

Generalised King linearity and new physics searches with isotope shifts

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Outline

- ◆ Isotope shift in atomic spectra
- ◆ King plot linearity
- ◆ Dark bosons and King plot nonlinearity
- ◆ Experiments: calcium and ytterbium ions
- ◆ Spurions and the generalised King plot
- ◆ Future directions

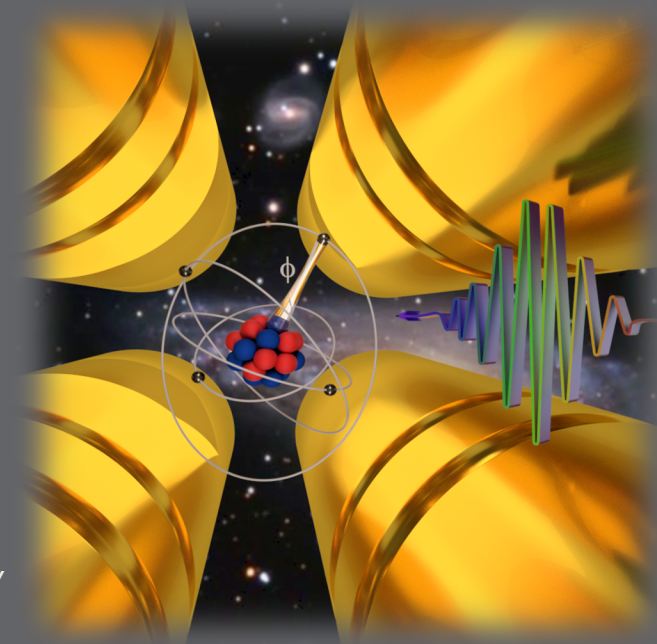


Image: Aarhus University

Isotope shift theory

- ◆ In the standard theory we express the difference between frequencies of isotope with mass number A and A' by

$$\delta\nu_i^{A',A} = \nu_i^{A'} - \nu_i^A = K_i \left(\frac{1}{A'} - \frac{1}{A} \right) + F_i \delta\langle r^2 \rangle^{A',A}$$

“Mass shift”

nuclear recoil

“Field shift”

charge distribution

- ◆ We can factorize the nuclear part and electron parts of each term (consequence of perturbation theory).
- ◆ $\delta\langle r^2 \rangle$ – change in nuclear charge radius – is hard to measure; we can extract it from isotope shift if the electronic factors (F_i, K_i) can be calculated accurately.

King plot

- ◆ Remove dependence on nuclear charge radius by cancelling it between two transitions.
- ◆ Introduce "modified" isotope shifts:

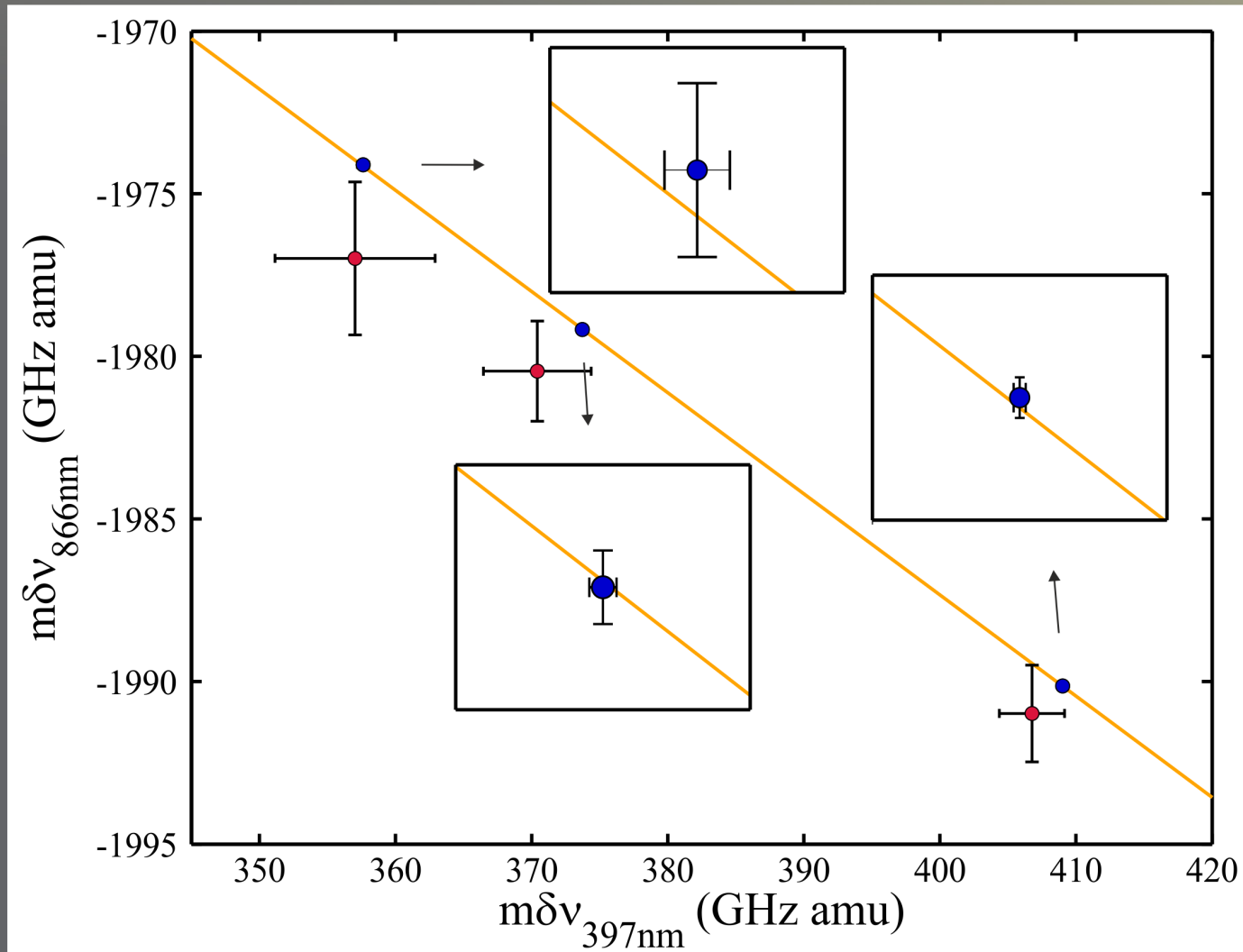
$$\delta\nu_i^{A',A} = K_i \left(\frac{1}{A'} - \frac{1}{A} \right) + F_i \delta\langle r^2 \rangle^{A',A}$$

