Investigating scattering rates in Thermal Field Theory beyond the leading log

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Quark-Gluon Plasma (QGP)

- State of matter, consisting of **deconfined** quarks and gluons
- Existed during the early universe
- Can be experimentally created in Heavy Ion Collisions (at RHIC and LHC)
- **Probes** necessary to gain insights into its behaviour

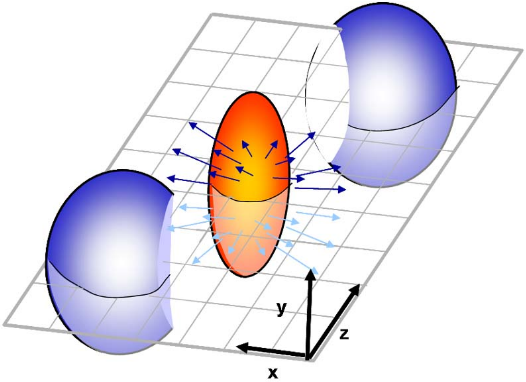
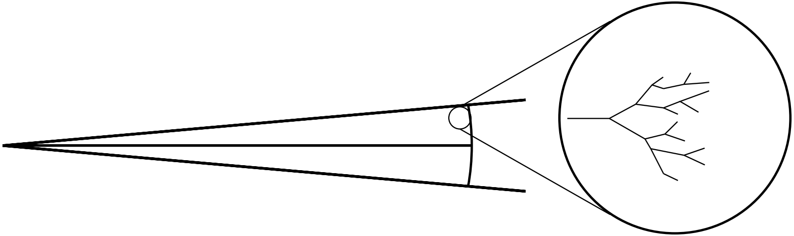


Figure: Collision of two Heavy Ions, creating a QGP

Jets

- During particle collisions, jets form: a **collimated** spray of **energetic particles**
- These jets evolve through multiple radiations (or splits) into partons of ever increasing lifetime, until partons finally hadronize
- Experimental measurements detect **hadrons** and cluster them to reconstruct the jet's collimated shower
- In collisions not producing a medium (e.g. e^+-e^-, e^--p), the jet evolution is well understood using vacuum splittings



Jet-medium interactions

- With a QGP present, **jet-medium interactions** can also decrease the jet parton's virtuality/energy and influence its evolution via scatterings (and medium-modified radiation)
- Leads to **transverse momentum broadening**, a broadening of the jet cone due to "kicks" from the medium

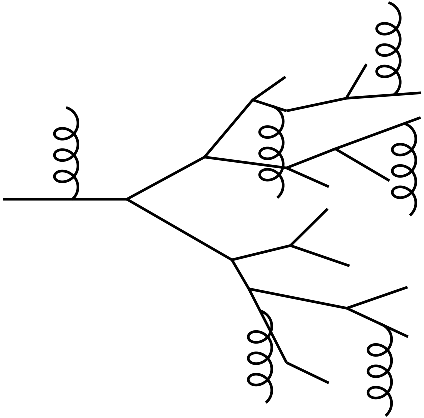
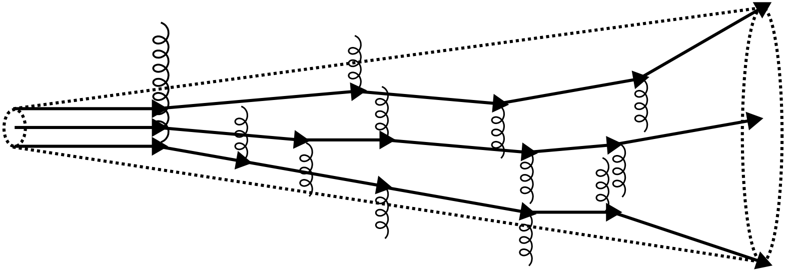


Figure: Showering of the jet parton with medium interactions

Transverse momentum broadening

- $\hat{q} := \frac{\langle q_\perp^2 \rangle}{L}$, transport-like coefficient characterizing jet broadening
- Analogous to Brownian motion, where the **spatial** diffusion coefficient is proportional to $\langle x^2 \rangle$
- $\hat{q}$ is a measure of **transverse momentum** diffusion
- Medium-specific quantity (like other QGP transport coefficients: viscosity, conductivity)



