor constraints**

Summary

→ Large code infrastructure for **high precision** calculations

→ Competitive constraints for $\mathbf{b_F}$ & $\boldsymbol{\varepsilon}_\mathbf{T}$ with *old data* in a **new way**

→ **Nuclear structure** uncertainties **do not play** significant role in **exotic tensor constraints**

Outlook

❖ Same study for **first forbidden transitions: hidden gems**

❖ Improvement needed in the estimation of atomic effects

❖ Condensed matter description of Behrens & Bühring for in-medium effects

❖ SALER/ASGARD will reduce the experimental uncertainties

❖ Together, constraints on $\mathbf{b_F} \leq \mathbf{0.1}\%$ are feasible!

Back-up

Standard Model (SM)

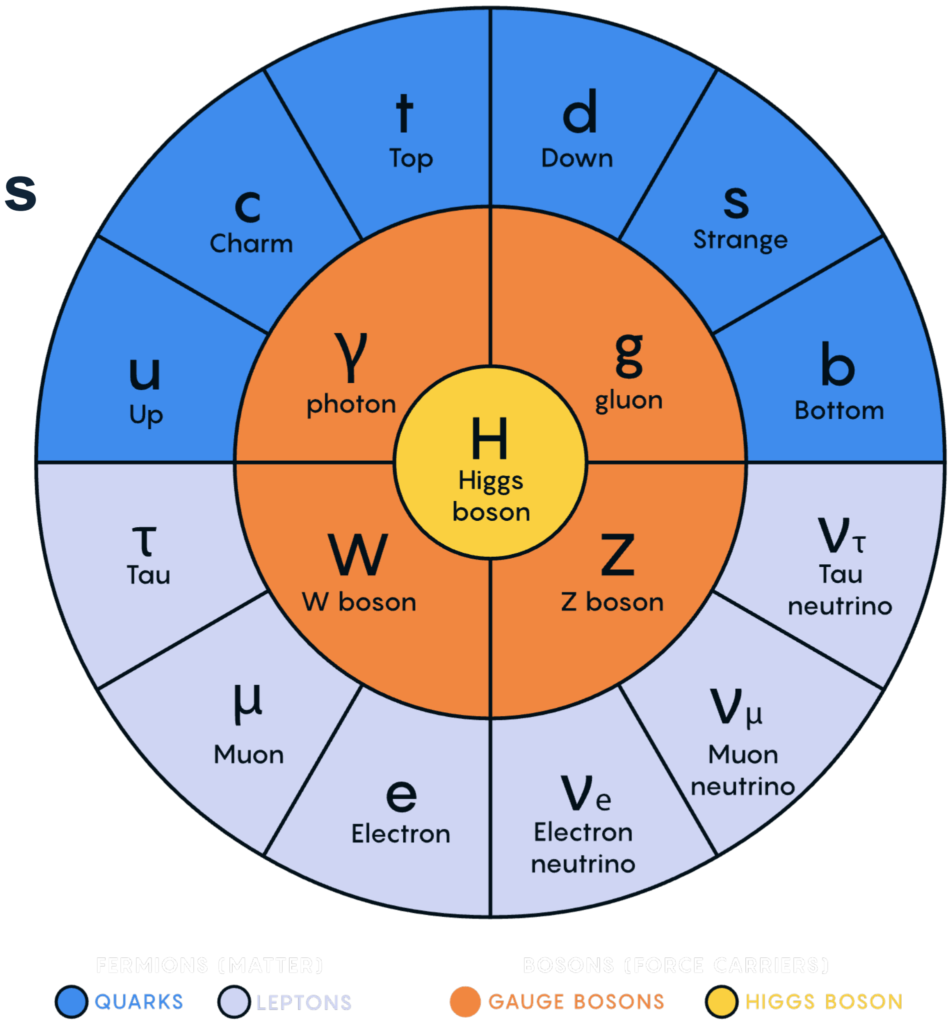
Describes **elementary particles** and **their interactions**

Describes 3 of 4 **fundamental forces**

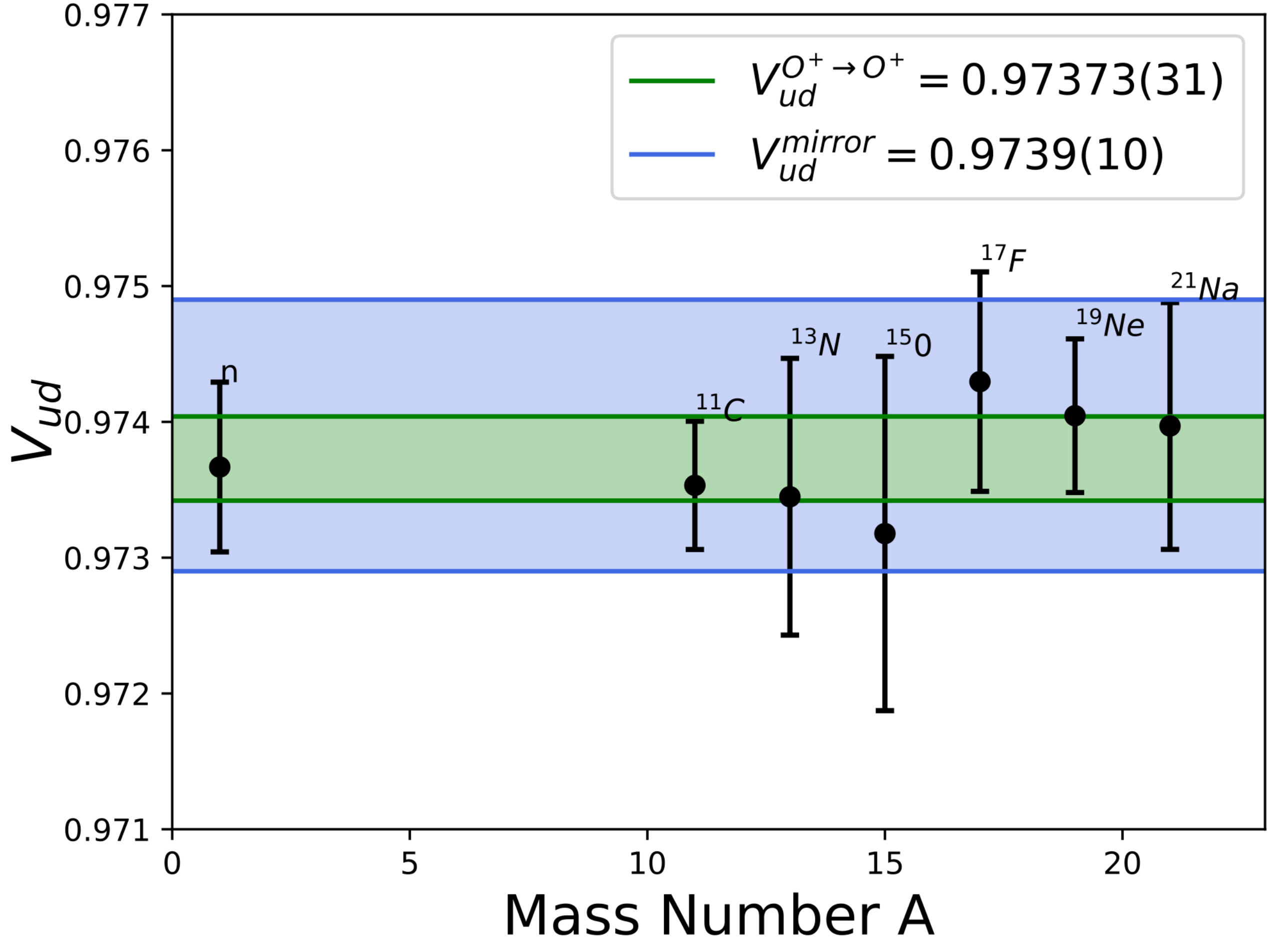
Strong model, consistent with constaints at **TeV** scale

<u>Open questions :</u>

- ➢ Gravity
- ➢ Asymmetry Matter/Anti-Matter
- ➢ Neutrino proprieties (masses, sterile, …)
- ➢ CKM unitarity
- ➢ Only Vector and Axial current (weak) ?



