Top Quark Physics
A Theoretical Overview

Rikkert Frederix
University of Zurich
85% of the total cross section

10 $tt$ pairs per day

60% of the time there is extra radiation so that $p_T(tt)>15$ GeV.

$tt$ are produced closed to threshold, in a $^3S_1$ state. Same spin directions. 100% correlated in the off-diagonal basis.

Worry because of the backgrounds: ($W$+jets, $WQ$+jets,$WW$+jets)

90% of the total cross section

Almost 70% of the time there is extra radiation so that $p_T(tt)>30$ GeV.

$tt$ can be easily produced away from threshold. On threshold they are $^1S_0$ state, with opposite spin directions. No 100% correlation.
**Single top**

- Named after the virtuality of the $W$ boson
- $t$-channel largest at Tevatron and LHC
- $Wt$ negligible at Tevatron, second largest at LHC
- Cross section is proportional to $|V_{tb}|^2$
Recent progress in top:

- Updates of total top pair cross section (NLO QCD + threshold resummation (NLL)) *Moch, Uwer; Cacciari et al; Kidonakis, Vogt*
- NNLL extensions at threshold: two slightly different definitions of threshold *Czakon et al.; Beneke et al.; Abrens et al.*
- Forward-Backward asymmetry from threshold resummation *Almeida et al; Abrens et al.; Antunano et al.*
- Top pair invariant mass very close to production threshold *Hagiwara et al; Kiyo et al.*
- Partial results towards top pair total rate at NNLO QCD *Czakon; Bonciani et al. ...*
- Top pair + jets @ NLO: top as a background to Higgs searches: *W⁺W⁻ -> H, and ttH*
  - pp -> tt+jet *Dittmaier et al.; Melikov, Schulze*
  - pp -> tt bb *Breidenstein et al.; Bevilacqua et al.*
  - pp -> tt jj *Bevilacqua et al.*
- tt spin correlations revisited *Mahlon, Parke; Bernreuther, Si*
- PDF updates *MSTW collaboration, ...*
- New features in NLO MC generators *MC@NLO, POWHEG*
- Wt production at NLO QCD in MC@NLO *Frixione et al.; White et al.*
- tt(+)jet production including decay at NLO QCD *Melnikov, Schulze*; including weak interference corrections *Bernreuther, Si*
- Single top t-channel production at NLO QCD in 5 and 4 flavor schemes *Campbell, RF, Maltoni, Tramontano*
- Single top including decay at NLO QCD *Falgarì et al.*
- Many, many pheno studies, including
  - boosted tops *Almeida et al.; Kaplan et al.; ...*
  - comprehensive determination of anomalous couplings in single top production and decay *Aguilar-Saavedra et al., ...*
  - BSM contributions to Forward-Backward asymmetry *Many contributions...
  - effects of a 4th generation or of heavy exotic quarks *Holdom et al.; Alwall et al; Kribs et al.; ...*
  - resonance studies, pp -> X -> tt, BSM Higgs, colored resonances, KK states, spin-2 *Barger et al.; RF, Maltoni; Bernreuther ...
  - BSM CP violation *Holdom et al.; Hou et al.*
  - etc...

Rikkert Frederix, University of Zurich
Selected Topics

- Theory updates on the top mass from the top pair cross section *(based on work by Langenfeld, Moch & Uwer 2009)*

- t-channel single top in 4-flavor scheme *(based on work by Campbell, RF, Maltoni & Tramontano 2009)*

- Top pair charge asymmetry *(based on work by Kühn & Rodrigo 1998; Aherns et al. 2009; Almeida et al. 2010)*

- BSM: effective field theory approach *(based on work by Zhang & Willenbrock 2010)*
SM -- top quark running mass
Given the $O(1 \text{ GeV})$ uncertainty on the top quark mass, 
$m_t=173.3 \pm 0.6 \text{ (stat)} \pm 0.9 \text{ (sys)} \text{ GeV}$
the question arises which mass we are measuring

Only when measuring the total cross section, the extraction of the top quark mass is unambiguous

However, this gives a measurement which has a larger uncertainty, e.g. DØ ’09:
$m_t=169.1 \pm 5.6 \text{ GeV}$
**Top pair cross section**

- Usually the top pair cross section is defined as a function of the top pole mass.
- This is based upon the (non-physical) concept of the top quark being a free parton.
From a theory point of view, a better definition for the mass is the MSbar mass.