➔
$$\delta\bar{\nu}_i^{A',A} = K_i + F_i \overline{\delta\langle r^2 \rangle}^{A',A}$$

- ◆ Plot $\delta\bar{\nu}_1$ vs $\delta\bar{\nu}_2$ for several pairs of isotopes (A', A):

$$\delta\bar{\nu}_1 = \frac{F_1}{F_2} \delta\bar{\nu}_2 + \left(K_1 - \frac{F_1}{F_2} K_2 \right)$$

King plot: Ca^+



King plot nonlinearities

- ◆ What if the King plot is not linear?
- ◆ May point to new physics in complex atoms that are beyond theoretical description at 7+ digits (e.g. Ca^+ , Sr/Sr^+ , Yb^+)
- ◆ Also many Standard Model “spurions”: sources of nonlinearities of unknown size in complex atoms. (We’ll come back to these.)

New physics searches with isotope shift

- ◆ Start with reformulation of King plot

$$\delta\overline{\nu}_i^{42,40} = K_i + F_i \delta\overline{\langle r^2 \rangle}^{42,40}$$

$$\delta\overline{\nu}_i^{44,40} = K_i + F_i \delta\overline{\langle r^2 \rangle}^{44,40}$$

$$\delta\overline{\nu}_i^{48,40} = K_i + F_i \delta\overline{\langle r^2 \rangle}^{48,40}$$

- ◆ There are N pairs of isotopes (A', A), so we can make a vector in N-dimensional "isotope pair space"

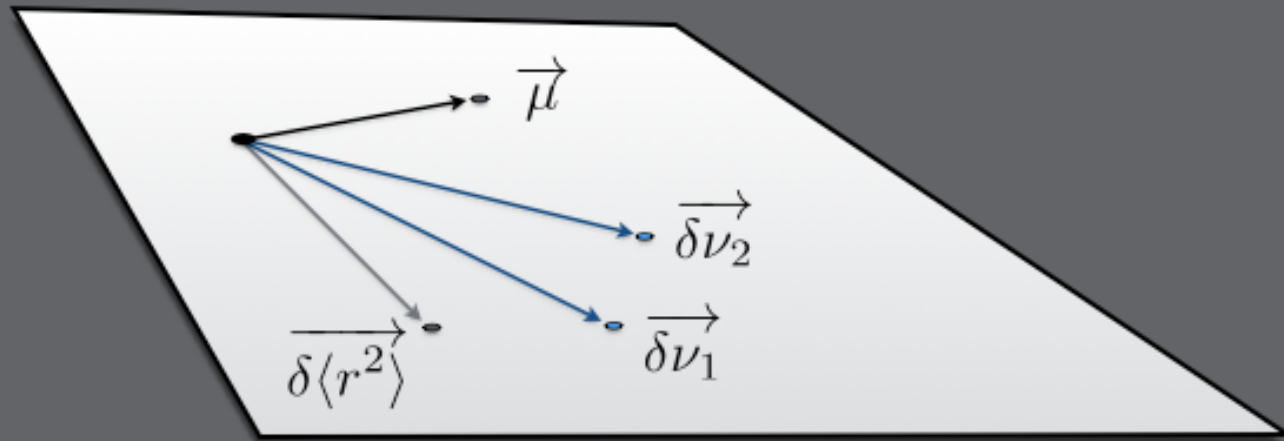
$$\overrightarrow{\delta\nu}_i = K_i \overrightarrow{\mu} + F_i \overrightarrow{\delta\langle r^2 \rangle}$$

New physics searches with isotope shift

- ◆ Start with reformulation of King plot

$$\vec{\delta\nu}_i = K_i \vec{\mu} + F_i \overrightarrow{\delta\langle r^2 \rangle}$$