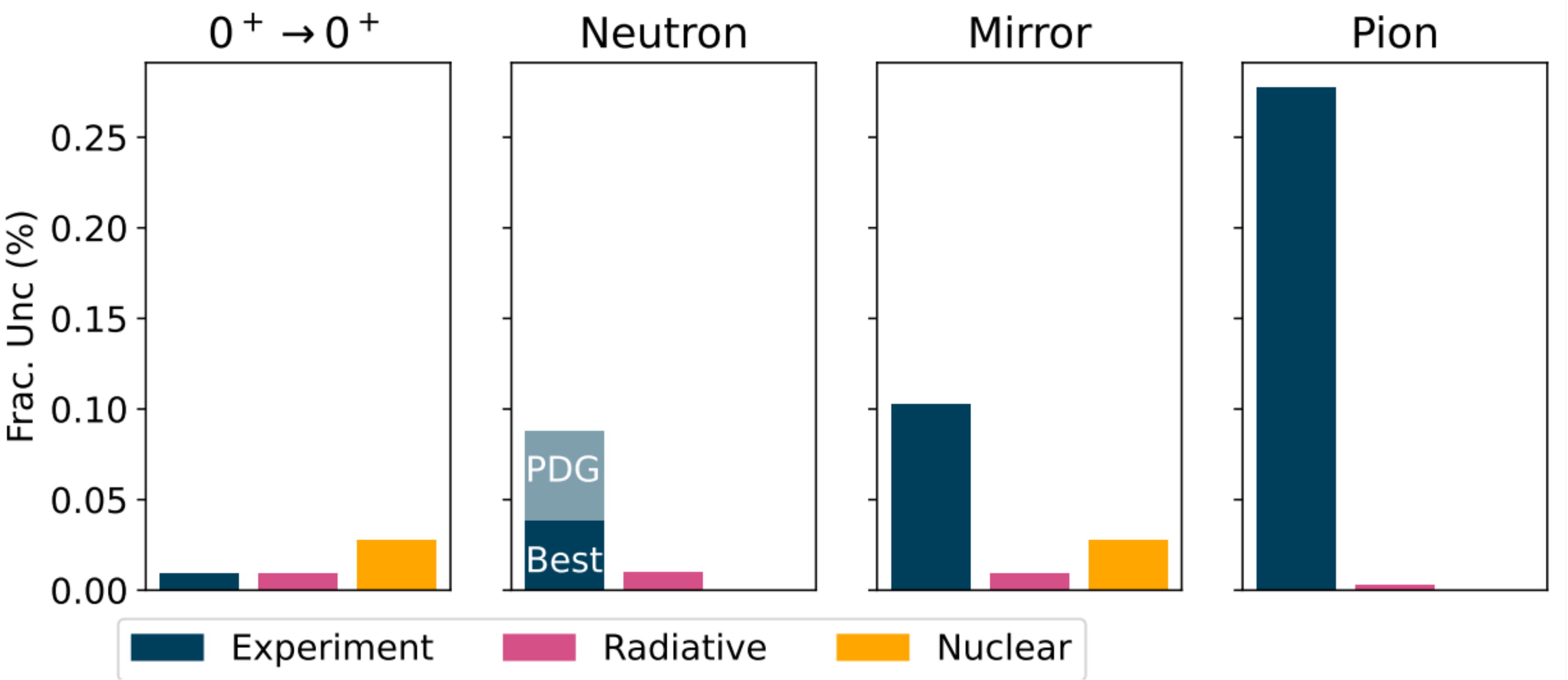
CKM matrix



$$\Delta^{CKM} - 1 = -0.00166(69)$$

Δ^{CKM} : exotic physics (new particle or **exotic currents**)

CKM precision



Pion Decay

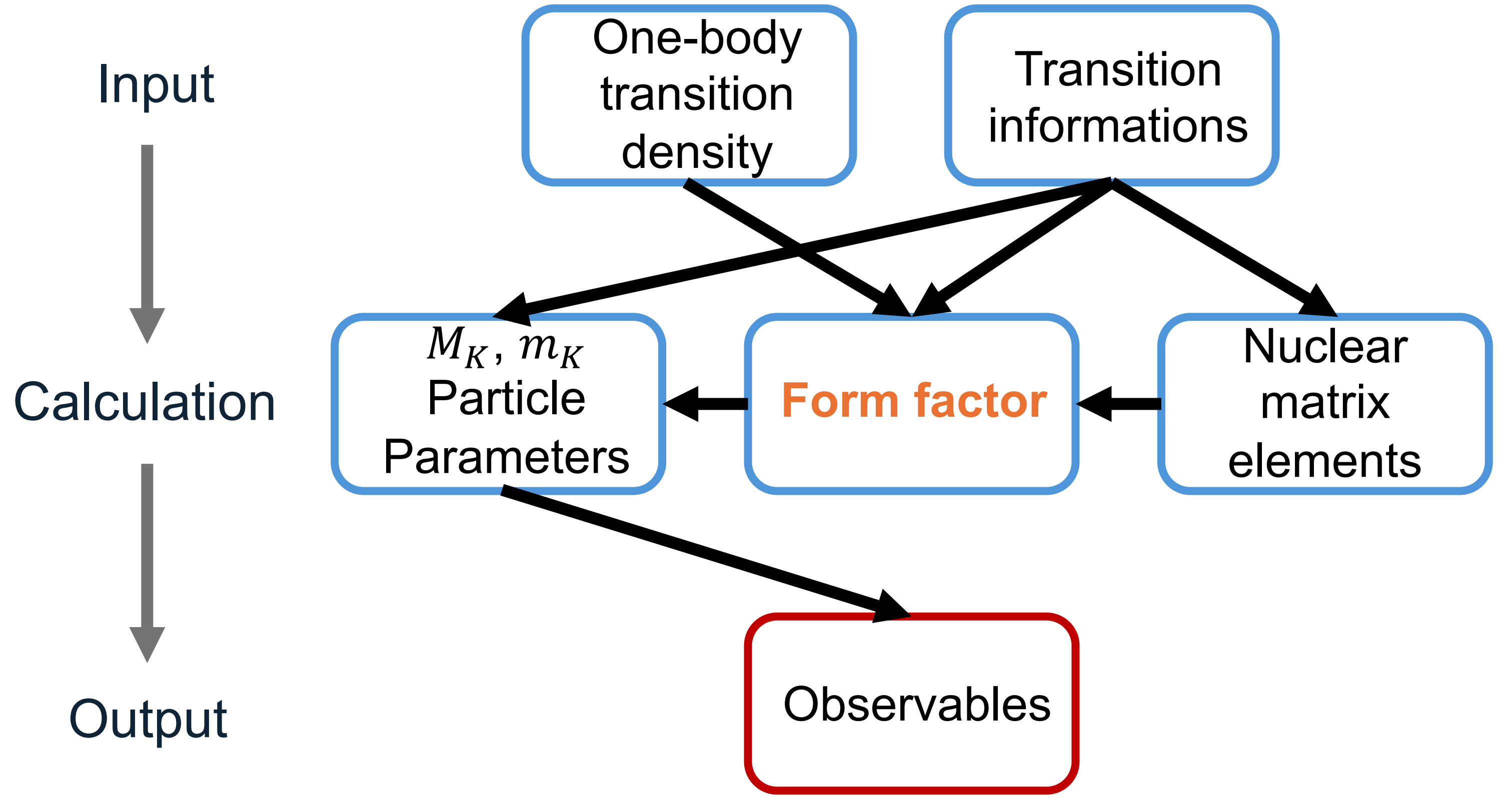
$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu} \qquad (99.9877\%)$$

$$\pi^{\pm} \rightarrow e^{\pm} + \nu_{e} \qquad (0.0123\%)$$

β-decay $\left\{ \pi^{\pm} \rightarrow \pi^{0} + e^{\pm} + \nu_{e} \right.$ $(\ 10^{-6}\%)$

Lack of stat

Code to reproduce Behrens & Bühring formalism



Possible observables

Spectra & Rates
- Half life
- **$\lambda_{EC}/\lambda_{\beta+}$ ratio**
- Shape factor $C(Z, E)$
- β-spectrum

Angular correlations
- $\beta - \alpha$
- $\beta - \gamma$
- $\beta - \nu$ (D time-reversal)

→ All observables are expressed in terms of form factors for ANY type of transitions

What transitions ?

Making hadronic theory easy

- $0^+ \rightarrow 0^+$ superallowed → $M_{GT} = 0$

Sensitive to ε_S, already well studied by Towner & Hardy

- T = 1/2 (mirror transitions MT) → $M_F = 1$

Sensitive to ε_S & ε_T

- Gamow-Teller → $M_F = 0$

Sensitive to ε_T

Validation of shell model interactions

Nucleus	Interaction	$t_{1/2}^{theo}$	$t_{1/2}^{exp}$
^{11}C	CKPOT YSOX	20.2406 m 21.8569 m	20.3401 m
^{13}N	CKPOT YSOX	10.290 m 10.641 m	9.967 m
^{15}O	CKPOT YSOX	133.462 s 137.630 s	122.268 s
^{17}F	USDB YSOX	74.512 s 75.989 s	64.368 s
^{18}F	USDB YSOX	110.773 m 125.562 m	109.734 m
^{19}Ne	USDB YSOX	16.9430 s 17.1596 s	17.2590 s
^{22}Na	USDB YSOX	2.0959 y 0.0546 y	2.6033 y
^{57}Ni	HO	213.70 h	209.41 h
^{58}Co	HO CA48MH1	4438.41 d 102.82 d	71.72 d
^{65}Zn	JUN45	350.98 d	489.03 d

Tricky cases : 22Na & 58Co

And ~30-40% for 65Zn & 58Co between theo & exp

Theoretical agree with experimental values
→ ~20% of theoretical calculation for **Nuclear Structure uncertainties**