2→2-scattering

- To calculate $\hat{q}$, **microscopic physics** of the scattering of jet-medium particles is needed
- The fundamental quantity to calculate is the $2 \to 2$ **scattering rate density** $\mathrm{d}R$, which gives a number of scatterings per unit time and volume: $\mathrm{d}R = \frac{\mathrm{d}^4 N}{\mathrm{d}^4 x}$

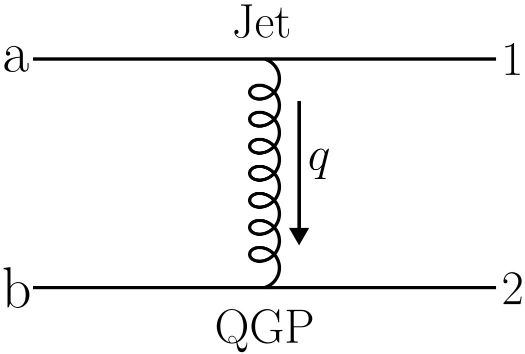


Figure: 2→2-scattering of a jet with a medium particle, mediated by a propagator of momentum q

Gluon-gluon scattering

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- This work considers gg-scattering, for now at tree-level, but other processes and diagrams work similar

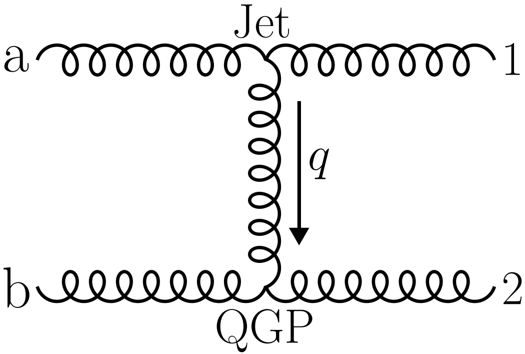


Figure: Gluon-gluon scattering of a jet with a medium particle, mediated by a propagator of momentum q

The Scattering Rate in the medium

$$\mathrm{d}R = \prod_{i=a,b,1,2} \left[\frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i} F_i(p_i) \right] \mathrm{d}^4 q \; \overline{|\mathscr{M}|^2} (2\pi)^4 \delta^{(4)} \left(p_a - p_1 - q \right) \delta^{(4)} \left(p_b - p_2 + q \right)$$

- $\overline{|\mathscr{M}|^2}$ is the matrix element, encapsulating the probability amplitude of the scattering process
- For **thermal equilibrium**, the initial medium particle obeys the Fermi-Dirac (for quarks), or Bose-Einstein (for gluons) statistics, while the final medium particle is either Fermi-suppressed or Bose-enhanced, giving

$$F_{\text{initial}}(p) = f_{\text{f/b}}(p) = \frac{e^{-\beta(u \cdot p)}}{1 \pm e^{-\beta(u \cdot p)}}$$

$$F_{\text{final}}(p) = 1 \mp f_{f/b}(p) = \frac{1}{1 \pm e^{-\beta(u \cdot p)}}$$

- The unknowns are $\beta = 1/T$, where T is the temperature (natural units were used, so $c = \hbar = k_B = 1$), and fluid flow u^μ

The Scattering Rate and $\hat{q}$ in the medium

$$\mathrm{d}R = \prod_{i=a,b,1,2} \left[\frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i} F_i(p_i) \right] \mathrm{d}^4 q \, \overline{|\mathscr{M}|^2} (2\pi)^4 \delta^{(4)} (p_a - p_1 - q) \, \delta^{(4)} (p_b - p_2 + q)$$

- $\overline{|\mathscr{M}|^2}$ is the matrix element, encapsulating the probability amplitude of the scattering process
- The F_i are **particle distributions** in thermal equilibrium
- $\hat{q}$ can be calculated from microscopic physics similar to the rate:

$$\hat{q} = \int \mathrm{d}q_\perp \, q_\perp^2 \frac{\mathrm{d}^4 R}{\mathrm{d}^3 p_a \mathrm{d}q_\perp}$$