- Mass depends on the scale of the process.
- Conversion between MSbar (running) mass and pole mass definitions in perturbation theory:
  \[ m_t = m(\mu_R)\left(1 + \alpha_s(\mu_R)d^{(1)} + \alpha_s(\mu_R)^2d^{(2)} + \ldots\right) \]

- Scale dependence greatly reduced and perturbative series better behaved.
MASS MEASUREMENT FROM RUNNING MASS

- Top pair cross section as a function of the running MSbar mass
- Theory uncertainties from renormalization and factorization scale dependence (varying independently by a factor 2 around \(m(m)\))

![Graph showing top pair cross section as a function of running MSbar mass](image)

- Determination of the mass from total cross section is more precise than using the pole mass
- MSbar mass: \(m(m)=160 \pm 3.3 \text{ GeV}\)
- Conversion to pole mass: \(m_t=168.9 \pm 3.5 \text{ GeV}\)

[Compared to DØ result: \(m_t=169.1 \pm 5.6 \text{ GeV}\]

Langenfeld, Moch, Uwer 2009
SM -- t-channel
SINGLE TOP
Initial state b quark

“Standard” way of looking at the t-channel single top process

But there is an equivalent description with no bottom PDF and an explicit gluon splitting to b quark pairs

5-flavor scheme

4-flavor scheme
At all orders both description should agree; otherwise, differ by:

- evolution of logarithms in PDF: they are resummed
- available phase space
- approximation by large logarithm
Use the 4-flavor (2 → 3) process as the Born and calculate NLO

Much harder calculation due to extra mass and extra parton

Spectator b for the first time at NLO

Compare to 5F (2 → 2) to assess logarithms and applicability

Process implemented in the MCFM-v5.7 parton-level NLO code

Starting point for future NLO+PS beginning at (2 → 3)
**Total rates and theory uncertainties**

- Estimate of the theory uncertainty:
  - independent variation of renormalization and factorization scales by a factor 2
  - 44 eigenvector CTEQ6.6 PDF's
  - Top mass: $172 \pm 1.7$ GeV
  - Bottom mass: $4.5 \pm 0.2$ GeV

<table>
<thead>
<tr>
<th>$\sigma_{t\rightarrow\text{ch}}^{\text{NLO}}(t + \bar{t})$</th>
<th>$2 \rightarrow 2$ (pb)</th>
<th>$2 \rightarrow 3$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tevatron Run II</td>
<td>$1.96 \pm 0.05 \pm 0.20 \pm 0.06 \pm 0.05$</td>
<td>$1.87 \pm 0.16 \pm 0.18 \pm 0.06 \pm 0.04$</td>
</tr>
<tr>
<td>LHC (7 TeV)</td>
<td>$62.6 \pm 1.1 \pm 1.4 \pm 1.1 \pm 1.1$</td>
<td>$59.4 \pm 2.1 \pm 1.4 \pm 1.0 \pm 1.3$</td>
</tr>
<tr>
<td>LHC (14 TeV)</td>
<td>$244 \pm 5 \pm 5 \pm 3 \pm 4$</td>
<td>$234 \pm 7 \pm 5 \pm 3 \pm 4$</td>
</tr>
</tbody>
</table>

Fac. & Ren. scale | top mass | b mass

PDF

Rikkert Frederix, University of Zurich
Jet defined by: $p_T > 15$ GeV, $\Delta R > 0.7$

Some differences, but typically of the order of $\sim 10\%$ in the regions where the cross section is large

Shapes are very similar to LO predictions (not shown)
Dashes: $2 \rightarrow 2$ at “NLO”, with massive (when final state) $b$ quark: the same shape as the $2 \rightarrow 3$ at LO