Table of results

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

Isotope	Prob. ratio	ρ_{exp}	ρ^{I}_{th}	ρ^{II}_{th}	$\delta^{exp}_{NS}(\%)$	$\delta^{I}_{NS}(\%)$	$\delta^{II}_{NS}(\%)$	$(\lambda_{EC}/\lambda_{\beta^+})_{exp}$	$(\lambda_{EC}/\lambda_{\beta^+})_{th}$
$^{11}\mathrm{C}^{a,c}$	K/β^+	-0.7544(8)	-0.7728	-0.6927	0.32	-0.016	-0.019	0.00225(15)	0.00210(6)
$^{22}\mathrm{Na}^{b,c}$	ϵ/β^+	-	-	-	-1.07	-0.336	0.570	0.1083(9)	0.1105(9)
$^{22}\mathrm{Na}^{b,c}$	K/β^+	-	-	-	-1.07	-0.336	0.570	0.105(9)	0.1034(9)
$^{57}\mathrm{Ni}^{d}$	ϵ/β^+	-	-	-	-	0.03	-	1.460(47)	1.427(11)
$^{58}\mathrm{Co}^{d,e}$	ϵ/β^+	-	-	2.4997	-	0.03	-0.85	5.61(8)	5.59(8)
$^{58}\mathrm{Co}^{d,e}$	K/β^+	-	-	2.4997	-	0.03	-0.849	4.94(6)	4.92(7)
$^{65}\mathrm{Zn}^{f}$	K/β^+	-	-	-	-	0.37	-	30.1(5)	29.1(7)
$^{13}\mathrm{N}^{a,c}$	K/β^+	-0.560(1)	-0.5480	-0.5077	0.12	-0.103	-0.105	0.00168(12)	0.00178(5)
$^{15}\mathrm{O}^{a,c}$	K/β^+	0.630(2)	0.5555	0.5185	0.16	-0.151	-0.148	0.00107(6)	0.00091(2)
$^{18}\mathrm{F}^{b,c}$	K/β^+	-	-	-	0.69	-0.031	-0.031	0.030(18)	0.0312(5)
$^{19}\mathrm{Ne}^{b,c}$	K/β^+	-1.602(1)	-1.6338	-1.6197	0.82	-0.190	-0.190	0.00096(3)	0.00093(1)

	CKPOT^a	USDB^b	YSOX^c	HO^d	CA48MH1^e	JUN45^f	SN132^g
Model Space	$1p_{3/2}$, $1p_{1/2}$	$1s_{1/2}$, $1d_{5/2}$ $1d_{3/2}$	$1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$ $1d_{5/2}$, $1d_{3/2}$	N: $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$ P: $1f_{7/2}$	$1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$ P: $1f_{7/2}$, N: $1g_{9/2}$	$1f_{7/2}$, $1g_{9/2}$ $2p_{3/2}$, $2p_{3/2}$	$1g_{9/2}$, $2d_{5/2}$, $2d_{3/2}$ $3s_{1/2}$, $1h_{11/2}$

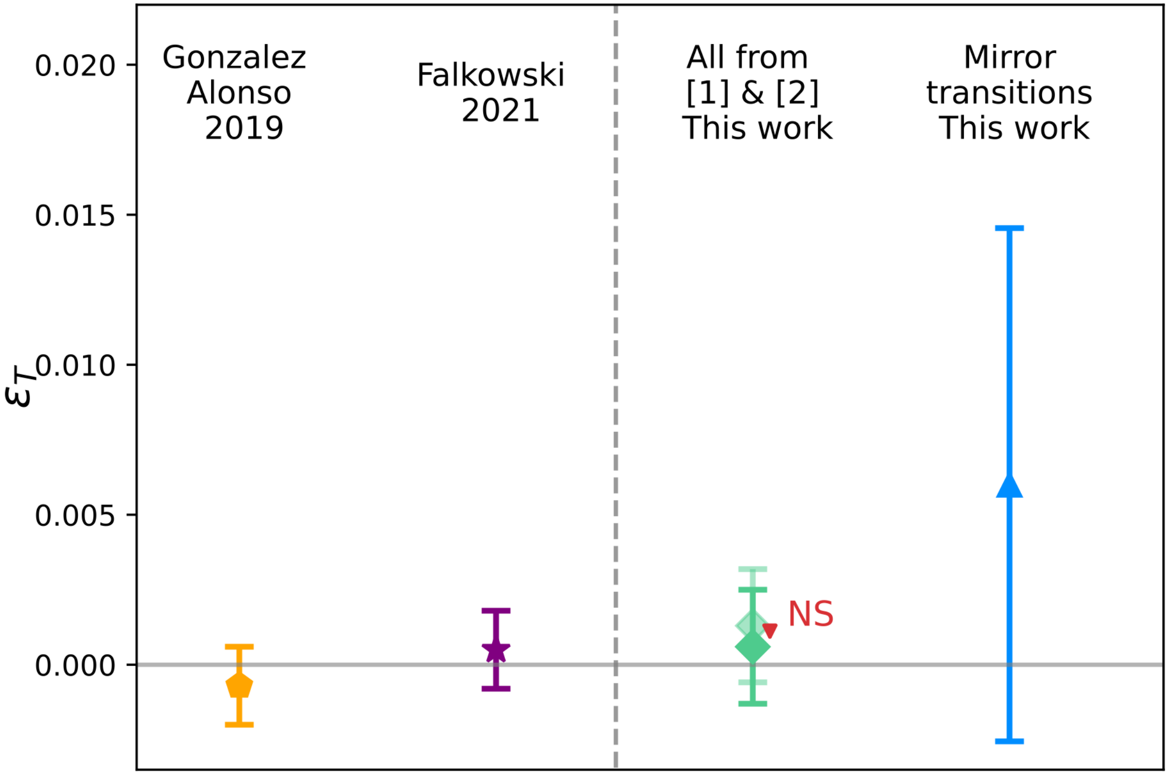
→ Nuclear structure corrections ~ 1% - 0.1%

Result for ε_T constraints

[1] X. Mougeot, Applied Radiation and Isotopes, 2019
[2] W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

Extraction of ε_T by using the most recent contraint on ε_S and g_i from *FLAG2024**

*) 2411.04268



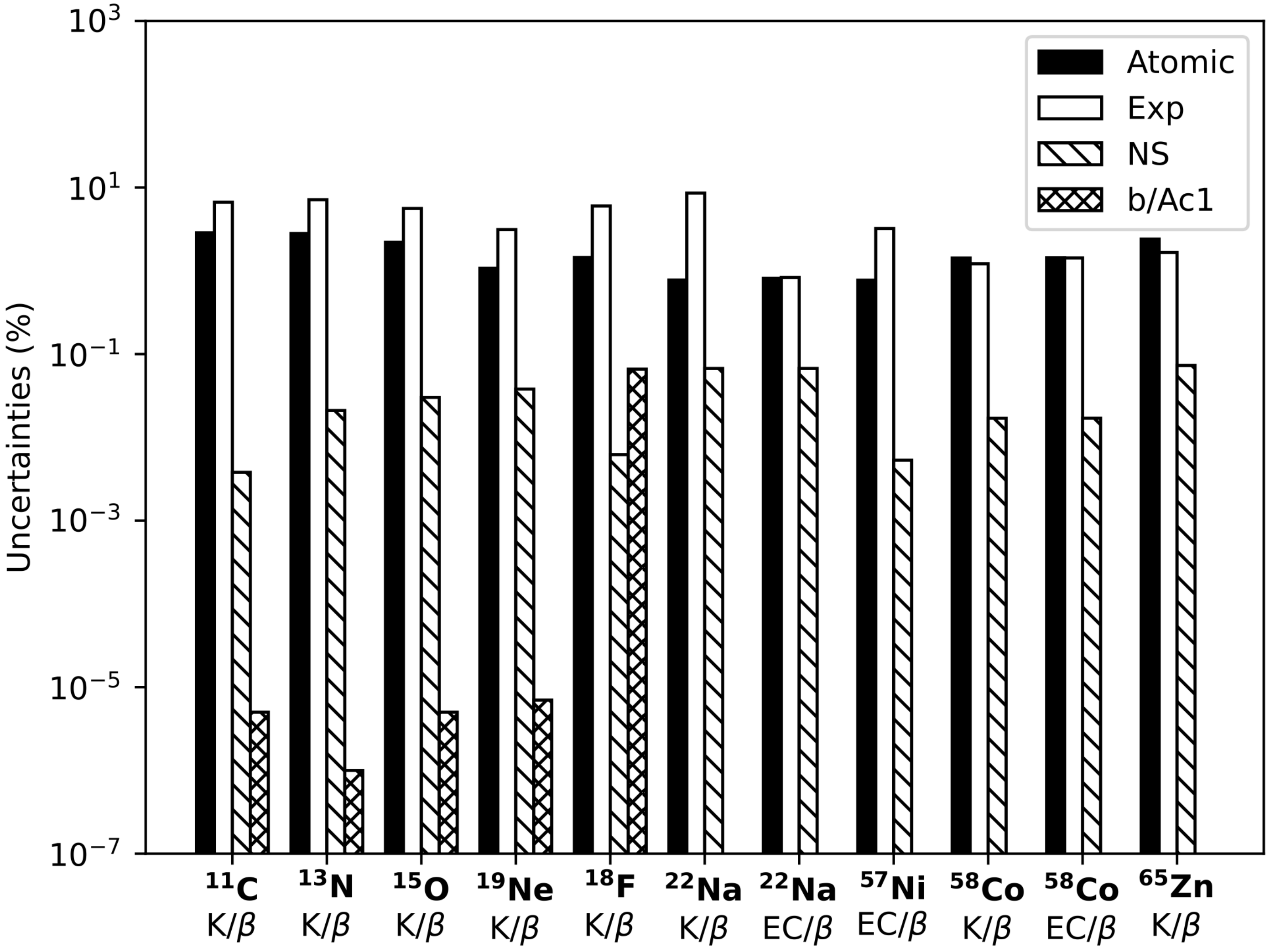
$$b_F = \pm \frac{2\gamma}{1+|\rho|^2} \mathrm{Re}\left(\frac{\epsilon_S g_S}{g_V} + |\rho|^2 \frac{8\epsilon_T g_T}{-2g_A} \right)$$