- $q_\perp = |\vec{q}| \sin(\theta)$, with θ being the angle relative to $\vec{p}_a$
- **Leading log approximation** widely used in QGP jet physics

The leading log

- Assumes a **small energy-momentum transfer** of the propagator: $p_a \approx p_1 \Rightarrow q \sim q_\perp \ll p_a, p_1$
- Incoming jet scatters mostly in the forward direction, deposits only a small fraction of energy-momentum into the medium
- All integrals solved analytically with this approximation give the following behaviour for gluon-propagators:

$$\frac{\mathrm{d}^3 R}{\mathrm{d}^3 p_a} \sim \frac{3\alpha_S^2 T^3}{\pi^2}\left[\frac{1}{m_D^2} - \frac{1}{E_a^2} + \mathcal{O}\left(e^{-\beta E_a}\right)\right]$$

$$\hat{q} \sim \frac{3\alpha_S^2 T^3}{\pi^2}\left[\ln\left(\frac{E_a}{m_D}\right) + \mathcal{O}\left(e^{-\beta E_a}\right)\right]$$

Beyond the leading log

- **Approximations** done to reach the leading log result **not necessary**
- Integral up to the last two can be done analytically
- Functional dependence can be gauged from there by expanding distribution function:

$$\frac{d^3R}{d^3p_a} \sim \frac{9\alpha_S^2 T}{2\pi^4}\left[C_1(m_D) - \frac{\pi^2 T}{6E_a}\text{arcsinh}\left(\frac{E_a}{m_D}\right) - \frac{C_2(m_D)T}{E_a} + \mathcal{O}\left(E_a^{-2}\right)\right]$$

$$\hat{q} \sim \frac{9\alpha_S^2 T^3}{2\pi^4}\left[4\zeta(3)\text{arcsinh}\left(\frac{E_a}{m_D}\right) + C_3(m_D) - \frac{C_4(m_D)T}{E_a}\text{arcsinh}\left(\frac{E_a}{m_D}\right) + \mathcal{O}\left(E_a^{-1}\right)\right]$$

- **Differences** for both quantities in their **functional dependence**
- $\hat{q}$ fares better at leading log order, but the leading order terms of the rate are missed

Graphical results - Rate

- Leading log approximation plateaus much earlier than the full rate
- Functional differences best seen by rescaling the leading log to match in the UV and plot the relative difference of full rate and approximation

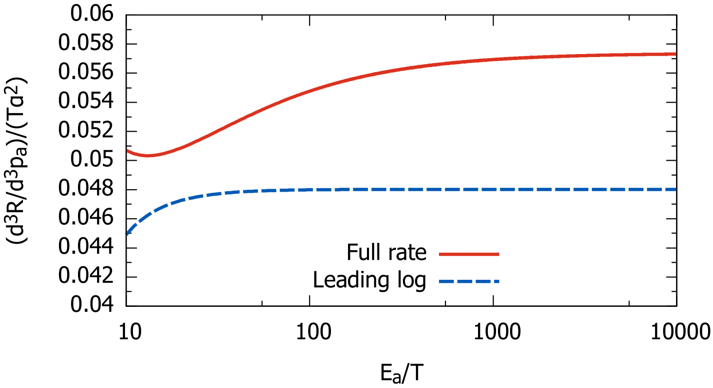


Figure: Comparing the full rate with the leading log

Graphical results - Rate

- Relative difference of full rate $\mathrm{d}R$ and rescaled leading log $\mathrm{d}R^*$
- $\mathrm{d}R^* = \mathrm{d}R_{LL} * \frac{\mathrm{d}R(10^4)}{\mathrm{d}R_{LL}(10^4)}$
- Discrepancy in functional shape easily seen, diverges towards the infrared
- Up to 11% difference, doesn't fall below 1% until late into the UV