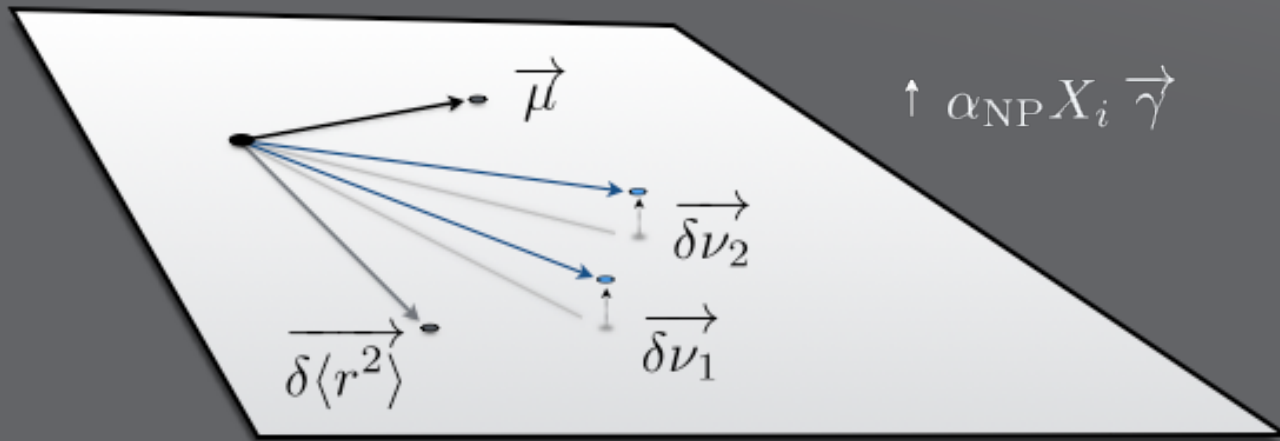
- ◆ Modified isotope shifts of all transitions $\vec{\delta\nu}_i$ lie in the same 2D plane in the N-dimensional isotope-pair space.



New physics searches with isotope shift

- ◆ Reduced isotope shift again:

$$\vec{\delta\nu}_i = K_i \vec{\mu} + F_i \overrightarrow{\delta\langle r^2 \rangle} + \alpha_{\text{NP}} X_i \vec{\gamma}$$



- ◆ No sensitivity to new physics if $\vec{\gamma} \sim \vec{\mu}$ or $X_i \sim F_i$

New physics searches with isotope shift

- ◆ Example: new force-carrying low-mass boson couples electrons to neutrons with Yukawa potential

$$V_\phi(r) = -\frac{y_e y_n}{4\pi} (A - Z) \frac{e^{-m_\phi r}}{r}$$

- ◆ Isotope shift has an extra term

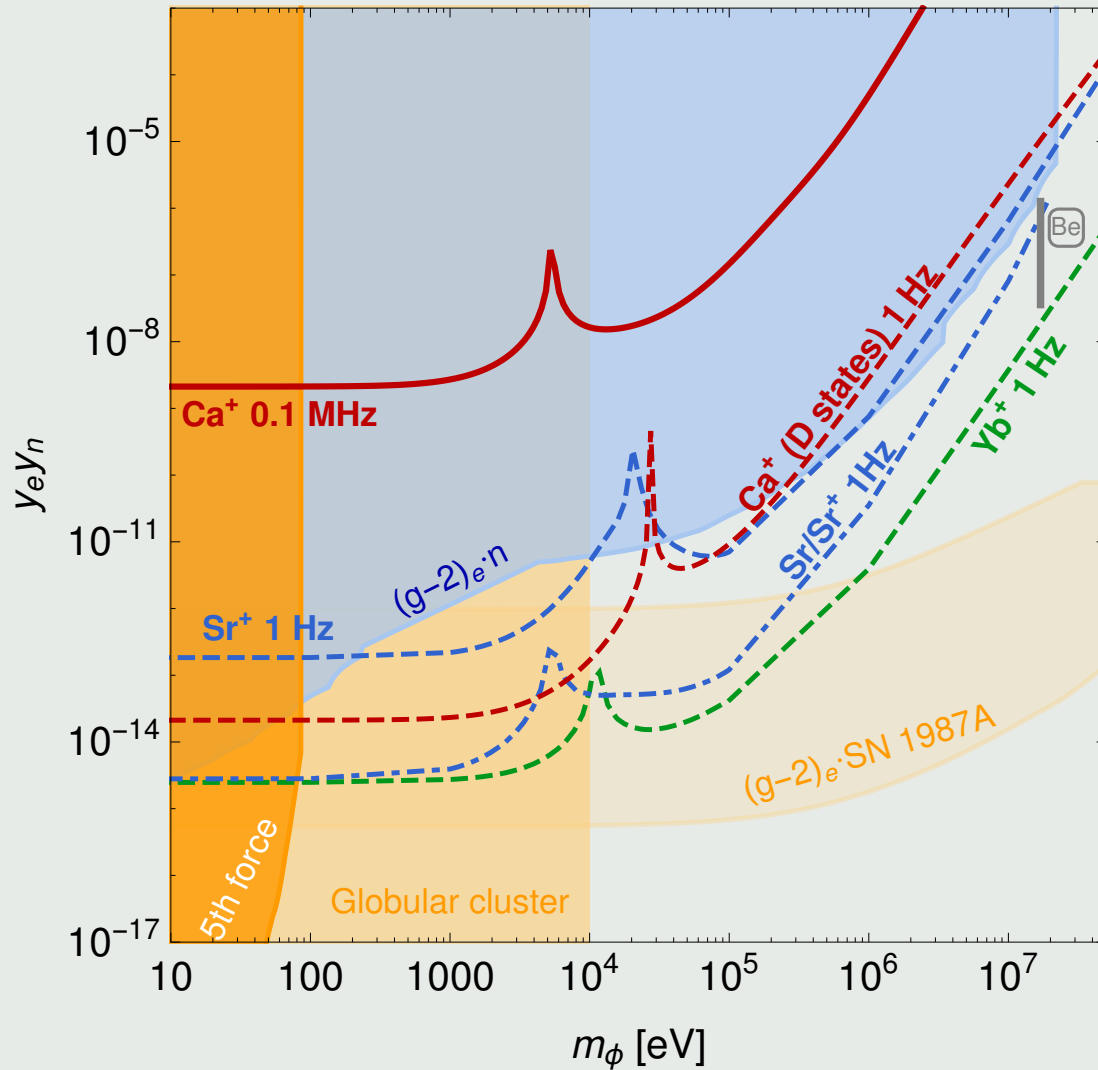
$$\delta\nu_i^{A',A} = K_i \left(\frac{1}{A'} - \frac{1}{A} \right) + F_i \delta\langle r^2 \rangle^{A',A} + \alpha_{\text{NP}} X_i \gamma^{A',A}$$