$$\varepsilon_T^{All} = 0.0006(15)_{exp}(12)_{atom}(1)_{NS} \text{ (90\%CL)}$$

$$\varepsilon_T^{MT} = 0.0060(80)_{exp}(28)_{atom}(6)_{NS} \text{ (90\%CL)}$$

→ Nuclear structure uncertaintes **do not play** significant role in constraints

Result for ε_T constraints



22Na case

[1]: Firestone, McHarris, and Holstein, Phys.Rev. C 18, 2719 (1978)
[2]: Brown and Wildenthal, Phys. Rev. C 27, 1296 (1983)
[3]: Triambak and al., Phys.Rev. C 95, 035501 (2017)

Very **high** experimental precision (0.8%), but ~50 years theory disagreement

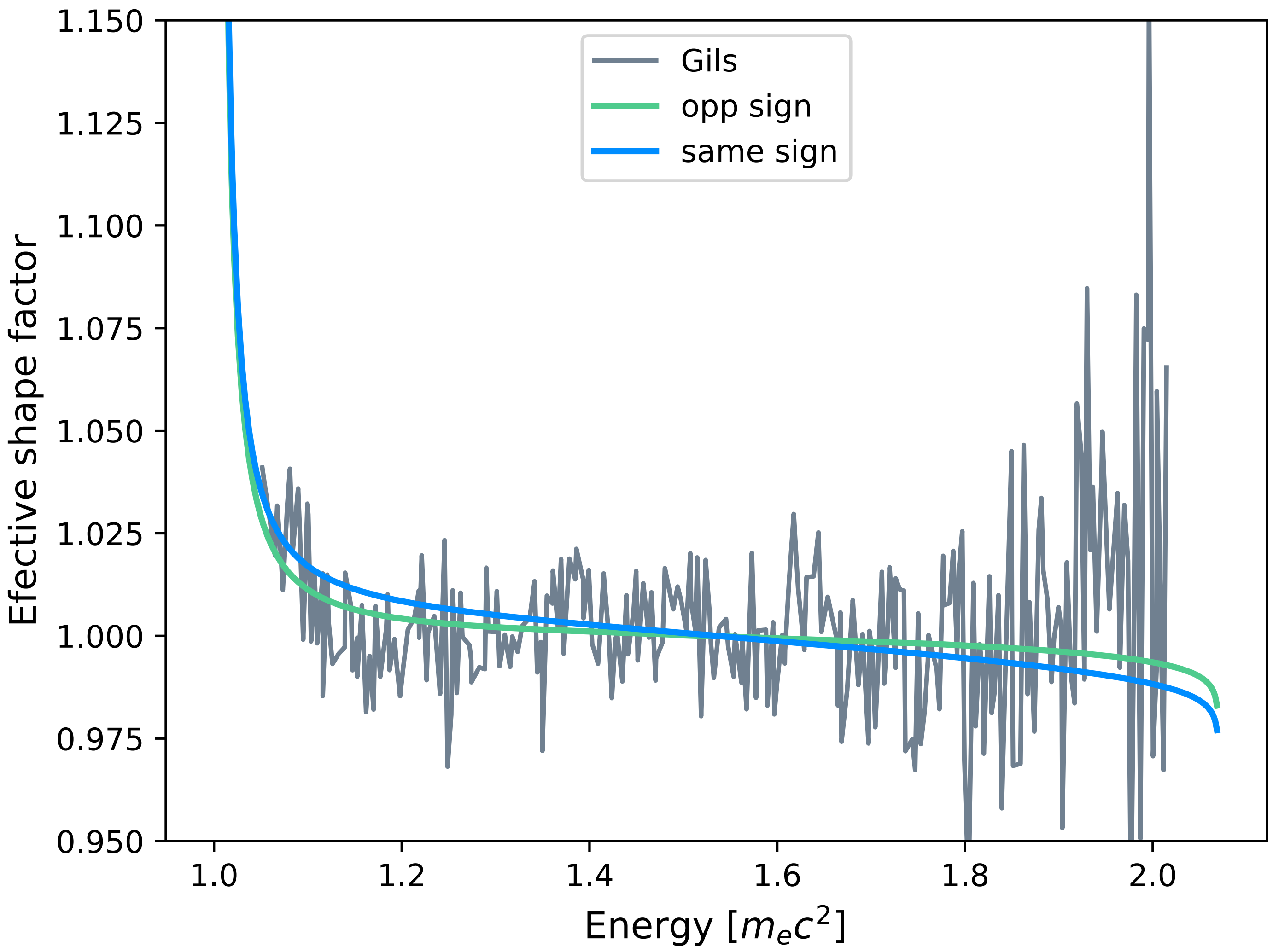
Different measurements: spectrum/shape factor, gamma anisotropy, beta polarization, beta-gamma (2017!)

Inconclusive with old shell model calculations

Form factor	FMH (norm.)[1]	BW (norm.)[2]	USDB	YSOX	Exp. [3]
c_1	-0.016	-0.016	0.018	-0.11	0.0153
b/Ac_1	+19	-10	-6.58	3.67	-8.9(1.2)
$c_2/c_1 R^2$	+0.37	-0.062	0.007	0.140	-
d/Ac_1	+3.2	+2.5	-3.83	-0.86	3(6)

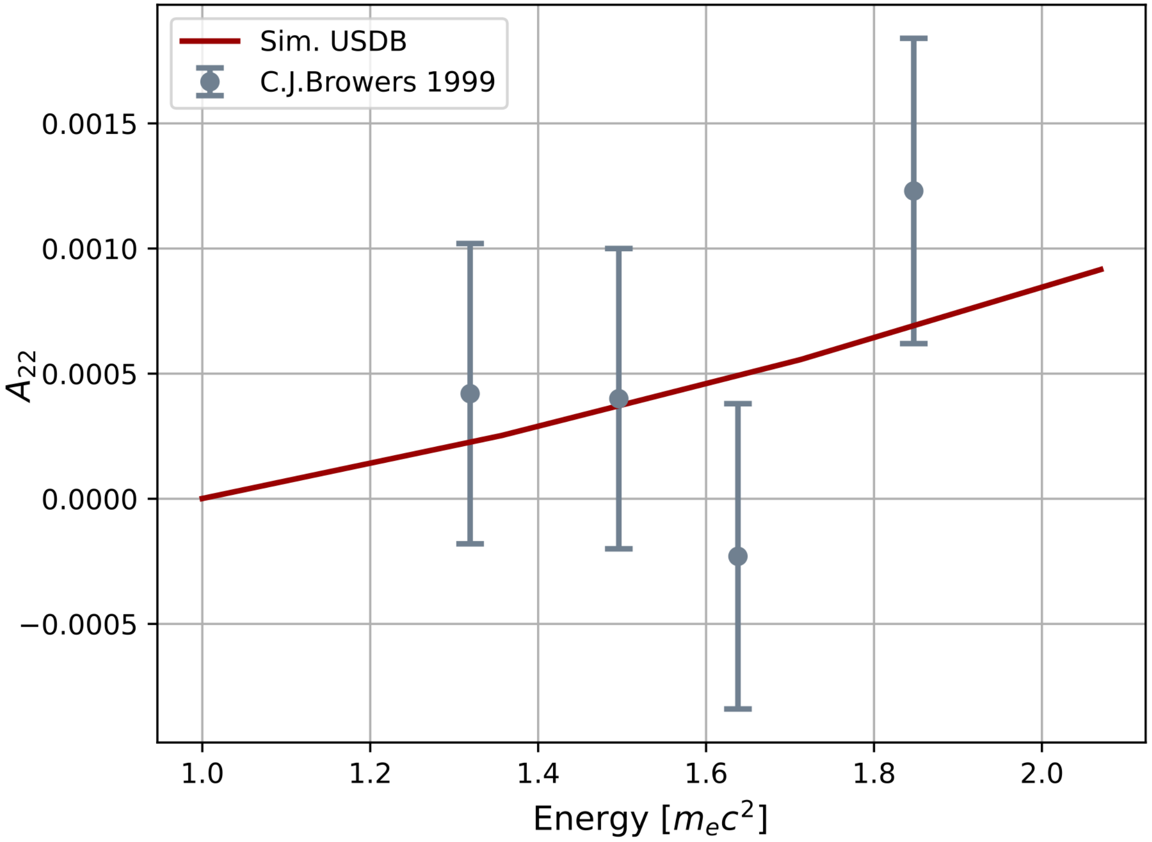
→ USDB interaction is fine

22Na case


Gils
opp sign
same sign
Efective shape factor
1.150
1.125
1.100
1.075
1.050
1.025
1.000
0.975
0.950
1.0
1.2
1.4
1.6
1.8
2.0
Energy [$m_e c^2$]