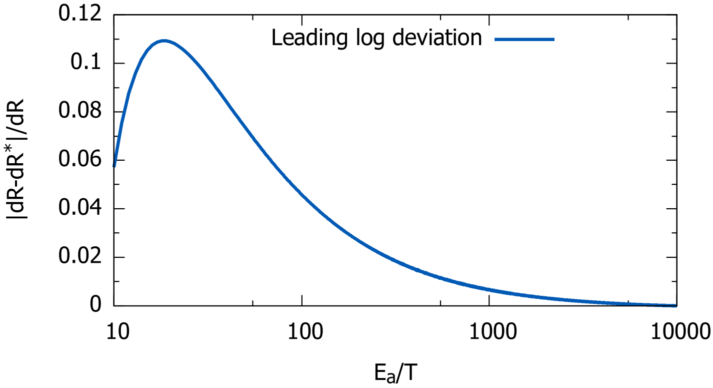


Figure: Difference of the functional shape of full rate and leading log

Graphical results - $\hat{q}$

- Leading log works better for $\hat{q}$ than the rate
- Subtle difference with argument and overall scale of logarithm (shown on log-scale as intercept and slope)
- Again best to rescale leading log and compare the two

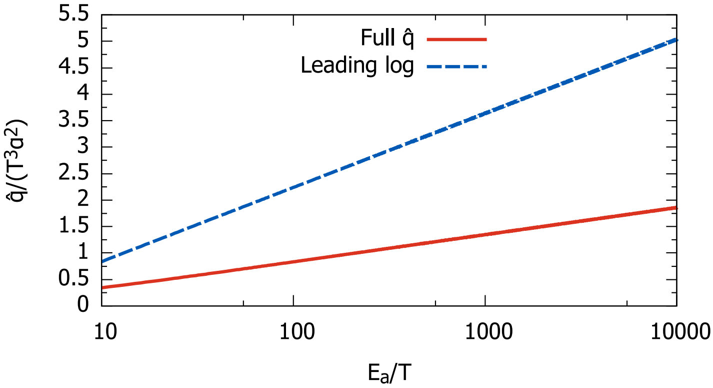


Figure: Comparing the full $\hat{q}$ with the leading log

Graphical results - $\hat{q}$

- Relative difference of full $\hat{q}$ and rescaled leading log $\hat{q}^*$
- $\hat{q}^* = \hat{q}_{LL} * \frac{\hat{q}(10^4)}{\hat{q}_{LL}(10^4)}$
- Discrepancy again diverges towards the infrared, but falls off faster than for the rate (reaches 1% earlier)
- Remaining difference due to constant shift/argument inside the log

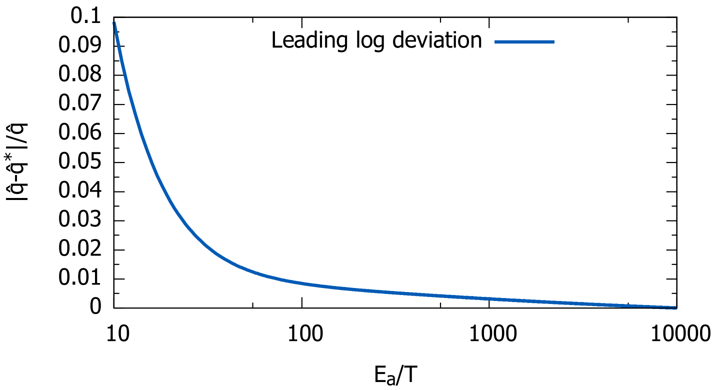


Figure: Difference of the functional shape of full $\hat{q}$ and leading log

Bayesian analysis

- **Bayesian analyses** model in-medium jet propagation, take model parameter to simulate data
- Bayes theorem then gives **probability distribution** to put constraints on model parameter (i.e. $\hat{q}$)
- Forcing the leading log onto the data leads to **biased** constraints
- Not accounting for this **theoretical uncertainty**, the error bands become unreliable

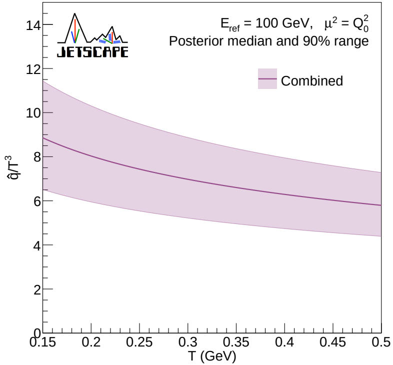


Figure: Bayesian constraints on $\hat{q}$ from JETSCAPE [Phys. Rev. C 111, 054913]

