Lifetime of a muon bound to a light nucleus

WNPPC-2026 Conference

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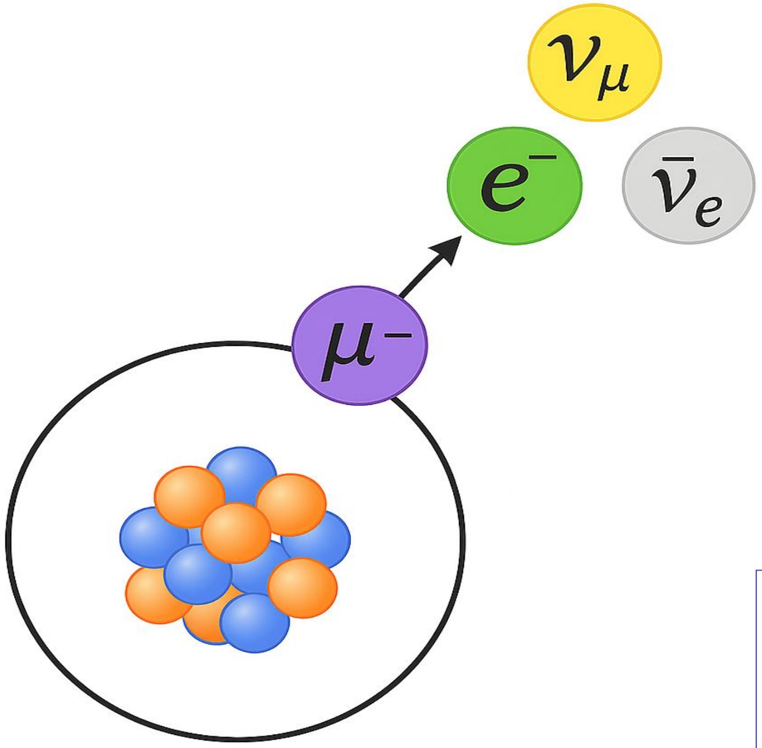

UNIVERSITY OF ALBERTA

CINP ICPN
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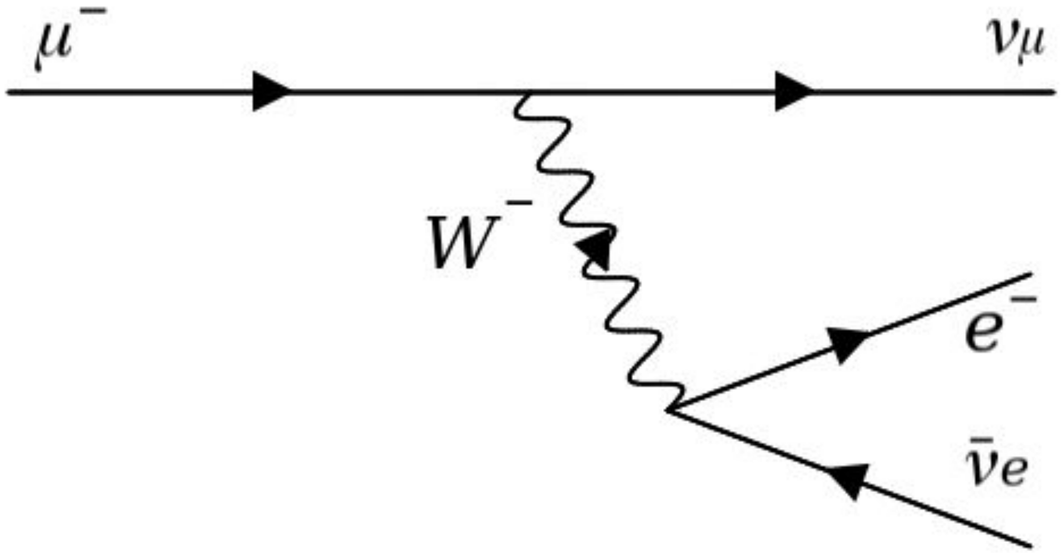
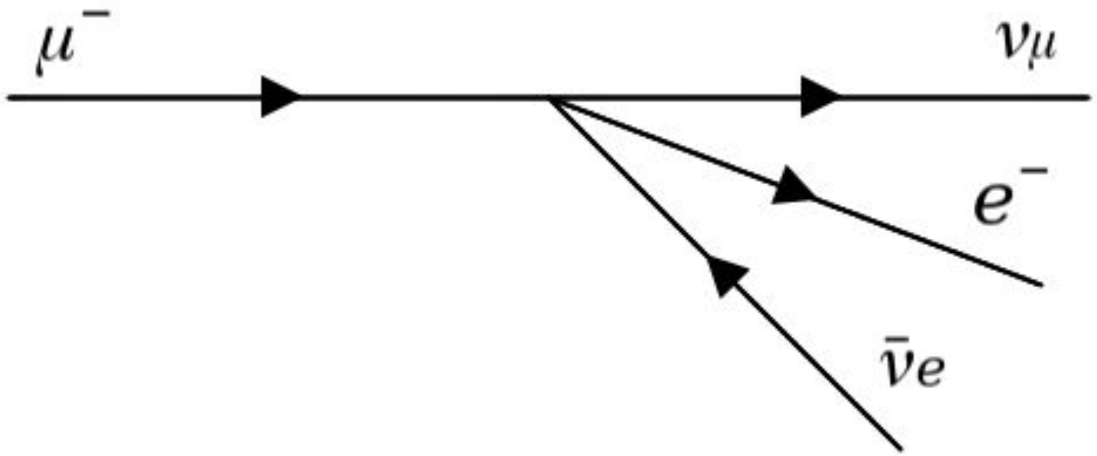
Contents