22Na case

For $\beta - \gamma$ corr. calculations must be pushed to higher orders!! (more than K=1)



$$^{22}Na(3+) \xrightarrow{\beta^+} {}^{22}Ne(2+) \xrightarrow{\gamma} {}^{22}Ne(0+)$$

$$W(\theta) \propto 1 + \frac{3}{2} A_{22} \cos^2\theta$$

And **everything agrees. 50 years mystery solved!**

W.Bambynek et al., Rev. Mod. Phys. 49, 77 (1977)

Experimental Method

Method	Source	Detectors	Estimated accuracy (%)	Nucleus
Spectroscopy of K events and positrons (no K x-ray escape)	Internal gaseous	pc	6	18F, 58Co(K/B+)
Spectroscopy of K events and positrons (no K x-ray escape)	Internal gaseous	apc	3	11C, 13N, 15O, 19Ne
Spectroscopy of K events and positrons	Internal solid	NaI(Tl)	2	22Na(K/B+), 58Co(K/B+)
Spectroscopy of K Auger electrons and positrons	External solid	NaI(Tl), Si(Li), pc	1	58Co(K/B+)
Measurement of positron-gamma ray coincidences	External solid	Pc, pl, NaI(Tl), Ge(Li)	2.5	22Na(EC/B+), 57Ni, 58Co(EC/B+), 65Zn
Measurement of positron-gamma ray N and positron-gamma ray S coincidences	External solid	Pc, NaI(Tl)	0.3	22Na(EC/B+)
Measurement of gamma ray-(511 keV) beta+ annihilation photon coincidences	External solid	NaI(Tl), Ge(Li)	3	58Co(K/B+)

pc : proportional counter / apc : anticoincidence proportional counter

Experimental Method

Nucleus	References
11C, 13N, 15O, 18F, 19Ne	Bambynek et al 1977
22Na	+ Galan 2009
57Ni	+ Konijin 1958 + Bakhru and Preiss1967
58Co	+ Bé 2016
65Zn	+ Bé et al 2005

Shell Model interactions

CKPOT

A : 8 - 16
Model space : $1p_{3/2}$, $1p_{1/2}$
Inert core : ${}^{4}_{2}He$
Characteristic : Obtained by adjustment
Coulomb effect : estimated from the observed energy differences of mirror nuclei
Isospin : non-relativistic charge-independent form

YSOX

A : 8 - 30
Model space : $1p_{3/2}$, $1p_{1/2}$, $2s_{1/2}$, $1d_{5/2}$, $1d_{3/2}$
Inert core : ${}^{4}_{2}He$
Characteristic :Adjust macroscopically, Based on universal monopole interaction (VMU), SFO and SDPF-M
Coulomb effect : like CKPOT
Isospin : Assumed

USDB

A : 16 -
Model space : $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$
Inert core : ${}^{16}_{8}O$
Characteristic : Empirical, based on a renormalised G matrix (from the NN potential)
Coulomb effect : Obtained from isobaric analogue states
Isospin : Assumed

HO

A : 48 -
Model space : $2p_{3/2}$, $2p_{1/2}$, $1f_{5/2}$
Inert core : ${}^{48}_{20}Ca$
Characteristic : Proton-neutron interactions are determined by fitting to the observed spectra
Coulomb effect :
Isospin : Assumed

JUN45

A : 56 -
Model space : $2p_{3/2}$, $2p_{1/2}$, $1f_{5/2}$, $1g_{9/2}$
Inert core : ${}^{56}_{28}Ni$
Characteristic : Empirical, derived from realistic interaction (Bonn-C potential)
Coulomb effect : via Cole formula
Isospin : Assumed

Radiative corrections in EC/B+ ratio

$$\delta_{RC}^{EC} = \frac{\alpha}{2\pi}\left(3\ln\frac{m_p}{m_e} - \frac{27}{4}\right) + \frac{\alpha}{12\pi}(W_0 + 1)^2$$

$$\delta_{RC}^{tot} = \frac{\alpha}{2\pi}\left(3\ln\frac{m_p}{m_e} - \frac{27}{4}\right) + \frac{\alpha}{12\pi}(W_0 + 1)^2 - \overline{\delta_{RC}^{\beta}}$$

Overlap and exchange corrections

Overlap effect :

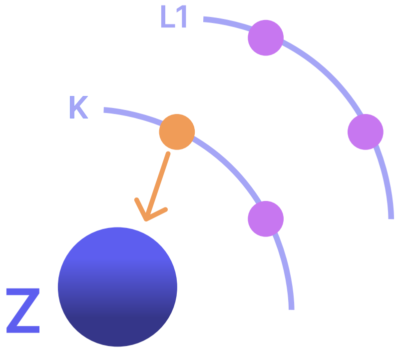
Imperfect overlap between initial a final atomic wave functions

→ Variation of nuclear charge

Exchange effect :

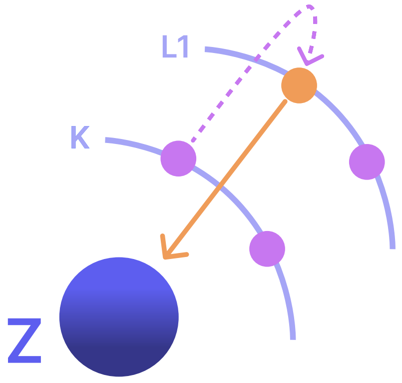
K-shell capture contribution

Regular K capture



\+

Virtual K capture



L1 capture + L1-K exchange

\+ M1, …

Overlap and exchange corrections B

$$B_{n\kappa} = \left| \frac{b_{n\kappa}}{\beta_{n\kappa}} \right|^2 \qquad b_{n\kappa} = t_{n\kappa} \left[\prod_{m \neq n} \left\langle (m,\kappa)' | (m,\kappa) \right\rangle \right] \left[\beta_{n\kappa} - \sum_{m \neq n} \beta_{n\kappa} \frac{\left\langle (m,\kappa)' | (n,\kappa) \right\rangle}{\left\langle (m,\kappa)' | (m,\kappa) \right\rangle} \right]$$

Overlap

Exchange

Bachall

$$t_{n\kappa} = 1$$

Shake-up and shake-off effects are roughly accounted for, though certain probabilities tend to be **underestimated** while others are **overestimated**.