Summary and outlook

Results:

- **Leading log** for gg-scattering found to **not accurately** describe the scattering rate
- $\hat{q}$ fares better, but subtle differences found
- Current **Bayesian analyses** of $\hat{q}$ do not include these terms as **theoretical uncertainties**, yielding an imprecise uncertainty on $\hat{q}$

Further improvements:

- **Other processes**: since leading log calculations depend mostly on the propagator's form, for quark propagator diagrams results are expected to change
- Instead of the naive cutoff, actual **resummations** have to be taken into account in the IR region (Hard Thermal Loops)
- **Mass and viscous corrections**
- **NLO** contributions

Section 1

Backup Slides

Bayesian analysis of nuclear medium properties

- Large scale Bayesian analyses model jet propagation after the initial collision
- $\hat{q}$ and the rate $\frac{d^3R}{d^3p_a}$ are used as input to the model to simulate the nature of jet-medium interactions
- Bayesian analysis extract the posterior probability distribution giving constraints on transport coefficients
- Uncertainties in the theory have started to be accounted for (Bayesian model averaging, theory uncertainty via Gaussian process), but leave room for theoretical improvement

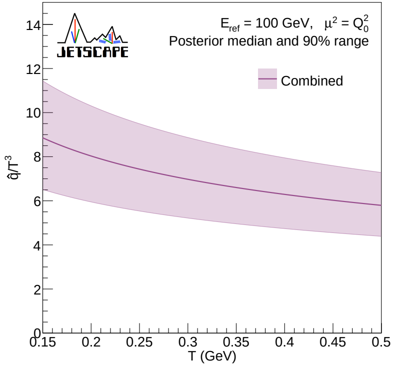
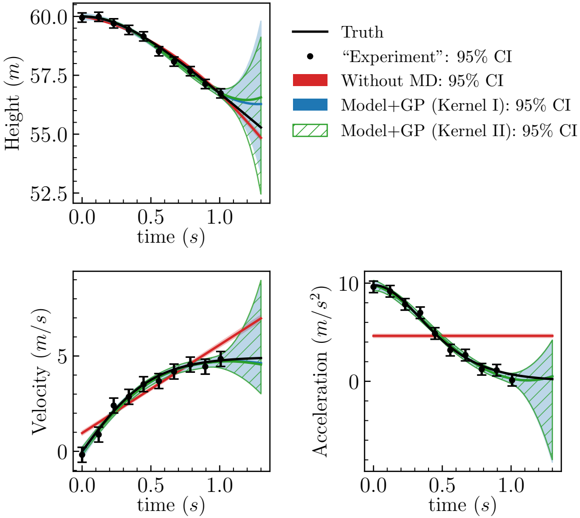


Figure: Bayesian constraints on $\hat{q}$ from JETSCAPE [Phys. Rev. C 111, 054913]

Theoretical uncertainty through Gaussian process - a simple example

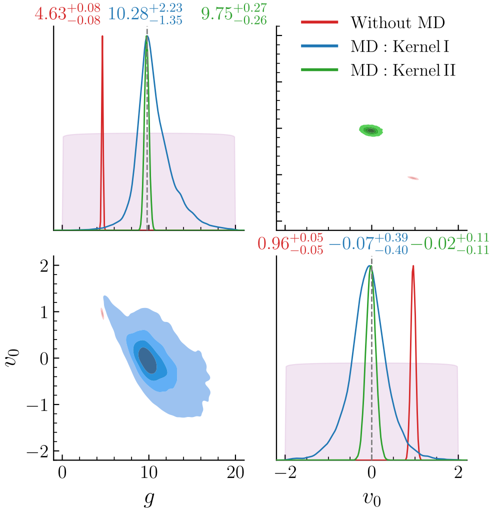
- Simple example of a Bayesian analysis, including theory uncertainty for a free-falling ball with air drag, from [arXiv:2504.13144]
- For a model not incorporating this friction term, predictions of observables can be quite off
- Can be remedied by including a Model Discrepancy Gaussian Process (GP) into the Bayesian analysis



