Applications of *ab initio* nuclear theory to electroweak processes in atomic nuclei

Irène Joliot-Curie

Laboratoire de Physique des 2 Infinis

NGUYEN Hoang Dang Khoa

CNRS/IJCLab — Université Paris-Saclay

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Laboratoire de Physique des 2 Infinis Irène Joliot-Curie
IJCLab - **UMR9012** - Bât. 100 - 15 rue Georges Clémenceau
91405 Orsay cedex

Outline

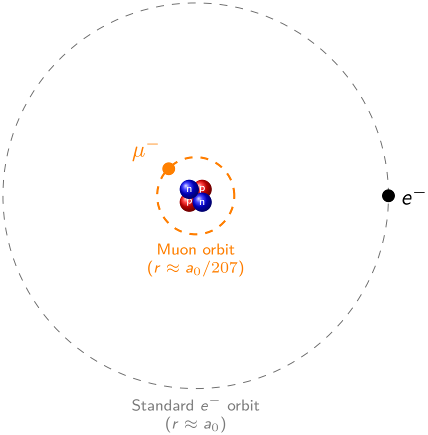
Muonic Atom

The Concept:

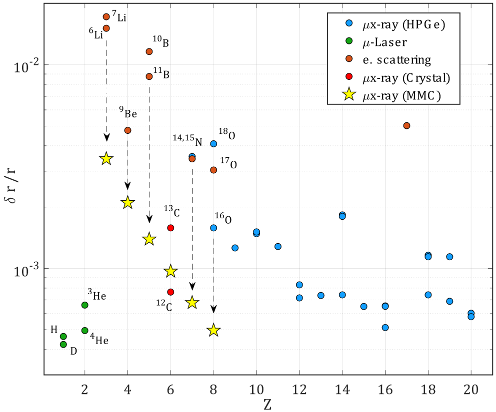
- An exotic atom where one (or more) electrons are replaced by a muon (μ^-).

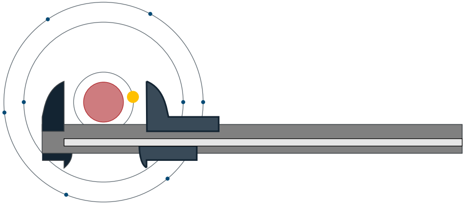
Physical Implication:

- The muon orbits roughly 200 times closer to the nucleus.
- This makes muonic atoms incredible laboratories for probing nuclear structure and testing Quantum Electrodynamics (QED).



QUARTET collaboration’s experiment



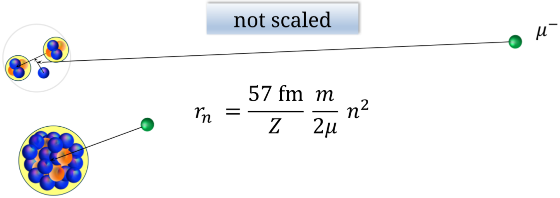


- aims to improve the measurement of absolute charge radii of light nuclei by a factor of 10

Challenges

A 2-scale problem

- Nuclear size $\approx 10^{-5}$ a.u.
- Muonic 1s Bohr radius $\approx 10^{-3}$ a.u.



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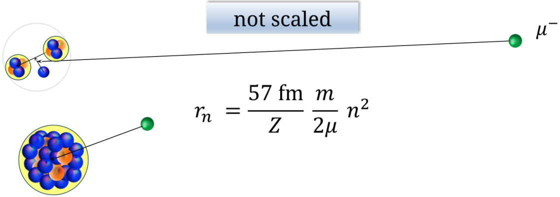

$$r_n = \frac{57\ \mathrm{fm}}{Z}\frac{m}{2\mu}n^2$$

Gauss' law must only be used with caution due to finite nuclear size
$\Longrightarrow$ An accurate charge distribution extending up to several tens of fm is needed

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...with Continuum!

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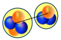
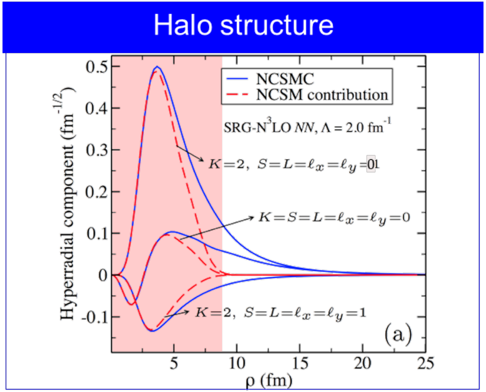
- Drawback of NCSM: HO basis has an inherent Gaussian cutoff
- No-Core Shell Model with Continuum (NCSMC) provides a natural solution

$$\left|\Psi^{(A)}_{NCSMC}\right\rangle = \sum_{\lambda} c_{\lambda} \left|A \lambda J^{\pi} T\right\rangle + \sum_{\nu} \int \mathrm{d}r \, \frac{\gamma_{\nu}(r)}{r} \hat{\mathcal{A}}_{\nu} \left|\Phi_{\nu}\right\rangle$$

- $\left|A \lambda J^{\pi} T\right\rangle = \Bigg|$  $\Bigg\rangle$: NCSM basis
- $\left|\Phi_{\nu}\right\rangle = \Bigg|$  $\Bigg\rangle$: binary-cluster basis