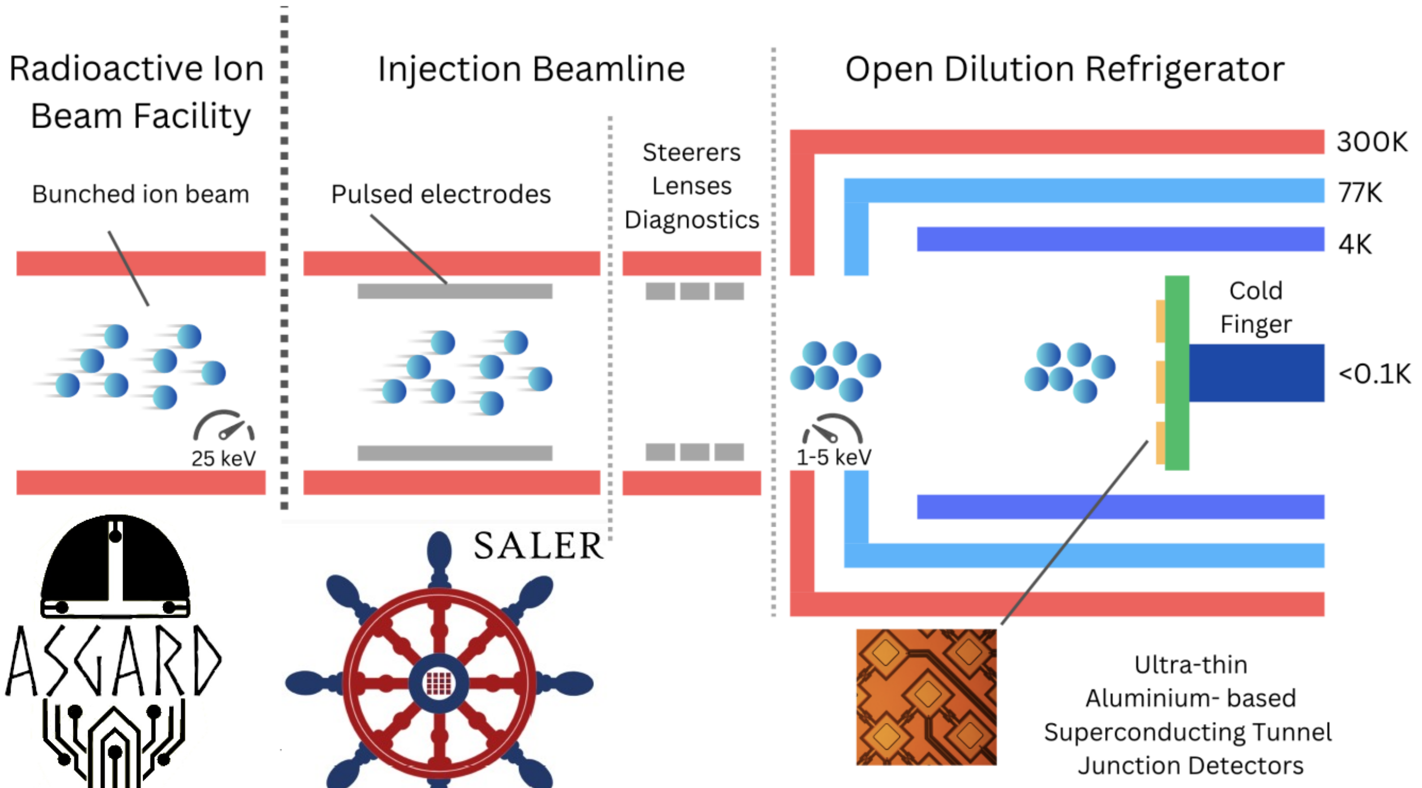
Vatai

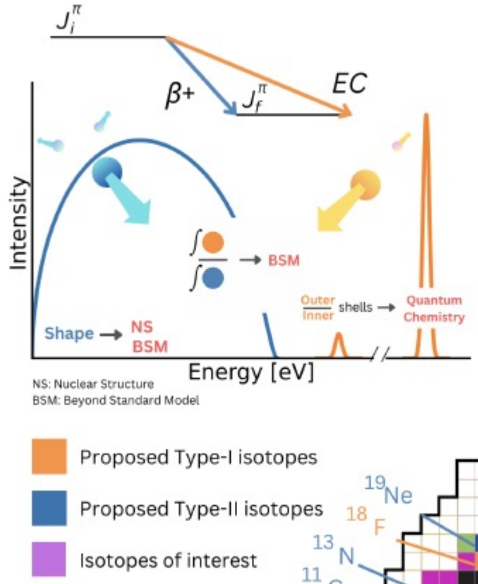
$$t_{n\kappa} = \left\langle (n,\kappa)' | (n,\kappa) \right\rangle^{n_{n\kappa} - 0.5|\kappa|} \left[\prod_{m \neq n} \left\langle (m,\kappa)' | (m,\kappa) \right\rangle^{n_{m\kappa} - 1} \right] \left[\prod_{\substack{m,\mu \\ \mu \neq \kappa}} \left\langle (m,\mu)' | (m,\mu) \right\rangle^{n_{m\mu}} \right]$$

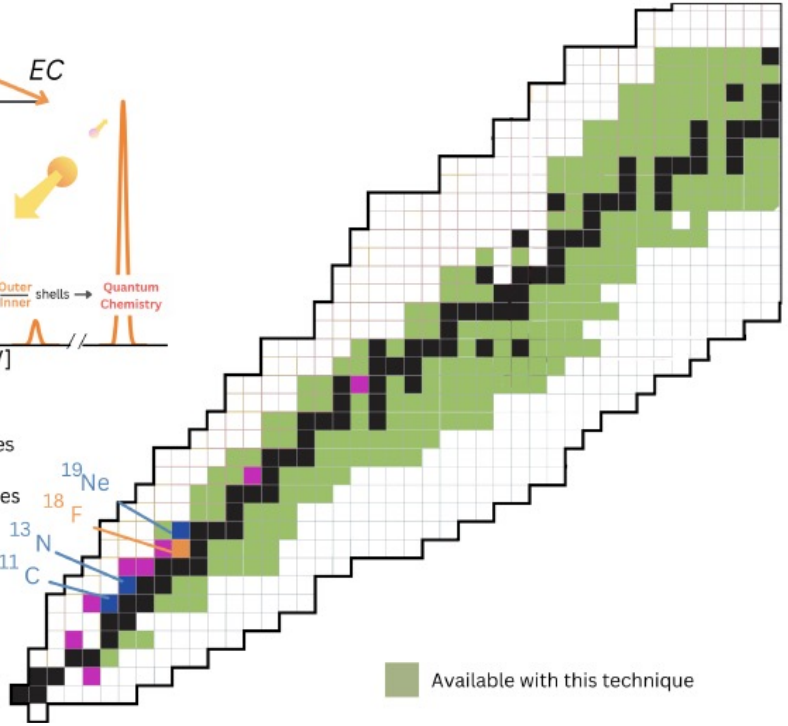
No Shake-up and shake-off effects, but this approach is more comprehensive.

Experimental opportunities with recoil spectroscopy

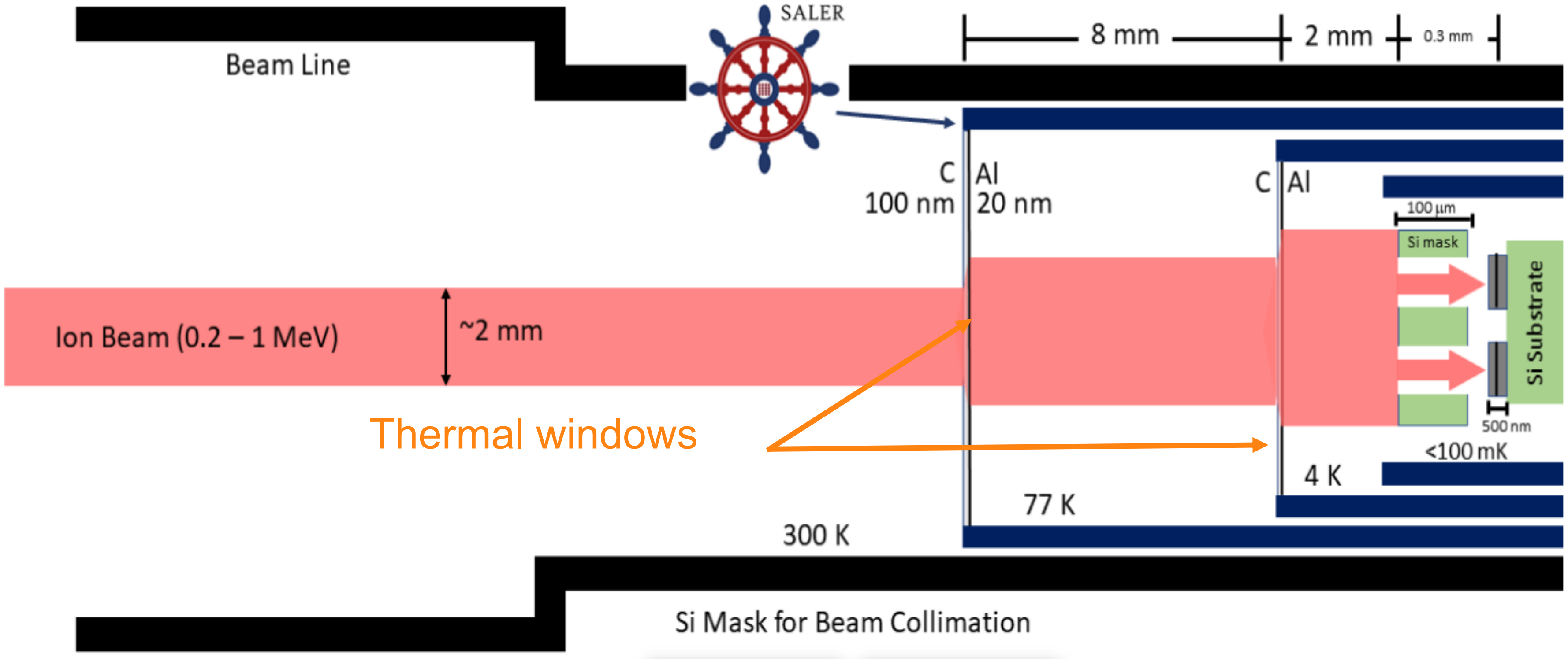
Measurement of recoil energy (<keV) with STJ (~1 eV) → precision spectroscopy
Implementation of nucleus in the STJ
→ BeEST already competitive for sterile neutrino searches