Solid: $2 \rightarrow 3$ at NLO: first NLO predictions for these observables

More forward and softer in $2 \rightarrow 3$, particularly at the Tevatron

Mild deviations up to $\sim 20\%$
More bottoms in 4F

* However, there are large differences between 5F (2 → 2) and 4F (2 → 3) schemes for more exclusive quantities in the spectator b quark

* Event though b quarks in the 4F (2 → 3) scheme are more forward and softer, we expect to see more b’s than in the 5F (2 → 2)

* In 5F (2 → 2) only a subset of real emission diagrams have a final state b quark

* Define “acceptance” as the ratio of events that have a central, hard b over inclusive cross section:

\[ \frac{\sigma(|\eta(b)| < 2.5, p_T(b) > 20 \text{ GeV})}{\sigma_{\text{inclusive}}} \]
Acceptance

- In the Monte Carlo samples used by CDF (based on ZTOP), almost half as many b-jets (not from top decay) compared to best NLO predictions.
- What is the impact on the recent measurements for single top?
- DØ predictions are consistent with best theory prediction.

Best theory prediction: 30.5%

Value from ZTOP: 16.7%
**Impact on measurement**

- Naively:
  
  Because
  
  - s-channel has one more b-jet in the final state compared to the 5 flavor t-channel, and
  
  - in the 4 flavor more t-channel events have the same # of b-jets as s-channel,
  
  many t-channel events were assigned to the s-channel
Naively:

Because

- s-channel has one more b-jet in the final state compared to the 5 flavor t-channel, and
- in the 4 flavor more t-channel events have the same # of b-jets as s-channel,
- many t-channel events were assigned to the s-channel
In practice...
It’s slightly more complicated:

Dominating categories are compensating each other. Large differences for channels with only minor contributions

Jan Lueck et al. @ CDF
Impact on measurement

- So that the effects on the final results are negligible
- The 2 sigma deviation remains
Top pair charge asymmetry
TOP PAIR CHARGE ASYMMETRY

At leading order: top and anti-top have identical angular distributions

Real emission corrections: negative contribution

Virtual corrections: positive contribution

Real emission flavor excitations: negligibly small
FIG. 1. Origin of the QCD charge asymmetry in hadroproduction of heavy quarks: interference of final-state (a) with initial-state (b) gluon bremsstrahlung plus interference of the box (c) with the Born diagram (d). Only representative diagrams are shown.

FIG. 2. Origin of the QCD charge asymmetry in hadroproduction of heavy quarks through flavor excitation.

Let us briefly discuss a few important aspects of this calculation. The box amplitude for $q\bar{q}\rightarrow Q\bar{Q}$ is ultraviolet finite and the asymmetric contribution to the cross section of order $\alpha_s^3$ is therefore not affected by renormalization, an obvious consequence of the symmetry of the lowest order reaction. The same line of reasoning explains the absence of initial state collinear singularities in the limit $m_q\rightarrow 0$ which would have to be absorbed into the (symmetric) lowest order cross section. Infrared singularities require a more careful treatment. They are absent in the asymmetric piece of the process in eq. (3). However, real and virtual radiation (Fig. 1), if considered separately, exhibit infrared divergences, which compensate in the sum, corresponding to the inclusive production cross section.

The charge asymmetry in the partonic reactions (1) and (3) implies for example a forward-backward asymmetry of heavy flavor production in proton-antiproton collisions. In particular, it leads to a sizeable forward-backward asymmetry for top production which is dominated by reaction (1), and can, furthermore, be scrutinized by studying $t\bar{t}$ production at fixed longitudinal momenta and at various partonic energies $\hat{s}$. However, the charge asymmetry can also be observed in proton-proton collisions at high energies. In this case one has to reconstruct the $t\bar{t}$ restframe and select kinematic regions, which are dominated by $q\bar{q}$ annihilation or flavor excitation $gq\rightarrow t\bar{t}X$. Alternatively, one may also study the difference in the one-particle inclusive rapidity distribution of top versus antitop, which again integrates to zero.