Free muon decay

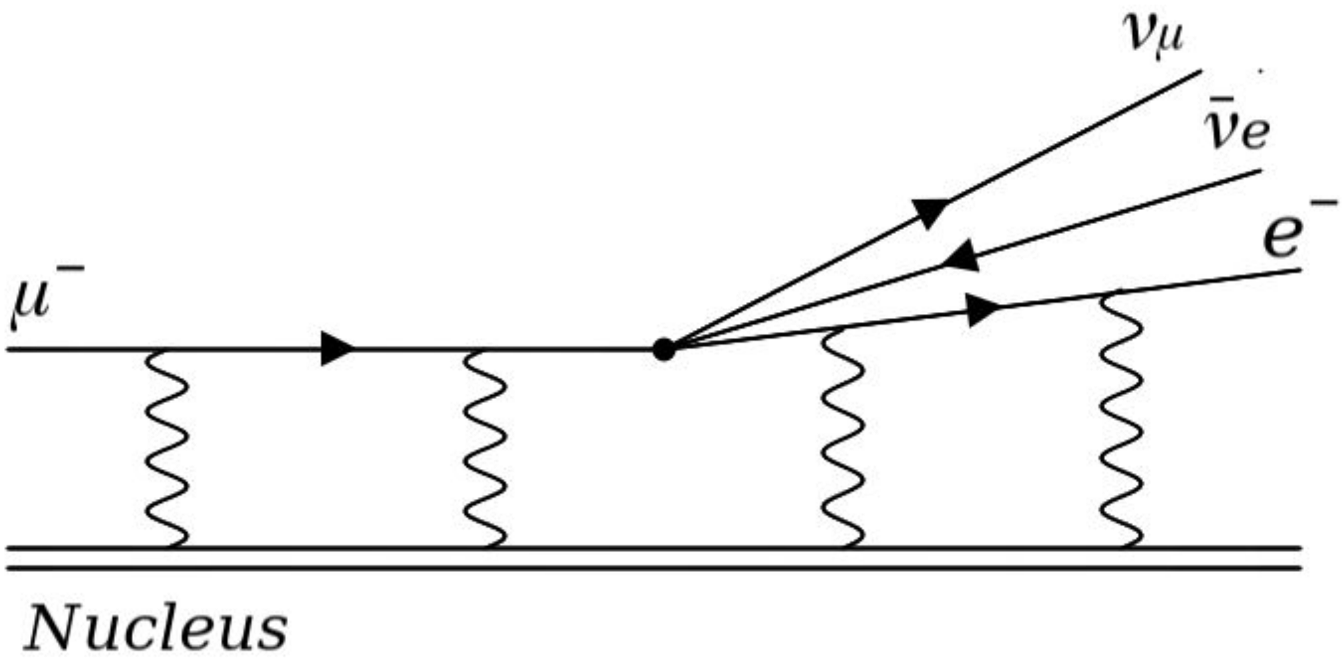
$$m_\mu \approx 200 m_e$$





$$\mu^- \longrightarrow e^- \bar{\nu}_e \nu_\mu$$

Bound muon decay



$$(N\mu^-) \longrightarrow Ne^-\bar{\nu}_e\nu_\mu$$

$$N = N(Z)$$