Theoretical uncertainty through Gaussian process - a simple example

- Observables as function of input space are modeled as $y(x_i) = \zeta(x_i) + \epsilon_i = \eta(x_i, \vec{\theta}) + \delta(x_i) + \epsilon_i$ with the true value ζ, experimental uncertainty ϵ, model η and model discrepancy δ, and model parameters $\vec{\theta}$
- δ is given as a Gaussian process whose covariance kernel depends on the assumed model discrepancy
- Simple kernel in this example: $\sim (x_i x_j)^r \exp\left(-\frac{|x_i - x_j|^2}{l^2}\right)$

Inference with height, velocity, acceleration



Bayesian re-analysis

- Need to account for theory uncertainty in the rate and in $\hat{q}$ owing to truncations in the systematic expansion (e.g. leading-log vs leading order, NLO vs LO and so on)
- This is best done in the ultraviolet regime of QCD, where perturbation theory works best
- Once theory corrections are obtained, they can serve as theoretical systematic uncertainties to previous Bayesian analysis that didn't include these theory corrections

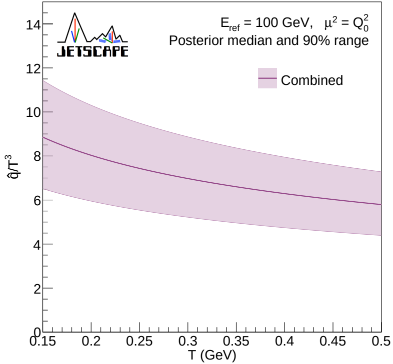


Figure: Bayesian constraints on $\hat{q}$ from JETSCAPE [Phys. Rev. C 111, 054913]

Graphical results - Rate

- Option 1: Rescale approximation via $dR^* = dR_{\text{app}} \cdot \frac{A}{B}$ for $A := dR(10^4)$, $B := dR_{\text{app}}(10^4)$

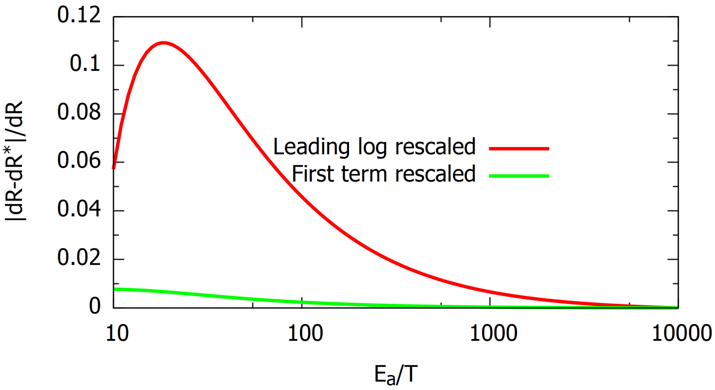


Figure: Comparing the difference of the full rate and two approximations

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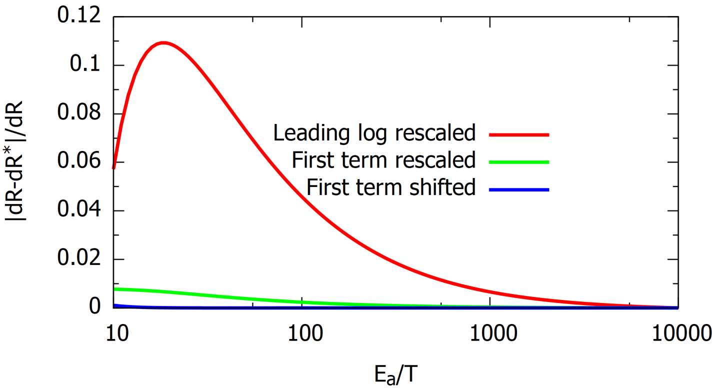
- Option 2: Shift approximation via $dR^* = dR_{\text{app}} + (A - B)$