The analysis of these effects allows to improve our understanding of the QCD production mechanism. At the same time it is important for the analysis of single top production through $Wb$ fusion. This reaction is charge asymmetric.

At leading order: top and anti-top have identical angular distributions

Real emission corrections: negative contribution

Virtual corrections: positive contribution

Corrections from the virtuals are larger than the real emission corrections:
Top quarks are preferentially emitted in the direction of the incoming quark
Quantitative description

Due to CP invariance, charge asymmetry is the same as forward-backward asymmetry. The precise definition is frame dependent

\[ A_{fb}(lab) = \frac{\int_{y>0} N_t(y) - \int_{y>0} N_{\bar{t}}(y)}{\int_{y>0} N_t(y) + \int_{y>0} N_{\bar{t}}(y)} \]

\[ A_{fb}(ttbar) = \frac{\int N(\Delta y > 0) - \int N(\Delta y < 0)}{\int N(\Delta y > 0) + \int N(\Delta y < 0)} , \quad \Delta y = y_t - y_{\bar{t}} \]

Theory (NLO+EW, Kühn, Rodrigo):
\[ A_{fb}(lab) = 0.051 \pm 0.006 \]
\[ A_{fb}(ttbar) = 0.078 \pm 0.009 \]

Results are very stable when including threshold logarithms
\[ A_{fb}(ttbar) = 0.073 + 0.011 - 0.007 \] (NLO+NNLL, Ahrens et al. 2010)
Results

- **CDF (5.3 fb-1):**
  \[ A_{fb}(\text{lab}) = 0.073 \pm 0.028 \text{ (uncorrected)} \]
  \[ A_{fb}(tt\bar{b}) = 0.057 \pm 0.028 \text{ (uncorrected)} \]

- **Corrected (bkg, and parton level):**
  \[ A_{fb}(\text{lab}) = 0.150 \pm 0.050 \text{ stat} \pm 0.024 \text{ syst} \]
  \[ A_{fb}(tt\bar{b}) = 0.158 \pm 0.072 \text{ stat} \pm 0.017 \text{ syst} \]

- **DØ (4.3 fb-1):**
  \[ A_{fb}(tt\bar{b}) = 0.08 \pm 0.04 \text{ stat} \pm 0.01 \text{ syst (uncorrected)} \]

- **Theory:**
  \[ A_{fb}(\text{lab}) = 0.051 \pm 0.006 \text{ (NLO+EW, Kühn, Rodrigo)} \]
  \[ A_{fb}(tt\bar{b}) = 0.073 + 0.011 - 0.007 \text{ (NLO+NNLL threshold resum, Ahrens et al.)} \]

**Difference between theory and experiment is sizable, but below 2 sigma**
Many BSM models studied

Djouadi, Moreau, Richard, Singh, Jung, Murayama, Pierce, Wells, Cheung, Keung, Yuan, Frampton, Shu, Wang, Tait, Arhrib, Benbrik, Chen, Ferrario, Rodrigo, Dorsner, Fajfer, Kamenik, Košnik, Ko, Lee, Nam, Cao, Heng, Wu, Barger, Yu, Antunano, Kuhn, McKeen, Rosner, Shaughnessy, Wagner, ... and many more ...

Not trivial to find a model: invariant mass agrees well with SM predictions

Need for full NNLO, i.e. first complete corrections to charge asymmetry
BSM -- effective theory approach
BSM in top physics

New Physics

Energy

SM

Λ_{NP}
BSM in top physics

New Physics

Energy

SM

\[ \Lambda_{NP} \]
**BSM in Top Physics**

New Physics

Energy

SM

**Effective 4-fermion interaction**

$\Lambda_{NP}$
**Dimension-6 Operators for Top**

A systematic description of all dimension-6 operators relevant for top quark physics. There are only 15 relevant operators.