$$1 - \frac{\Gamma(Z)}{\Gamma(0)} =?$$

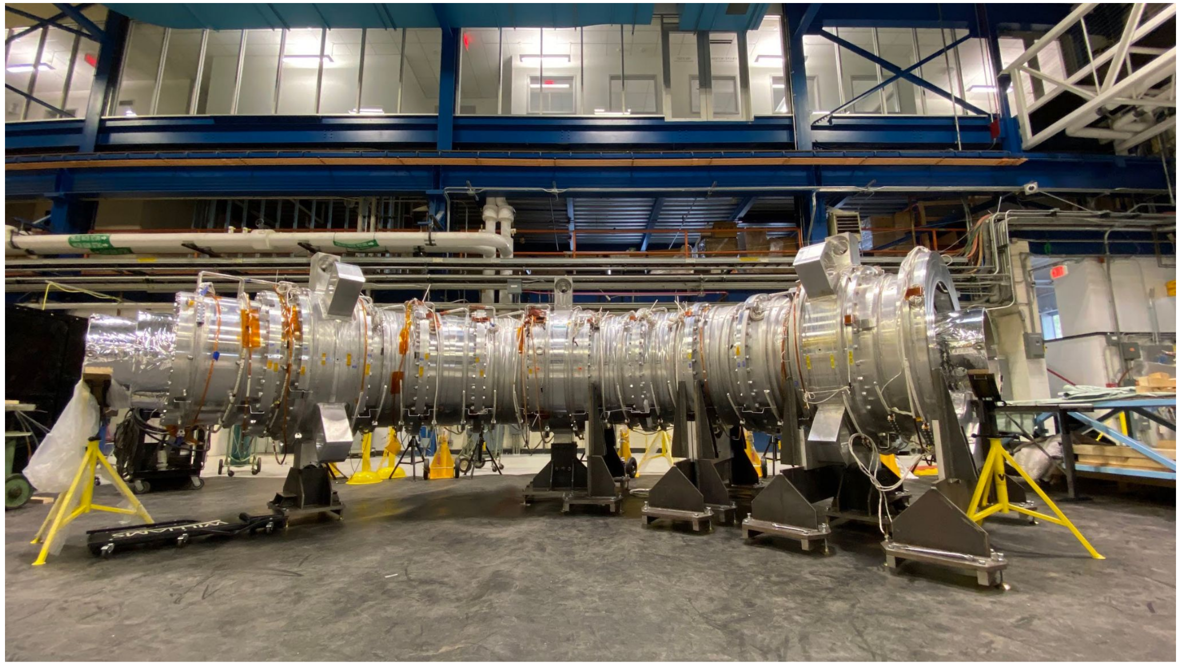
$$\Gamma \sim \frac{1}{\tau}$$

Current and upcoming experiments

Looking for the neutrinoless decay

$$(N\mu^-) \longrightarrow Ne^-$$