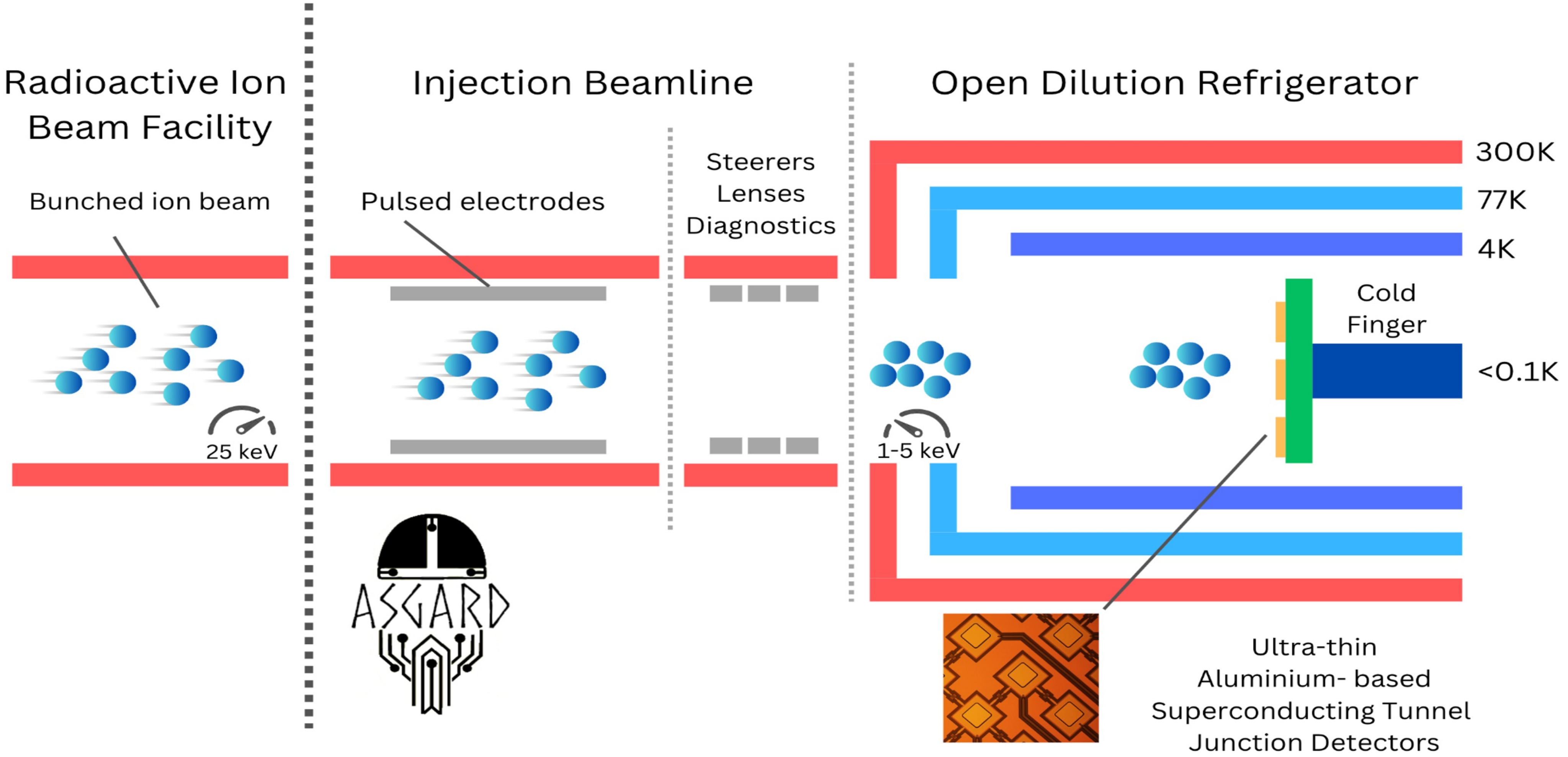




SALER


SALER
Beam Line
8 mm
2 mm
0.3 mm
C
100 nm
Al
20 nm
C
Al
100 µm
Si mask
Si Substrate
500 nm
Ion Beam (0.2 – 1 MeV)
~2 mm
Thermal windows
<100 mK
4 K
77 K
300 K
Si Mask for Beam Collimation

ASGARD



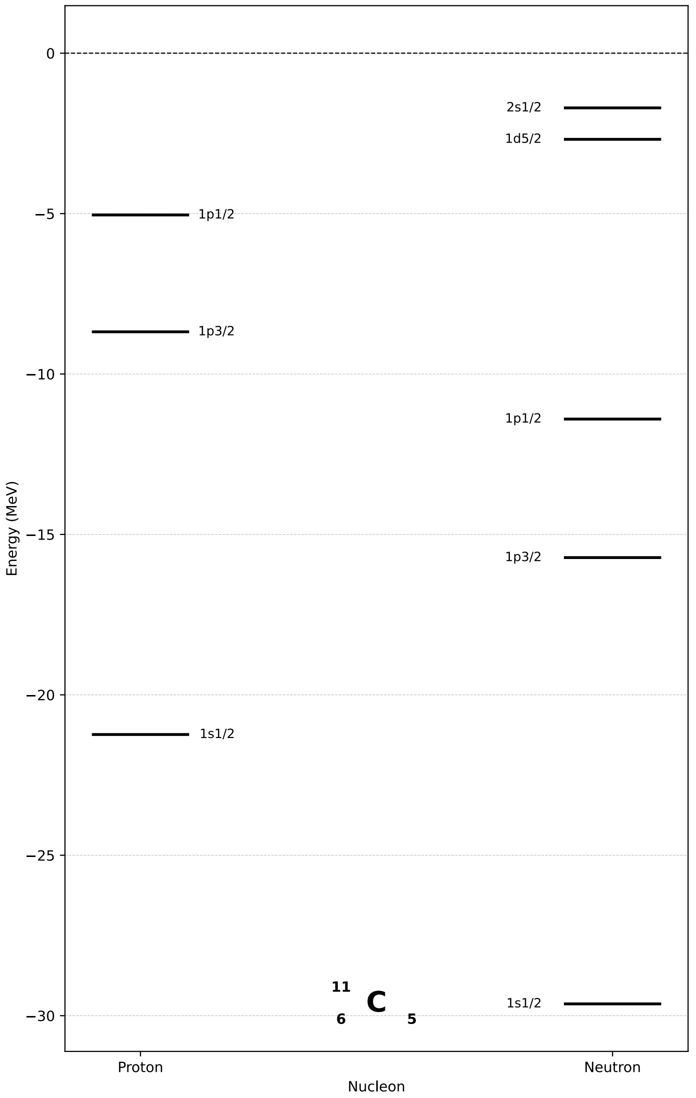
Radioactive Ion Beam Facility
Injection Beamline
Open Dilution Refrigerator
Bunched ion beam
Pulsed electrodes
Steerers
Lenses
Diagnostics
25 keV
1-5 keV
300K
77K
4K
Cold Finger
<0.1K
ASGARD
Ultra-thin Aluminium- based Superconducting Tunnel Junction Detectors

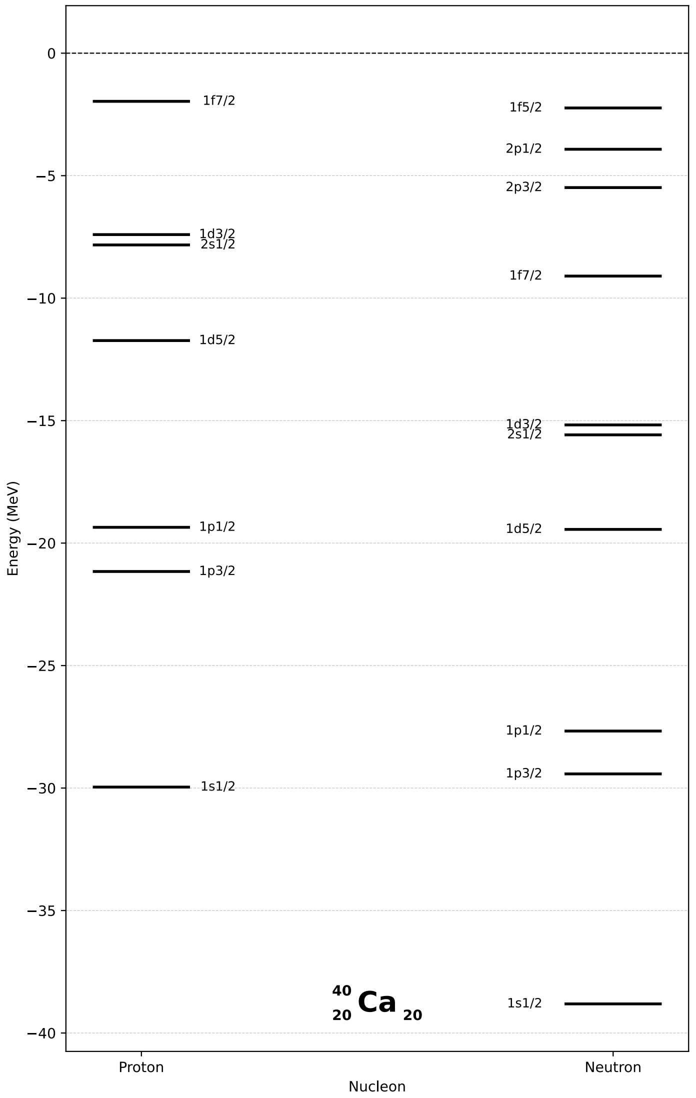
CVC Hyp.