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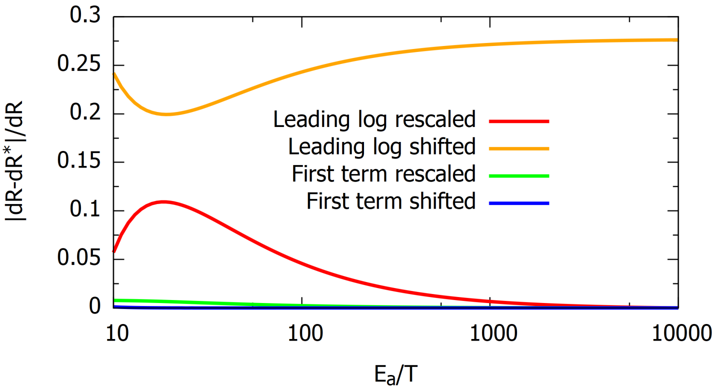
- Option 2: Shift approximation via $dR^* = dR_{\text{app}} + (A - B)$
- Rescaling works better for leading log
- Shifting more in line with full expansion, as next terms will mostly be constants



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Graphical results - $\hat{q}$

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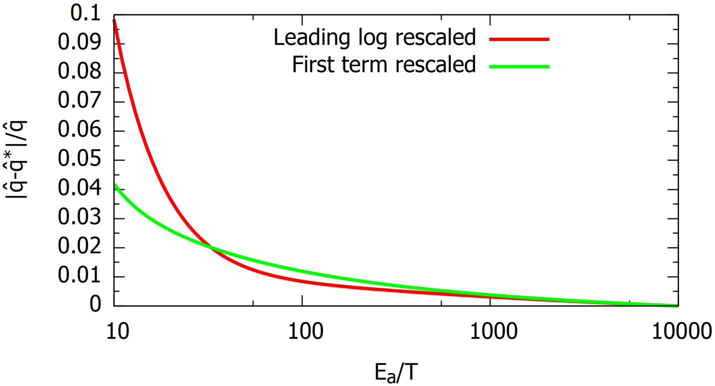

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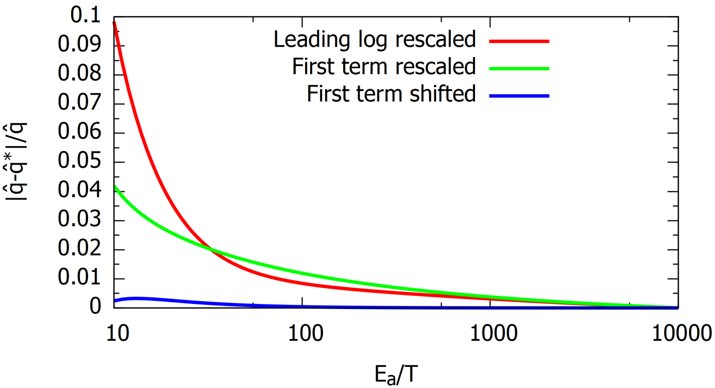

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Graphical results - $\hat{q}$

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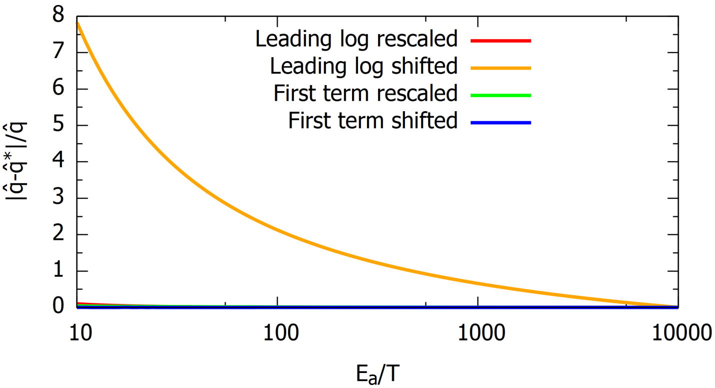
- Option 2: Shift approximation via $\hat{q}^* = \hat{q}_{\text{app}} + (A - B)$
- Subtle difference in argument and overall factor of leading log and log from the first term of full expansion



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