- ◆ For scalar boson case

$$\gamma^{A',A} = A' - A$$

$$X_i \approx \int \frac{e^{-m_\phi r}}{r} (|\psi_b|^2 - |\psi_a|^2) d^3r$$

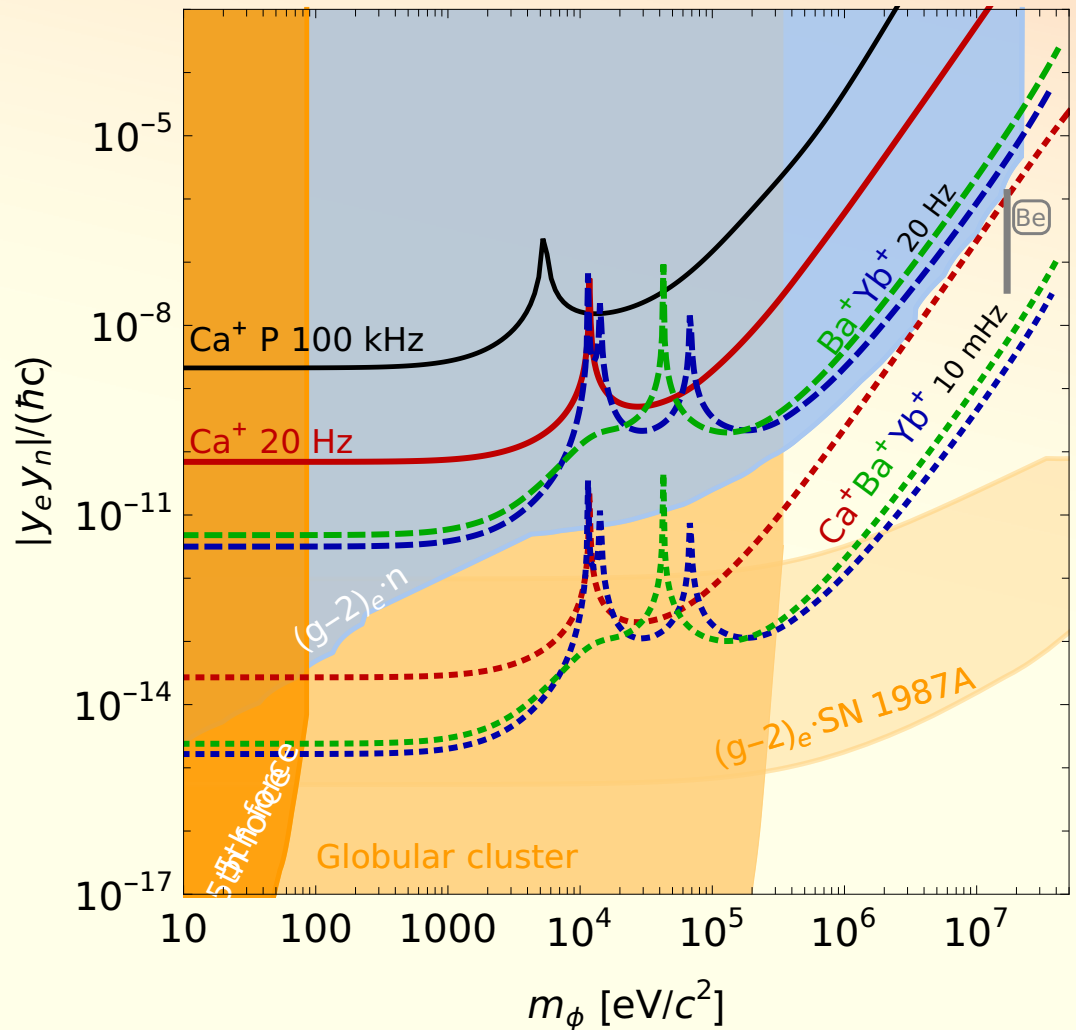
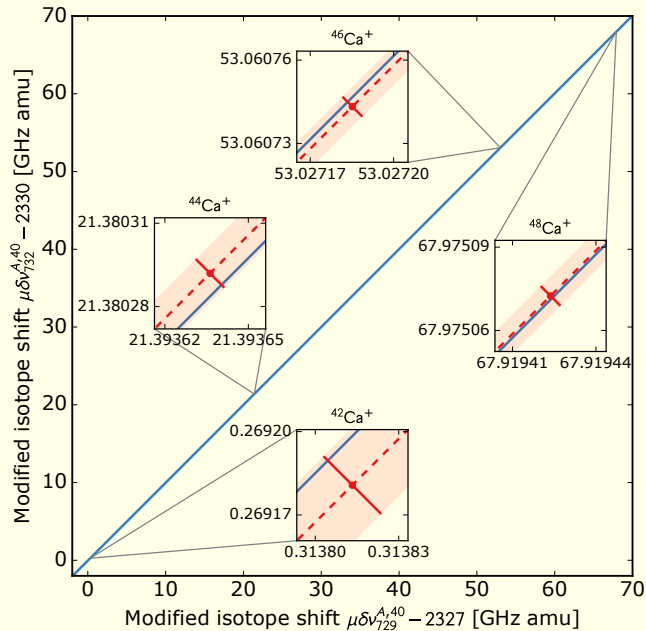
Predicted exclusion plot