### CP-even

<table>
<thead>
<tr>
<th>Operator</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O^{(3)}<em>{\phi q} = i(\phi^+ \tau^I D</em>\mu \phi)(\bar{q}\gamma^\mu \tau^I q)$</td>
<td>top decay, single top</td>
</tr>
<tr>
<td>$O_{tW} = (\bar{q}\sigma^{\mu\nu} \gamma^\nu t)\phi W^{I\mu}$ (with real coefficient)</td>
<td>top decay, single top</td>
</tr>
<tr>
<td>$O^{(1,3)}<em>{qq} = (\bar{q}i\gamma</em>\mu \tau^I q^j)(\bar{q}^j\gamma^\mu \tau^I q)$</td>
<td>single top</td>
</tr>
<tr>
<td>$O_{tG} = (\bar{q}\sigma^{\mu\nu} \lambda A t)\phi G_{\mu\nu}^A$ (with real coefficient)</td>
<td>single top, $q\bar{q}, gg \to t\bar{t}$</td>
</tr>
<tr>
<td>$O_G = f_{ABC} G_{\mu\nu}^A G_{\nu}^{B\rho} G_{\rho}^{C\mu}$</td>
<td>$gg \to t\bar{t}$</td>
</tr>
<tr>
<td>$O_{\phi G} = \frac{1}{2}(\phi^+ \phi) G_{\mu\nu}^A G^{A\mu\nu}$</td>
<td>$gg \to t\bar{t}$</td>
</tr>
<tr>
<td>7 four-quark operators</td>
<td>$q\bar{q} \to t\bar{t}$</td>
</tr>
</tbody>
</table>

### CP-odd

<table>
<thead>
<tr>
<th>Operator</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_{tW} = (\bar{q}\sigma^{\mu\nu} \gamma^\nu t)\phi W^{I\mu}$ (with imaginary coefficient)</td>
<td>top decay, single top</td>
</tr>
<tr>
<td>$O_{tG} = (\bar{q}\sigma^{\mu\nu} \lambda A t)\phi G_{\mu\nu}^A$ (with imaginary coefficient)</td>
<td>single top, $q\bar{q}, gg \to t\bar{t}$</td>
</tr>
<tr>
<td>$O_G = g_s f_{ABC} G_{\mu\nu}^A G_{\nu}^{B\rho} G_{\rho}^{C\mu}$</td>
<td>$gg \to t\bar{t}$</td>
</tr>
<tr>
<td>$O_{\phi G} = \frac{1}{2}(\phi^+ \phi) G_{\mu\nu}^A G^{A\mu\nu}$</td>
<td>$gg \to t\bar{t}$</td>
</tr>
</tbody>
</table>
A systematic description of all dimension-6 operators relevant for top quark physics. There are only 15 relevant operators.

<table>
<thead>
<tr>
<th>Operator</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O^{(3)}<em>{\phi q} = i(\phi^+ \tau^1 D</em>{\mu} \phi)(\bar{q}\gamma^\mu \tau^q q)$</td>
<td>top decay, single top</td>
</tr>
<tr>
<td>$O_{tW} = (\bar{q}\sigma^{\mu\nu}\tau^I t)\tilde{\phi}W^I_{\mu\nu}$ (with real coefficient)</td>
<td>top decay, single top</td>
</tr>
<tr>
<td>$O_{qq}^{(3)} = (q^I_{\mu}</td>
<td>I_q</td>
</tr>
<tr>
<td>$O_{tG} = (\bar{q}\sigma^{\mu\nu}A^\mu t)\tilde{\phi}G^A_{\mu\nu}$ (with real coefficient)</td>
<td>single top, $q\bar{q}, gg \rightarrow t\bar{t}$</td>
</tr>
<tr>
<td>$O_G = f_{ABC}G^A_{\mu\nu}G^B_{\rho\nu}G^C_{\rho\mu}$</td>
<td>$gg \rightarrow t\bar{t}$</td>
</tr>
<tr>
<td>$O_{\phi G} = \frac{1}{2}(\phi^+ \phi)G^A_{\mu\nu}G^A_{\mu\nu}$</td>
<td>$gg \rightarrow t\bar{t}$</td>
</tr>
<tr>
<td>7 four-quark operators</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operator</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_{tW} = (\bar{q}\sigma^{\mu\nu}\tau^I t)\phi W^I_{\mu\nu}$ (with imaginary coefficient)</td>
<td>top decay, single top</td>
</tr>
<tr>
<td>$O_{tG} = (\bar{q}\sigma^{\mu\nu}A^\mu t)\phi G^A_{\mu\nu}$ (with imaginary coefficient)</td>
<td>single top, $q\bar{q}, gg \rightarrow t\bar{t}$</td>
</tr>
<tr>
<td>$O_{\tilde{G}} = g_s f_{ABC}G^A_{\mu\nu}G^B_{\rho\nu}G^C_{\rho\mu}$</td>
<td>$gg \rightarrow t\bar{t}$</td>
</tr>
<tr>
<td>$O_{\phi \tilde{G}} = \frac{1}{2}(\phi^+ \phi)G^A_{\mu\nu}G^A_{\mu\nu}$</td>
<td>$gg \rightarrow t\bar{t}$</td>
</tr>
</tbody>
</table>