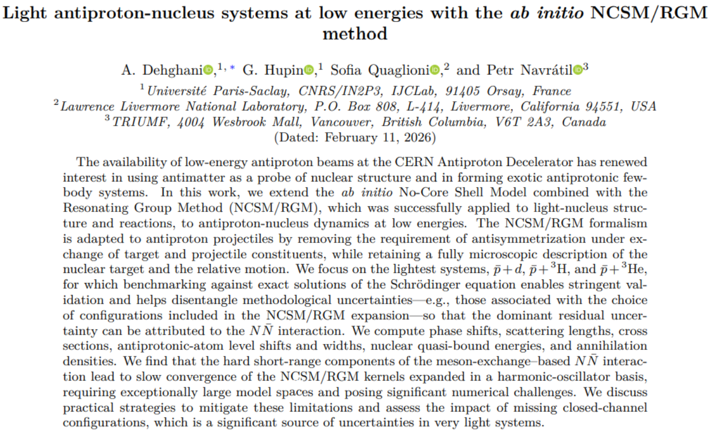
Example: ^{6}He

A $N_{\max} = 12$ model space for NCSM corresponds to $r \approx 8$ fm HO "spatial box".
NCSMC extend much further providing correct asymptotics and internal part of the wave function.



C. Romero-Redondo et al. (2016). Phys. Rev. Lett. 117

Example: light antiproton-nucleus system



Light antiproton-nucleus systems at low energies with the *ab initio* NCSM/RGM method

A. Dehghani,[1,*] G. Hupin,[1] Sofia Quaglioni,[2] and Petr Navrátil[3]

[1] *Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France*
[2] *Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, California 94551, USA*
[3] *TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, V6T 2A3, Canada*

(Dated: February 11, 2026)

The availability of low-energy antiproton beams at the CERN Antiproton Decelerator has renewed interest in using antimatter as a probe of nuclear structure and in forming exotic antiprotonic few-body systems. In this work, we extend the *ab initio* No-Core Shell Model combined with the Resonating Group Method (NCSM/RGM), which was successfully applied to light-nucleus structure and reactions, to antiproton-nucleus dynamics at low energies. The NCSM/RGM formalism is adapted to antiproton projectiles by removing the requirement of antisymmetrization under exchange of target and projectile constituents, while retaining a fully microscopic description of the nuclear target and the relative motion. We focus on the lightest systems, $\bar{p}+d$, $\bar{p}+{}^3\mathrm{H}$, and $\bar{p}+{}^3\mathrm{He}$, for which benchmarking against exact solutions of the Schrödinger equation enables stringent validation and helps disentangle methodological uncertainties—e.g., those associated with the choice of configurations included in the NCSM/RGM expansion—so that the dominant residual uncertainty can be attributed to the $N\bar{N}$ interaction. We compute phase shifts, scattering lengths, cross sections, antiprotonic-atom level shifts and widths, nuclear quasi-bound energies, and annihilation densities. We find that the hard short-range components of the meson-exchange–based $N\bar{N}$ interaction lead to slow convergence of the NCSM/RGM kernels expanded in a harmonic-oscillator basis, requiring exceptionally large model spaces and posing significant numerical challenges. We discuss practical strategies to mitigate these limitations and assess the impact of missing closed-channel configurations, which is a significant source of uncertainties in very light systems.

We can even compute atomic states for an antiprotonic system with 400 fm of channel radius.

Nuclear AIMs and Atomic AIMs

- General matrix element of type

$$\left\langle \Psi_f^{(A)} \middle| \hat{O} \middle| \Psi_i^{(A)} \right\rangle$$

- Nuclear charge density operator

$$\hat{\rho}(\boldsymbol{r}) = e \sum_{i=1}^{A} \left(\frac{1}{2} - t_{i3} \right) \delta(\boldsymbol{r} - \boldsymbol{r}_i)$$

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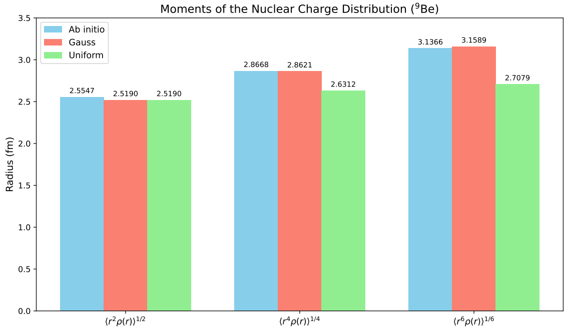
- **Objective**: Collaboration with experts in Multi-Configuration Dirac-Fock methods [Cre: Paul Indelicato (LKB)] to use these nuclear calculations of charge and magnetic density as input

Example: ^{9}Be muonic

Preliminary test calculations from P. Indelicato. The nuclear size effect is included in the Dirac equation potential.

1s binding energy (eV)

Ab initio	Gauss	Uniform
-4**3**277.235	-4**4**377.730	-4**4**377.430



Conclusion

Summary:

- Muonic atom performs very well in probing nuclear structure
- NCSMC provides solutions that extend to large distances
- ...and can be used as MCDF input to achieve hyperfine splitting predictions

Outlook:

- Completing the charge and magnetic density calculation

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Thank you for your attention!