Experiment

Ca⁺ exclusion plot

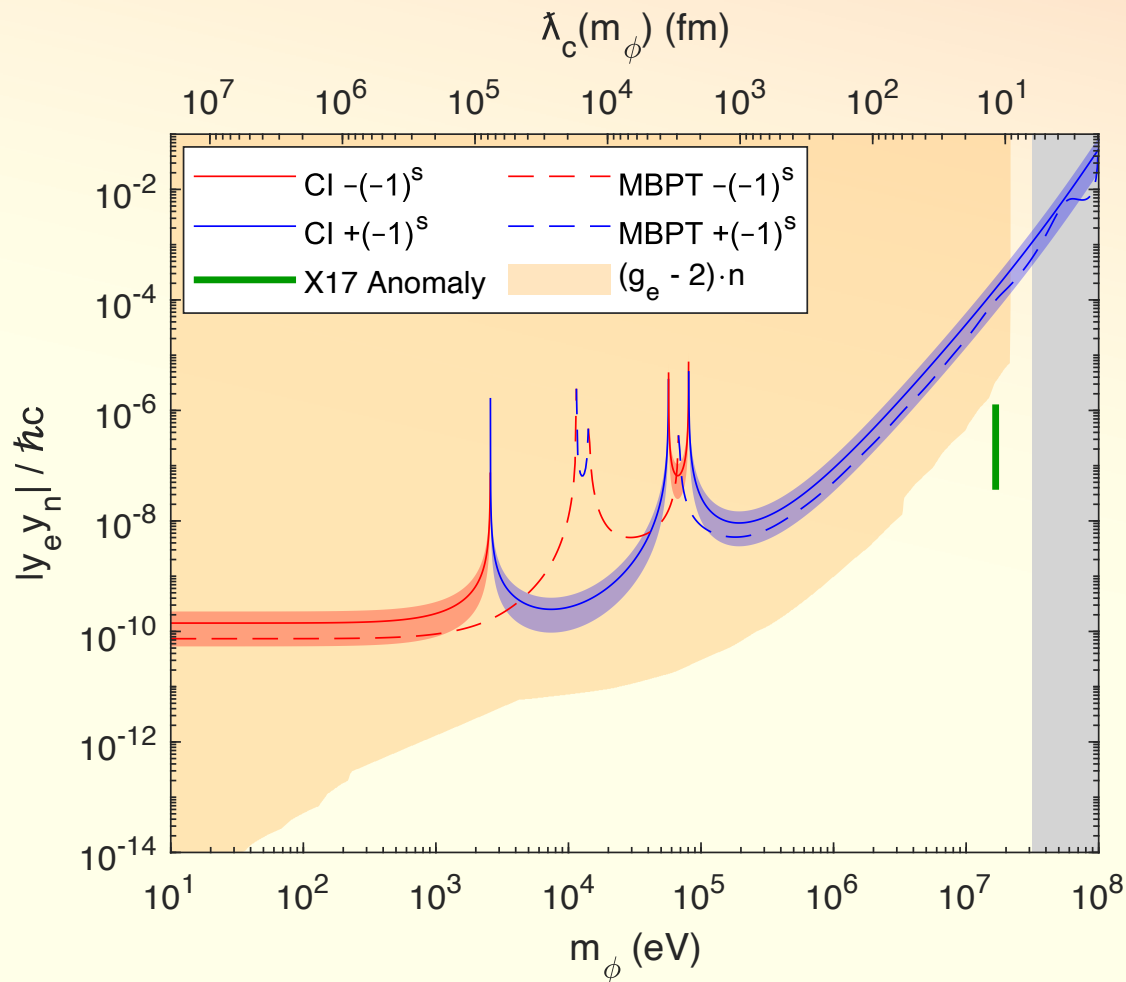
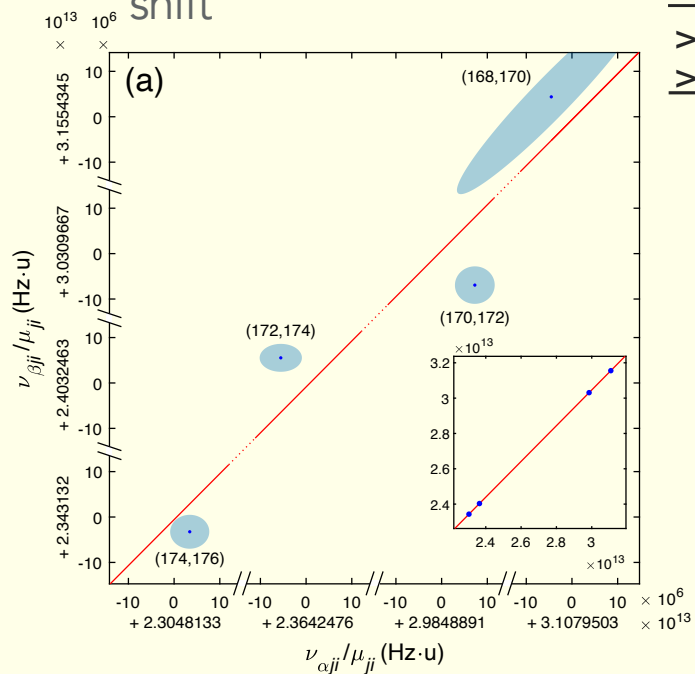
- ◆ 4s – 3d_{3/2} (732 nm) data at 1 kHz accuracy (Knollman et al. 2019)
- ◆ d_{3/2} – d_{5/2} fine-structure data at 20 Hz accuracy (Solaro et al. 2020)