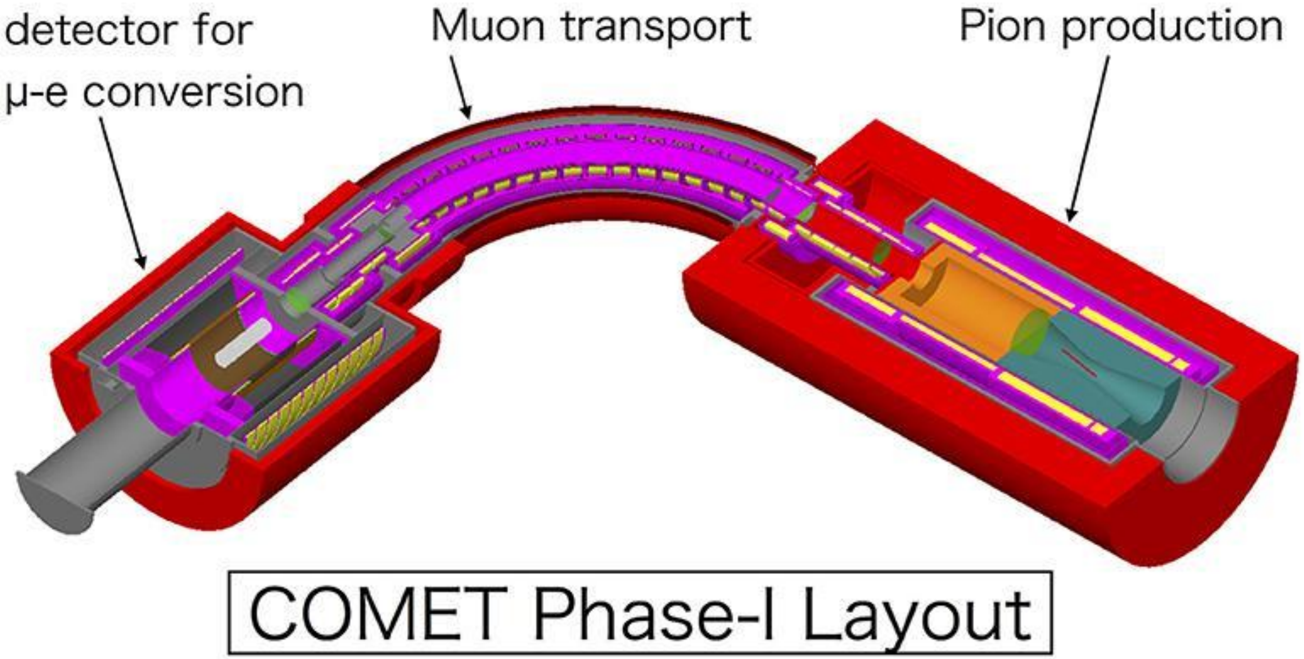
Mu2e, Fermilab



Serves as a background

$$(N\mu^-) \longrightarrow Ne^-\bar{\nu}_e\nu_\mu$$

COMET, J-PARC

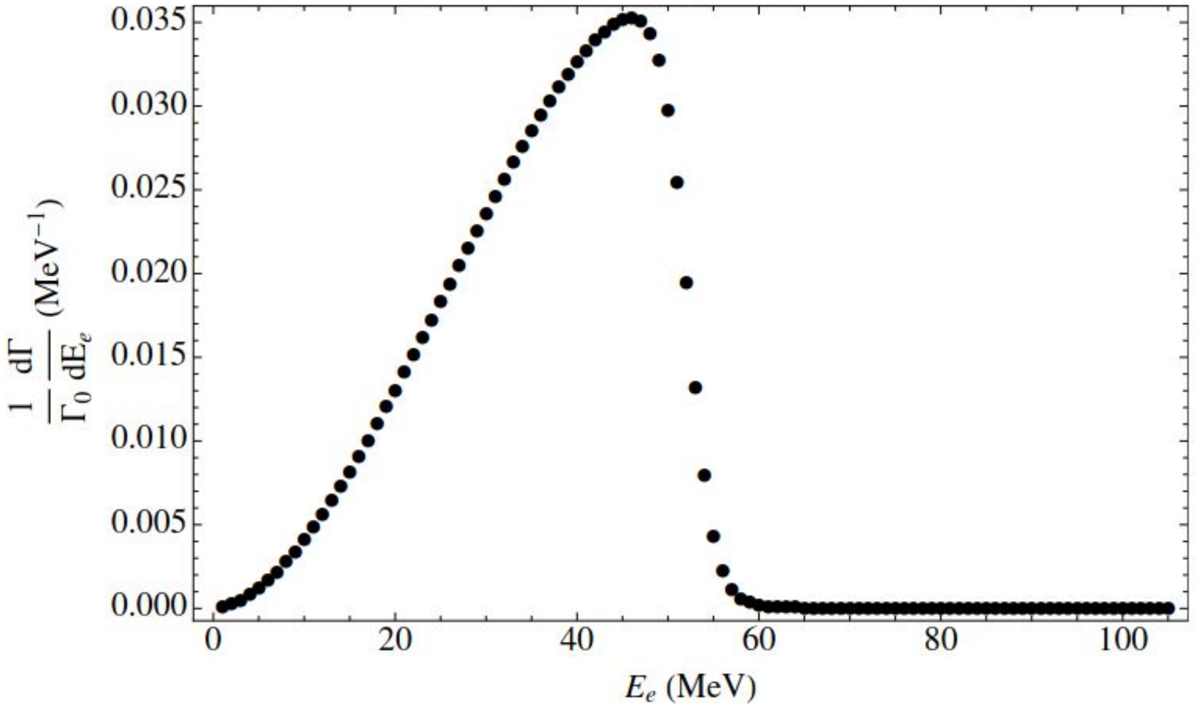


Current experiments

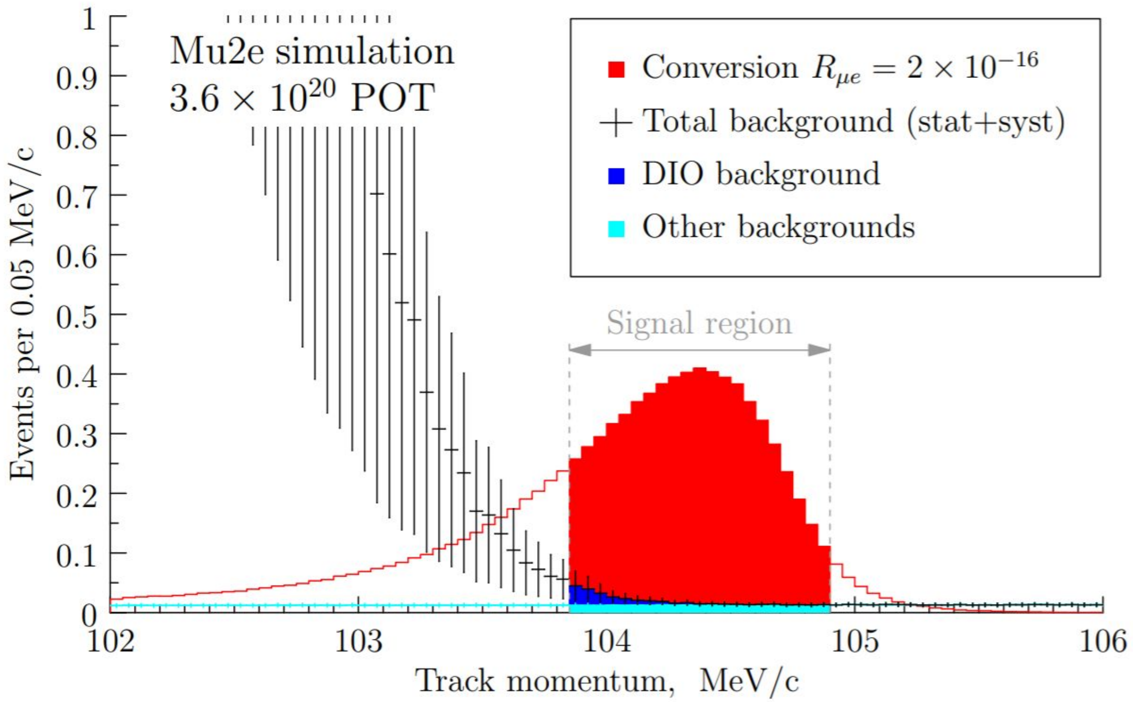
$$(N\mu^-) \longrightarrow Ne^-\bar{\nu}_e\nu_\mu$$

$$(N\mu^-) \longrightarrow Ne^-$$

Spectrum of background electrons



Peak in the tail region



$$E_e \approx E_\mu$$

Previous endeavors

What modifies the decay rate?