**CP-odd**

**CP-even**

---

Rikkert Frederix, University of Zurich

C. Zhang & S. Willenbrock
\[ O_{tW} = (\bar{q}\sigma^{\mu\nu}t^I t)\phi W^I_{\mu\nu} \quad \Rightarrow \quad L_{\text{eff}} = -2\frac{c_{tW}}{\Lambda^2} v\bar{b}\sigma^{\mu\nu} P_R t \partial_\nu W^-_{\mu} + h.c. \]

\[ t \quad b \]

\[ W^+ \]

\[ q, \nu_l \]

\[ \bar{q}, l^+ \]

\[ \blacklozenge \text{W-boson helicity fractions, } m_b = 0 \]

\[ F_0 = \frac{m_t^2}{m_t^2 + 2m_W^2} = 0.7 \]

\[ F_L = \frac{2m_W^2}{m_t^2 + 2m_W^2} = 0.3 \]

\[ F_R = 0 \]
$O_{tW} = (\bar{q} \sigma^{\mu\nu} \tau^I t) \phi W^I_{\mu\nu}$

$L_{\text{eff}} = -2 \frac{C_{tW}}{\Lambda^2} v_b \sigma^{\mu\nu} P_R t \partial_\nu W^-_{\mu} + \text{h.c.}$

\[ F_0 = \frac{m_t^2}{m_t^2 + 2m_W^2} - \frac{4\sqrt{2} \text{Re} C_{tW} v^2}{\Lambda^2 V_{tb}} \frac{m_t m_W (m_t^2 - m_W^2)}{(m_t^2 + 2m_W^2)^2} \]
\[ F_L = \frac{2m_W^2}{m_t^2 + 2m_W^2} + \frac{4\sqrt{2} \text{Re} C_{tW} v^2}{\Lambda^2 V_{tb}} \frac{m_t m_W (m_t^2 - m_W^2)}{(m_t^2 + 2m_W^2)^2} \]
\[ F_R = 0 \]

**W-boson helicity fractions, $m_b = 0$**
By measuring the W-boson helicity fractions in top decay, direct bounds can be set on the dimension-6 operator.

By a systematic analysis, bounds can be set on all dimension-6 operators relevant for top quark physics, using:

- **top decay:**
  - $O_{tW}$ and $O_{\phi q}^{(3)}$

- **single top s and t channel:**
  - $O_{tW}$, $O_{\phi q}^{(3)}$ and $O_{qq}^{(1,3)}$

- **Wt associated production:**
  - $O_{tW}$, $O_{\phi q}^{(3)}$ and $O_{tG}$

- **top pair production**
  - $O_{tG}$, $O_{\phi G}$ and $O_{G}$

- **gg channel:**
  - $O_{tG}$ and four-quark operators
Conclusions

- Many theory activities in top quark physics
  - I only had time to flash over a couple...
- Using the running mass in top pair production helps to reduce theory uncertainties
- 4-flavor (2 to 3) calculation for t-channel single top describes the spectator b quark for the first time at NLO
- Top quark charge asymmetry: 2 sigma deviation. No clear picture yet
- Effective field theory approach to BSM physics allows for a systematic way of putting bounds on BSM physics