Experiment

Yb⁺ exclusion plot

- ◆ 6s – 5d_{3/2} and 6s – 5d_{5/2} at 300 Hz accuracy.
- ◆ 3σ King-plot nonlinearity
- ◆ Non-trivial calculations
- ◆ Large second-order field shift



Counts, Hur, Aude Craik, Jeon, Leung, Berengut, Geddes, Kawasaki, Jhe, Vuletić, PRL 125, 123002 (2020)

Spurions

- ◆ Additional Standard Model effects include
 - ◆ 2nd order Field Shift
 - ◆ 2nd order Mass Shift
 - ◆ Nuclear polarizability
 - ◆ Infinitely many others
- ◆ Plus there is the possibility of being limited by uncertainties in nuclear mass measurements.

Spurions General Formalism

- ◆ Write the isotope shift including general spurions

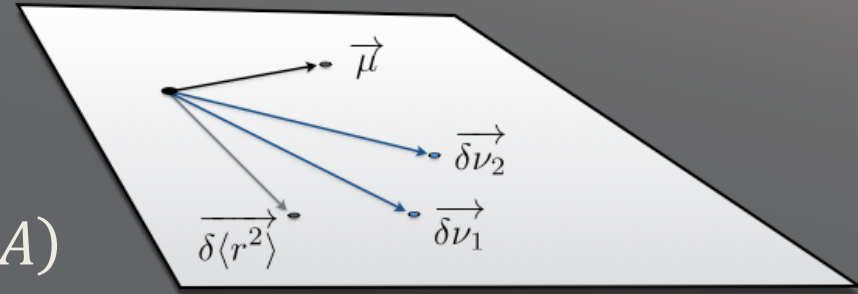
$$\delta\nu_i^{A',A} = K_i \mu^{A',A} + \sum_{l=1}^S F_{il} \lambda_l^{A',A}$$

- ◆ There are S spurions including field shift
- ◆ Each term is factorized into electronic and nuclear parts (this expansion can always be made, possibly at the expense of additional terms)
- ◆ Different spurions enter at different levels of experimental accuracy

Spurions General Formalism

$$\delta\nu_i^a = K_i \mu^a + \sum_{l=1}^S F_{il} \lambda_l^a$$

- ◆ $l = 1 \dots S$ spurions
- ◆ $i = 1 \dots m$ atomic transitions.
- ◆ $a = 1 \dots N$ pairs of isotopes (A', A)

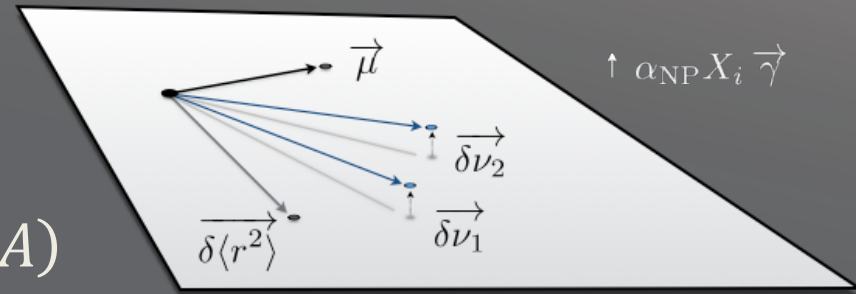


- ◆ In the absence of new physics all m isotope shift vectors (dimension N) and μ lie in a hyperplane of dimension $S + 1$ with basis vectors $\vec{\mu}$ and $\vec{\lambda}_l$.