❶ The relativistic time dilation: $\tau_{\text{free}}/\tau_{\text{bound}} = \sqrt{1-(\alpha Z)^2}$ $\left\langle \frac{v}{c} \right\rangle = \alpha Z$ $\alpha \approx \frac{1}{137}$

❷ The phase space reduction: $E_\mu = m_\mu - E_b < m_\mu$

❸ The electron wave function enhancement: $\int d^3 r \phi_\mu^\dagger \hat{O} \phi_e$

Theory - Uberall (1960)

$$1 - \Gamma(Z)/\Gamma(0) = \frac{1}{2}(\alpha Z)^2 + \ldots$$

$$1 - \Gamma(8)/\Gamma(0) \approx 0.002$$

Numerical - Watanabe, et. al. (1993)

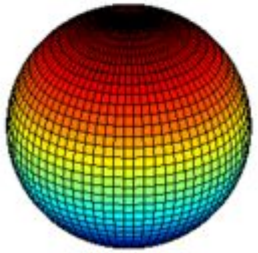
$$1 - \Gamma(8)/\Gamma(0) \approx 0.006$$

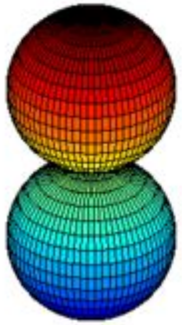
Mathematical background

Dirac Equation: $[-i\vec{\alpha}\cdot\vec{\nabla}+\beta m_{e,\mu}+V(r)]\phi_{e,\mu} = E_{e,\mu}\phi_{e,\mu}$ $\quad V(r) = -\frac{\alpha Z}{r}$

Solution for electron: $\phi_e = \sum_{\kappa} \begin{pmatrix} g_\kappa(r)\Omega_\kappa(\theta,\phi) \\ f_\kappa(r)\Omega_{-\kappa}(\theta,\phi) \end{pmatrix}$

Plane wave decomposition: $e^{i\vec{k}\cdot\vec{r}} \sim \sum_l i^l j_l(kr) Y_l^*(\hat{r}) Y_l(\hat{k})$

$Y_0 \sim 1$ 

$Y_1 \sim \cos\theta$ 

Mathematical background

Decay width:

$$\Gamma \sim \int d\Phi_3 |\mathcal{M}|^2$$

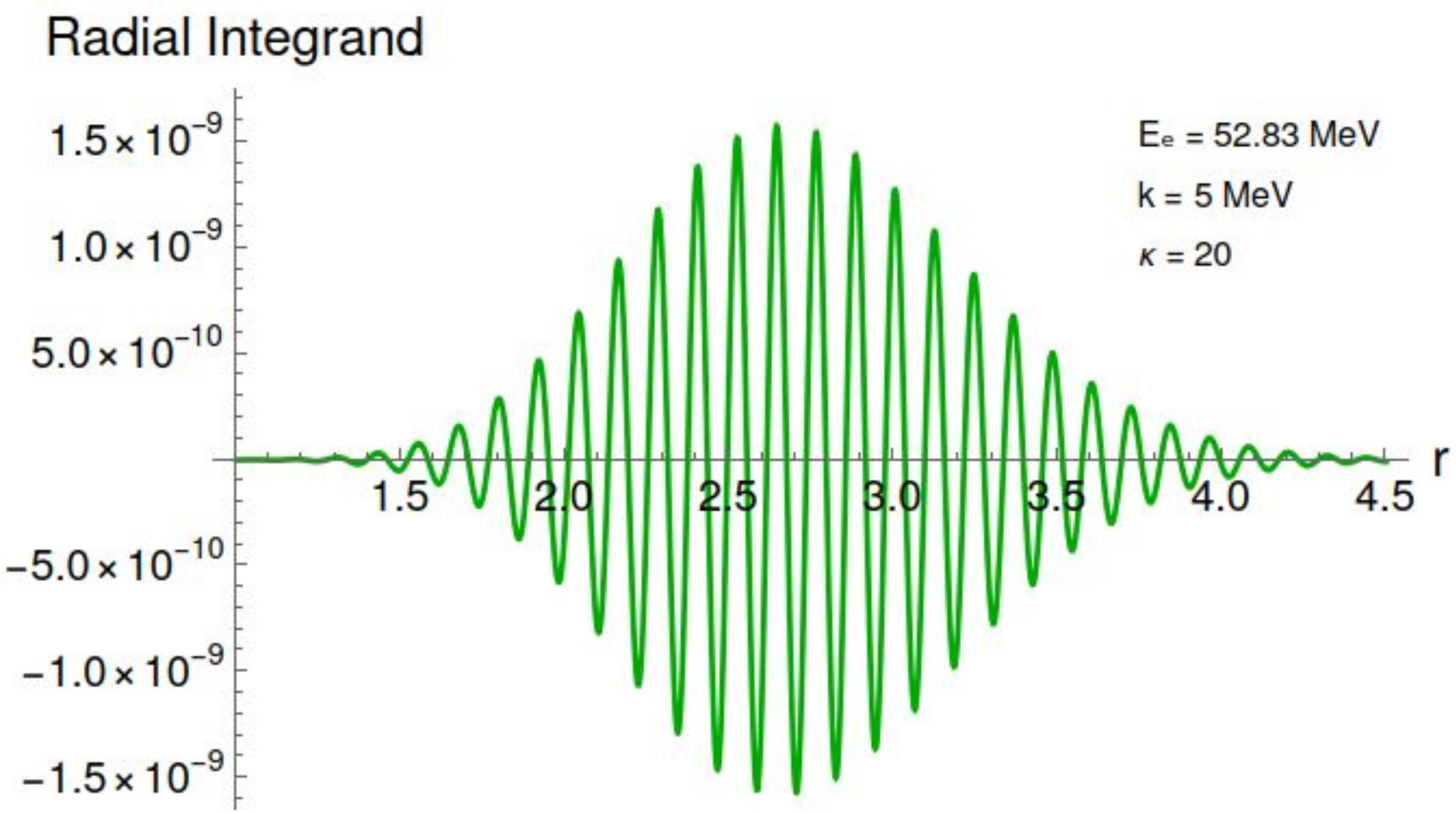
Matrix element:

$$\mathcal{M} \sim \int d^3 r \phi_\mu^\dagger \hat{O} \phi_e$$

Partial summation:

$$\Gamma = \sum_\kappa \Gamma_\kappa$$

$$\phi_e = \sum_\kappa \begin{pmatrix} g_\kappa(r)\Omega_\kappa(\theta,\phi) \\ f_\kappa(r)\Omega_{-\kappa}(\theta,\phi) \end{pmatrix}$$