$$-(2K+1+2N)\sqrt{\frac{K}{2K+1}}\,{}^{V}F^{N}_{KK-11}-2N\sqrt{\frac{K+1}{2K+1}}\,{}^{V}F^{N-1}_{KK+11}$$

$$=(W_0\mp 2.5)R\,{}^{V}F^{N}_{KK0}\pm\alpha Z\left(\int\left(\frac{r}{R}\right)^{K+2N}U(r)T_{KK0}\,j^{V}_{0}\right).$$

Naive Shell Model





Shell Model : interactions

$$\mathcal{H} = \sum_{i}^{A} T_i + \sum_{i<j}^{A} V_{ij} + (\sum_{i,j,k}^{A} V_{ijk})$$

3-body term can be omit

$$\mathcal{H} \approx \mathcal{H}_{\text{core}} + \mathcal{H}_{\text{valence}} + \mathcal{H}_{\text{core-valence}}$$

~ constant

Single-particle energy

$$\mathcal{H}^{\text{eff}}_{\text{valence}} = \sum_{i} \epsilon_i a_i^\dagger a_i + \frac{1}{4} \sum_{i,j,k,l} < ij|V^{\text{eff}}|kl > a_i^\dagger a_j^\dagger a_k a_l$$

TBME

Lagrangian

$$\mathcal{L} = \sum [\overline{p}\mathcal{O}_i n] \times [\overline{e}\mathcal{O}_i(C_i + C_i\gamma^5)\nu]$$

$$\mathcal{L}_{\text{eff}} \propto \overline{e}\gamma_\mu\nu_L \cdot \overline{u}\gamma^\mu[C_V - (C_A - 2\epsilon_R)\gamma^5]d + \epsilon_S\overline{e}\nu_L \cdot \overline{u}d$$
$$- \epsilon_P\overline{e}\nu_L \cdot \overline{u}\gamma^5 d + \epsilon_T\overline{e}\sigma_{\mu\nu}\nu_L \cdot \overline{u}\sigma^{\mu\nu}(1 - \gamma^5)d$$

H.Behrens & W.Bühring formalism

Nuclear β-decay involves complex **many-body interactions** → Impulse approximation each particle is treated **independently**

BUT symmetries are conserved (angular momentum conservation) → multipole decomposition
The many-body physics (complex nuclear structure) are encoded into the form factor

Multipole decomposition : $\left\langle f \middle| H_0^{\text{hadr}} L_{\text{lep}}^0 \middle| i \right\rangle \propto \sum_{L,M} j_L(qR) Y_M^L(\hat{q}) F_L(q^2)$

→ « old school » model independent with $\mathrm{q} = \mathrm{p_f} - \mathrm{p_i}$
→ Lepton & Hadronic part are treated independently
→ QCD adds extra terms in weak vertex: induced currents
→ Relation between form factor and nuclear matrix element(s)

One-body transition density

Behrens & Bühring form factor notation $F_{KLs}^N(k_e, m, n, \rho) = \sum \mathcal{M}_{KLs}^N(k_e, m, n, \rho) \times \left\langle \zeta_f J_f \middle|\middle| [c_\alpha^\dagger c_\beta]_\lambda \middle|\middle| \zeta_i J_i \right\rangle$

Relativistic electron parameter

Nuclear matrix elements ${}^{V/A}\mathcal{M}^{N}_{KLs}$

Hadronic part :

2 types : - non-relatisvistic → coupling LNWFs

- relativistic → coupling LNWF with SNWF (using CVC theory)

$$\Psi_{nlm} = \begin{pmatrix} g_{nl}(r)\Omega_{lm}(\hat{r}) \\ i f_{nl}(r)\Omega_{-lm}(\hat{r}) \end{pmatrix}$$

$$\begin{pmatrix} m+V & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -m+V \end{pmatrix} \Psi_{nlm} = E\Psi_{nlm}$$

$$f_{nl}(r) = \frac{1}{2M}\left[\frac{d}{dr} + \frac{l+1}{r}\right] g_{nl}(r)$$

→ Non-relativistic approximation

$$E \approx m$$

Nuclear matrix elements ${}^{V/A}\mathcal{M}^{N}_{KLs}$

Leptonic part :

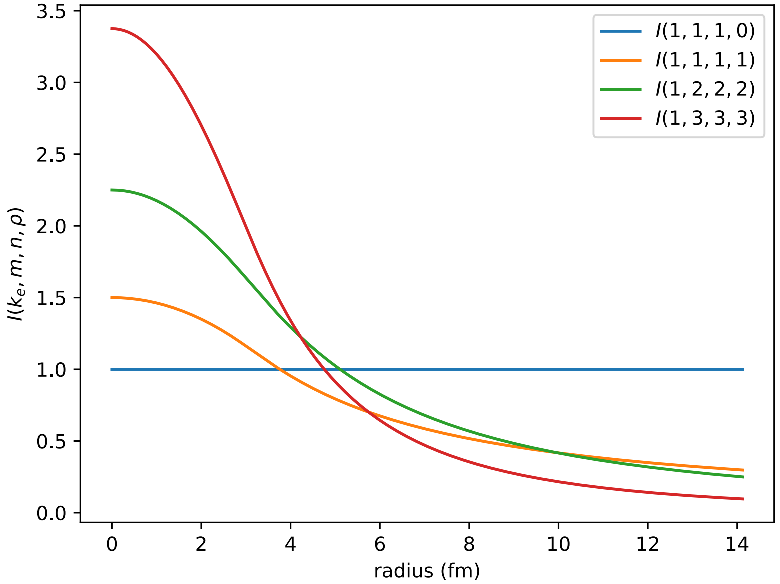
$$\begin{Bmatrix} f_\kappa(r) \\ g_\kappa(r) \end{Bmatrix} = \alpha_\kappa \frac{(pr)^{(k-1)}}{2k-1} \sum_{n=0}^{\infty} \begin{Bmatrix} a_{\kappa n} \\ b_{\kappa n} \end{Bmatrix} \left(\frac{r}{R}\right)^n$$

→ Exact solution (for central potential)

Correction on the electron wave central due to central potential

$$I(k, m, n, \rho; r)$$

$$(m_e R), (ER), (\alpha Z)$$



For uniform charge distribution

$$U(r) = \begin{cases} \frac{3}{2} - \frac{1}{2}\left(\frac{r}{R}\right)^2 & \text{if } r \leq R \\ \frac{R}{r} & \text{if } r > R \end{cases}$$

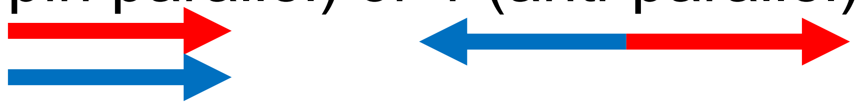
Form Factors

$$^{V/A}\mathcal{F}^{N}_{KLs} = \sum {}^{V/A}\mathcal{M}^{N}_{KLs} \times \mathrm{OBTD}$$

Nuclear matrix elements

KSHELL
or
NUSHELLX

N : form factor order
K : total angular momentum (rank)
L : orbital multipolarity
s : 0 (spin parallel) or 1 (anti-parallel)



The form factors depend on **induced currents**

$$g_M \approx 0.001$$

$$g_S = g_S = g_T = 0$$

NME and OBTD in the **same NWF basis** (HO, WS, …)
Phase convention (Condon-Shortley or Biedenharm-Rose)

Beta/EC Shape Factor

Theoritical Shape Factor :

Coulomb functions

$$C(E) = \sum_{k_e, k_\nu, K \geq 1} \lambda_{k_e} \left[M_K(k_e,k_\nu)^2 + m_K(k_e,k_\nu)^2 - \frac{2\gamma_{k_e}\mu_{k_e}}{k_e E} M_K(k_e,k_\nu) m_K(k_e,k_\nu) \right]$$
$$+\delta_{\Delta J,0} \left[M_0(1,1)^2 + m_0(1,1)^2 - \frac{2\gamma_{k_e}\mu_{k_e}}{k_e E} M_0(1,1) m_0(1,1) \right]$$

Beta

$$C_x = \sum_{k_e, k_\nu, K \geq 1} \lambda_{k_e} \left[M_K(k_e,k_\nu)^2 + m_K(k_e,k_\nu)^2 + \frac{\kappa_x}{k_x} M_K(k_e,k_\nu) m_K(k_e,k_\nu) \right]$$
$$+\delta_{\Delta J,0} \left[M_0(1,1)^2 + m_0(1,1)^2 - \frac{\kappa_x}{k_x} M_0(1,1) m_0(1,1) \right]$$

EC

$(\pi_i \pi_f) = (-1)^K$ $\quad {}^V F^N_{KK0}$ $\quad {}^A F^N_{KK1}$ $\quad {}^V F^N_{KK-11}$ $\quad {}^V F^N_{KK+11}$

$(\pi_i \pi_f) = (-1)^{K+1}$ $\quad {}^A F^N_{KK0}$ $\quad {}^V F^N_{KK1}$ $\quad {}^A F^N_{KK-11}$ $\quad {}^A F^N_{KK+11}$

Conserved Vector Current (CVC) hypothesis