Spurions General Formalism

$$\delta\nu_i^a = K_i \mu^a + \sum_{l=1}^S F_{il} \lambda_l^a + \alpha_{NP} X_i \gamma^a$$

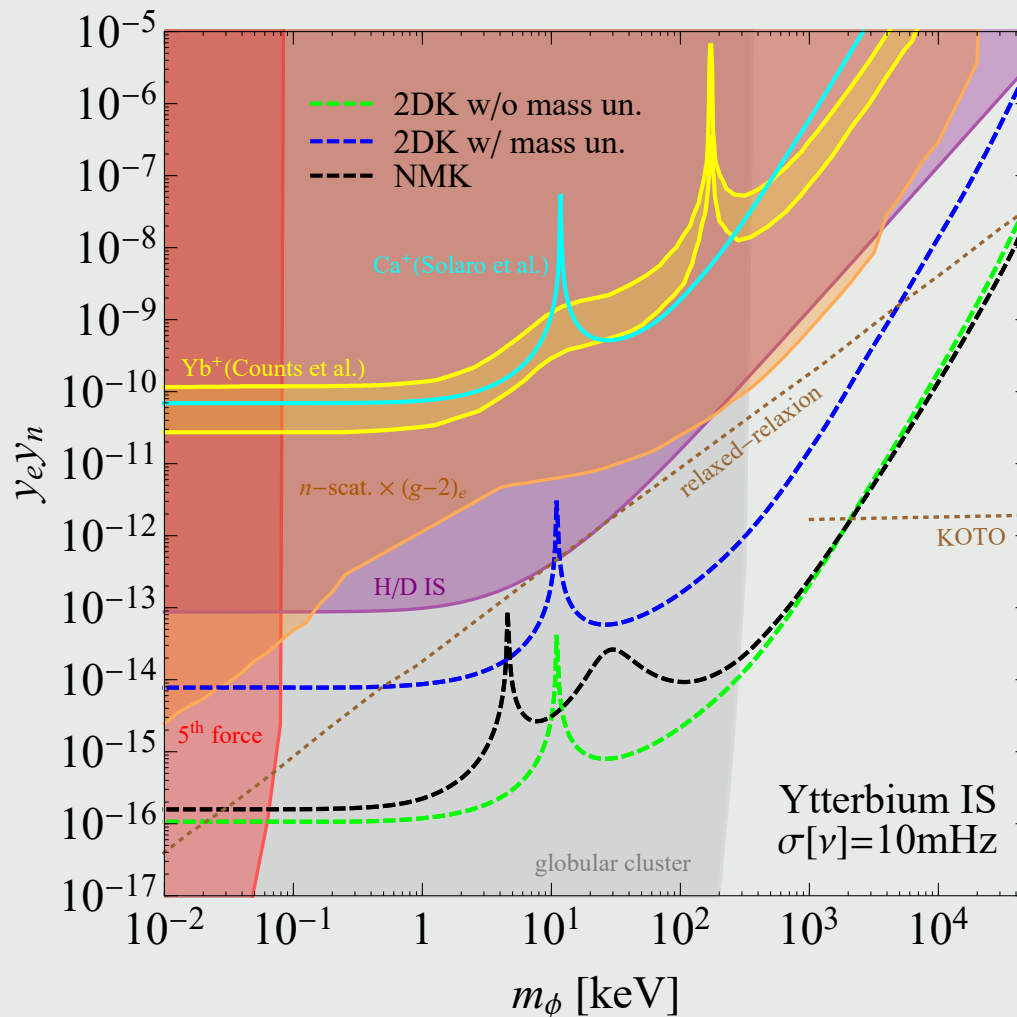
- ◆ $l = 1 \dots S$ spurions
- ◆ $i = 1 \dots m$ atomic transitions.
- ◆ $a = 1 \dots N$ pairs of isotopes (A', A)



- ◆ In the absence of new physics all m isotope shift vectors (dimension N) and μ lie in a hyperplane of dimension $S + 1$ with basis vectors $\vec{\mu}$ and $\vec{\lambda}_l$.
- ◆ To see new physics requires $N > m \geq S + 1$