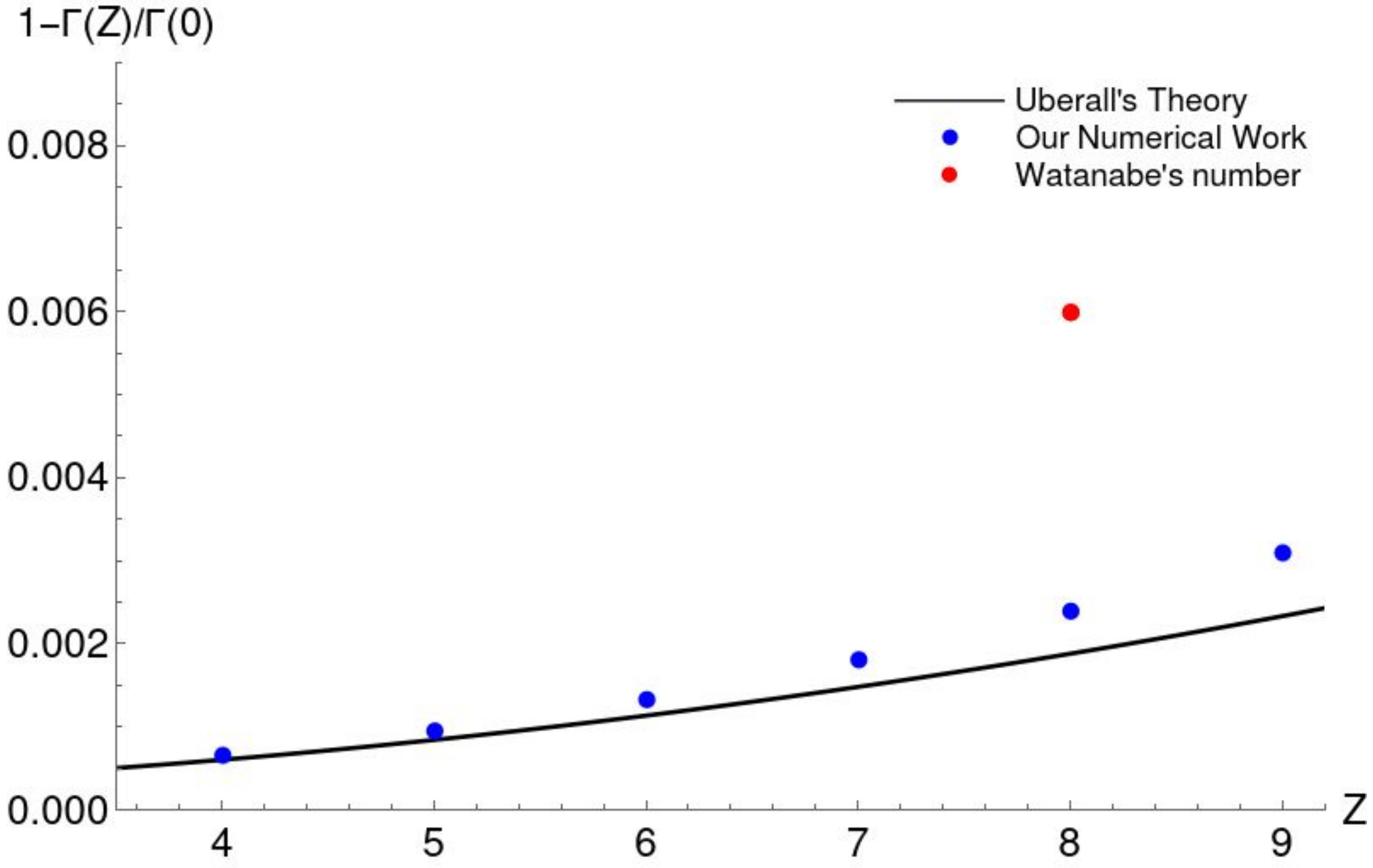


Numerical results

	Theory, Uberall (1960)	Numerical, Watanabe, et. al. (1993)	Current work
$1-\Gamma(8)/\Gamma(0)$	0.0019	0.006	0.0024
Truncation		$\kappa = 29$	$\kappa = 59$

1. The difference between our numerical result and Uberall's theoretical number can be attributed to higher order terms in (αZ) expansion.
2. Watanabe's numerical number is wrong due to incorrect truncation.

Numerical results



Z	$1 - \Gamma(Z)/\Gamma(0)$
4	0.0007
5	0.0010
6	0.0013
7	0.0018
8	0.0024
9	0.0031

[1] Kaygorodov et. al. arXiv:2506.02416 (2025)

[2] A. Czarnecki, A.O. Davydov, M.Y.Kaygorodov arXiv:2512.23023 (2025)

Summary

- The theoretical calculation done by Uberall is correct, meaning that the QED is a valid theory for high precision atomic calculations.
- Watanabe's numerical number is wrong due to incorrect truncation.
- The difference between our numerical result and Uberall's theoretical number can be attributed to higher order terms in $(\alpha \mathbf{Z})$ expansion.
- Correctness of the tail evaluation gives a solid base for conversion searches