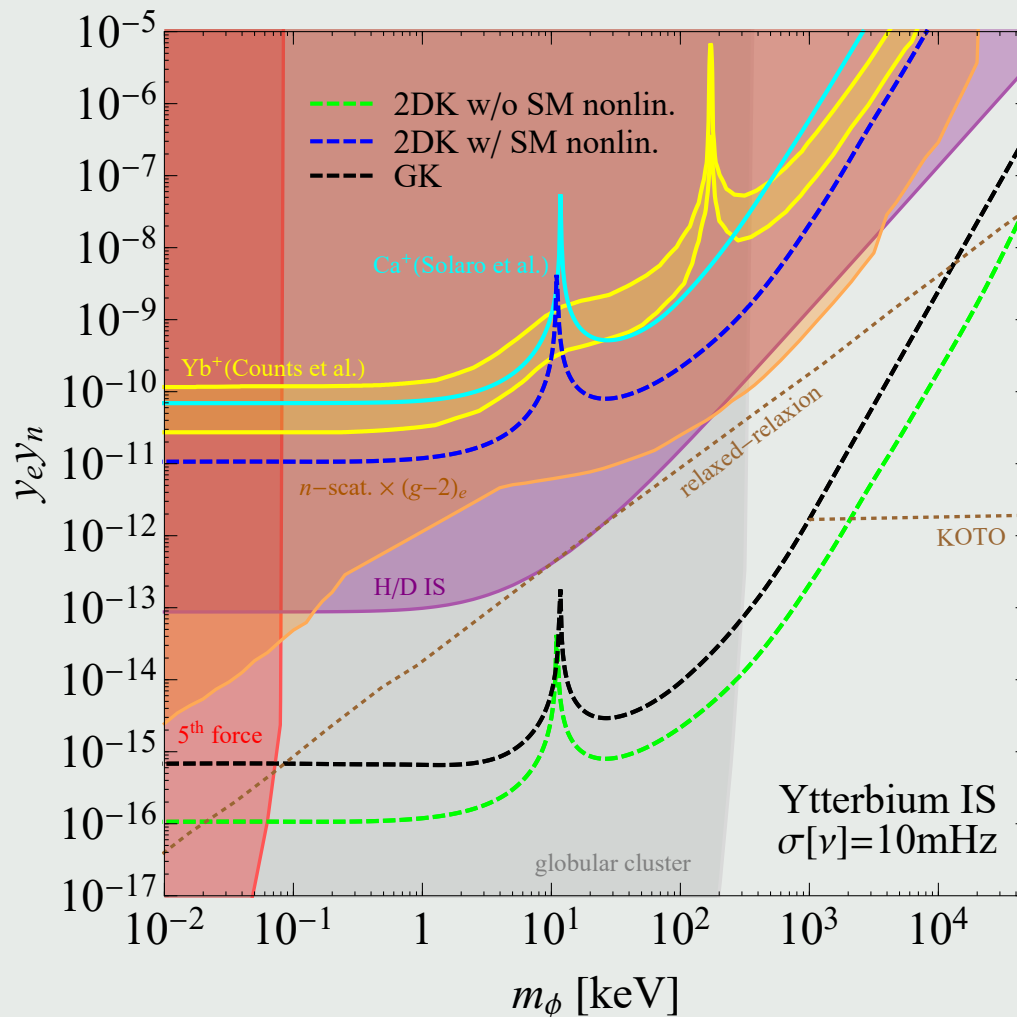
Nuclear mass measurements

- ◆ Nuclear masses must be measured better than isotope shifts by a factor of order $\frac{m_A}{(m_{A'} - m_A)}$, otherwise this uncertainty makes the mass shift just another spurion.
- ◆ One could use an additional transition rather than the mass difference measurement: the no-mass King (NMK) method
- ◆ In this case to see new physics requires $N \geq m \geq S + 2$



Projected: No-mass King plot

Yb⁺ with assumed 10 mHz measurement uncertainty based on E2 and E3 transitions, plus additional Yb S – P, assuming no additional SM nonlinearity



Projected: Generalised King plot

Yb⁺ with assumed 10 mHz measurement uncertainty based on E2 and E3 transitions, plus additional Yb S – P to remove second-order field shift effect

Future prospects

	calcium	ytterbium
Stable even isotopes	40, 42, 44, 46, 48	168, 170, 172, 174, 176
N (number of pairs)	4	4
Clock transitions	Ca ⁺ 4s – 3d _{3/2} Ca ⁺ 4s – 3d _{5/2} Ca 4s ² – 4s4p ³ P ₀ highly charged ions	Yb ⁺ 6s – 5d _{3/2} Yb ⁺ 6s – 5d _{5/2} Yb ⁺ 6s – 6s ² 4f _{7/2} ⁻¹ Yb 6s ² – 6s6p ³ P ₀
m (number of transitions)	3+	4+
Spurions		
2nd order field shift	10 ⁻² – 10 ⁻³ Hz	10 ³ – 10 ⁴ Hz
2nd order mass shift	3 Hz	28 Hz
nuclear polarizability	10 ⁻² Hz	38 Hz
S estimated number of spurions at 10 mHz	2+	4+

To see new physics requires $N > m > S$

Future prospects

	calcium	ytterbium
Stable even isotopes	40, 42, 44, 46, 48	168, 170, 172, 174, 176
N (number of pairs)	4	4
Clock transitions	$\text{Ca}^+ 4s - 3d$	$\text{Yb}^+ 6s - 5d$
Need more isotopes!		
m (number of transitions)	3+	4+
Spurions		
2nd order field shift	$10^{-2} - 10^{-3}$ Hz	$10^3 - 10^4$ Hz
2nd order mass shift	3 Hz	28 Hz
nuclear polarizability	10^{-2} Hz	38 Hz
S estimated number of spurions at 10 mHz	2+	4+

To see new physics requires $N > m > S$

Summary

- ◆ Can probe new physics with precision isotope shift spectroscopy of two different atomic transitions in the same element
- ◆ Unknown Standard Model effects (spurions, S) can be controlled using additional transitions
- ◆ Need more pairs of even isotopes (N) than transitions (m):

$$N > m > S$$

- ◆ Thanks to many collaborators and to you for listening